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Local observers design for a class of neural mass models

Mohammed H.A. Hamid, Romain Postoyan, Jamal Daafouz

Abstract—We present a model-based approach to estimate the mean membrane potentials (and their time-derivatives) of populations of neurons within cortical columns. We consider a general class of neural mass models for which we design local state observers. The synthesis relies on linear parameter-varying systems techniques and the observer gains are obtained by solving linear matrix inequalities. Simulations results are presented to illustrate the efficiency of the approach.

I. INTRODUCTION

Cortical mechanisms involved in epilepsy remain largely unknown nowadays. In this context, measurements of the cortical activity are essential to improve our understanding of the underlying processes. Among the different available methodologies, electroencephalograms (EEG) provide the best time resolution, and is therefore often favoured by the clinicians to investigate seizure generation and propagation. EEG recordings reflect membrane potential variations of pyramidal neurons grouped into populations within cortical columns. These cortical columns are also composed of excitatory interneurons and inhibitory interneurons (see Figure 1 for an illustration), which are also expected to contain important electrophysiological information but which are not measurable today. The purpose of this study is to estimate the mean membrane potential of all the populations of given cortical columns using (simulated) EEG signals and a model of the cortical columns dynamics.

Fig. 1: Functional relationship between neural populations for the model in [8].

Several works have investigated this problem in the recent years. Nonlinear Kalman filters are constructed in [6], [11], [13], [16] mainly for models of the type of [18], [19]. On the

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other hand, deterministic global observers with guaranteed convergence are designed in [3], [4] for a class of neural mass models which covers the models in [8], [14], [17] as particular cases. We focus in this paper on the same class of models as in [3], [4]. The objective is to go a step further towards the implementation of state observers for neural mass models. The models considered in [3], [4] generate outputs which are assumed to model EEG recordings. However, these output signals are not centered at the origin in general, contrary to real EEG signals, which a priori prevents the application of the observers of [3], [4] on real data. We therefore first revisit the models considered in [3], [4] so that they generate output signals with zero means, by using high-pass filters. It appears that the results in [3] are no longer applicable in this case, and that those in [4] can only be used for some specific model parameters as we show. To overcome those issues, we design local observers, *i.e.* observers which are ensured to converge when they are initialized sufficiently close to the initial condition of the system (see *e.g.*, [7], [10], [15], [20]). The design of local observers is expected to be more amenable compared to the synthesis of global observers. Indeed, in this case we are only interested in the behaviour of the estimation error around the origin where we can use first order approximations to analyse it. This leads to a linear time-varying systems which usually facilitates the design of the observer gains.

The originality of our work compared to existing local observer designs is that we apply tools from linear parameter varying (LPV) systems to construct the observer gains. We use the properties of the considered models to write the linearized estimation error as a LPV system, where the ‘parameter’ corresponds to a known bounded function of the state estimate which lies in a known hyper-rectangle. We then resort to standard analytical tools for LPV systems to design the local observer gains. We provide linear matrix inequalities (LMIs) to construct the observer gains at the vertices of the hyper-rectangle mentioned above. We then use the interpolation technique (see [1] for instance) to obtain the observer gain, which is therefore nonlinear in the state estimate. Contrary to [7], [10] where local observers are designed for general nonlinear systems, the proposed conditions can be verified a priori as they do not involve properties of the system trajectories which are unknown. Furthermore, we do not linearize the system around an equilibrium point to construct the observer, as it is done in [15], [20], only the state estimation error system is linearized at the origin, which is different. Simulation results are provided for a model of neural mass models inspired by the one in [8] and we characterize the set of model parameters for which the

required LMIs hold.

The paper is organized as follows. We present the considered class of neural mass models in Section II. The design of the local observers is addressed in Section III and simulation results are proposed in Section IV. Finally, Section V concludes the paper.

Notations. Let $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$, $\mathbb{R}_{> 0} := (0, \infty)$, $\mathbb{Z}_{> 0} := \{1, 2, \dots\}$. For $(x, y) \in \mathbb{R}^{n+m}$, the notation (x, y) stands for $[x^T, y^T]^T$. The identity matrix is denoted by $\mathbb{I}$. For a vector $x \in \mathbb{R}^n$, $|x|$ denotes the Euclidean norm of x . The notation $\mathcal{L}_\infty$ stands for the set of piecewise continuous functions $f : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}^n$, $n \in \mathbb{Z}_{> 0}$, such that $\|f\|_\infty := \sup_{\tau \geq 0} |f(\tau)| < r$, for some $r \in \mathbb{R}_{> 0}$.

II. NEURAL MASS MODELS

It is shown in [3] that the models in [8], [14], [17] can all be described by the following state-space model

$$\begin{aligned}\dot{\bar{x}} &= \bar{A}\bar{x} + \bar{\Phi}(\bar{x}, \theta) + \bar{\sigma}_1(u, \theta) + \bar{\sigma}_2(\bar{C}\bar{x}, \theta) \\ \bar{y} &= \bar{C}\bar{x},\end{aligned}\quad (1)$$

where $\bar{x} \in \mathbb{R}^{n_x}$ is the state which represents the mean membrane potential of different populations of neurons and, potentially, their time-derivatives, $u \in \mathbb{R}^{n_u}$ is a vector of inputs, which represents the external influences from the afferent neural populations and which is assumed to be known, $\theta \in \mathbb{R}^{n_\theta}$ is a known vector of parameters and $\bar{y} \in \mathbb{R}^{n_y}$ is the output. The matrix $\bar{A}$ is Hurwitz, $\bar{\Phi}$ is smooth, bounded and globally Lipschitz, $\bar{\sigma}_1$ and $\bar{\sigma}_2$ are smooth functions. An example of such a model is provided in Section IV.

In [3], [4], the output $\bar{y}$ of (1) is assumed to represent EEG signals. However, the output signals generated by (1) are not necessarily centered at the origin contrary to real EEG recordings, see for instance Figure 2. To overcome this issue, we add a high-pass filter to model (1). In particular, we consider the multi-dimensional filter below

$$\begin{aligned}\dot{x}_f &= -f_0 x_f + w_f \\ y_f &= -f_0 x_f + w_f,\end{aligned}\quad (2)$$

where $x_f := (x_{f_1}, \dots, x_{f_{n_y}})$ is the filter state, w_f is the vector of inputs to the filter and f_0 is the cutoff frequency. Since we want to remove the offset from the signals $\bar{y}$ generated by (1), f_0 will be taken sufficiently small. In view of (1) and (2), we obtain the augmented model below

$$\begin{aligned}\dot{\hat{x}} &= \bar{A}\bar{x} + \bar{\Phi}(\bar{x}, \theta) + \bar{\sigma}_1(u, \theta) + \bar{\sigma}_2(\bar{C}\bar{x}, \theta) \\ \dot{x}_f &= -f_0 x_f + \bar{C}\bar{x} \\ y_f &= -f_0 x_f + \bar{C}\bar{x},\end{aligned}\quad (3)$$

which we rewrite as

$$\begin{aligned}\dot{x} &= Ax + \Phi(x, \theta) + \sigma(u, \theta) \\ y_f &= Cx,\end{aligned}\quad (4)$$

where $x := (\bar{x}, x_f) \in \mathbb{R}^{n_x}$ is the augmented state, $y_f \in \mathbb{R}^{n_y}$ is the new output which represents the EEG signals and $A := \begin{bmatrix} \bar{A} & 0 \\ \bar{C} & -f_0 \mathbb{I} \end{bmatrix}$, $C := [\bar{C} \quad -f_0 \mathbb{I}]$, $\Phi(x, \theta) :=$

Fig. 2: Outputs of system (1) (solid blue line) and system (4) (dashed red line) for the neural mass model considered in Section IV with parameters values of Table I.

$(\bar{\Phi}(\bar{x}, \theta) + \bar{\sigma}_2(\bar{C}\bar{x}, \theta), 0)$ and $\sigma(u, \theta) := (\bar{\sigma}_1(u, \theta), 0)$. System (4) generates outputs whose mean values tend to zero as time grows, see for an example Figure 2. The first difference we note between (1) and (4) is that we no longer have an output injection term in the dynamics as $\bar{y}$ is no longer measured in (4). As a consequence, the results in [3] cannot be applied to (4) (the compensation of the output injection term plays a key role in the analysis of [3]). On the other hand, it can be noticed that the matrix A is Hurwitz and that Φ is still smooth, globally Lipschitz and bounded. We could therefore potentially apply the results in [4] to construct a global observer for system (4). Simulation results show that the required LMI condition in [4] is only valid for a restrictive set of model parameters for the model we consider in Section IV. We consequently have to provide an alternative solution to estimate the state x using the output y_f , the input u and model (4).

III. LOCAL OBSERVER SYNTHESIS

The objective of this section is to design a local observer for system (4), *i.e.* an observer whose convergence is ensured when its initial condition is sufficiently close to the initial condition of system (4) (a formal definition is provided below). For that purpose, we compute a first order approximation of the vector field of system (4) in the neighborhood of $\hat{x} \in \mathbb{R}^{n_x}$ using Taylor's theorem. We obtain

$$\begin{aligned}\dot{x} &= Ax + \Phi(\hat{x}, \theta) + \sigma(u, \theta) + A(x - \hat{x}) \\ &\quad + \left. \frac{\partial \Phi(x, \theta)}{\partial x} \right|_{\hat{x}} (x - \hat{x}).\end{aligned}\quad (5)$$

We concatenate each non-zero component of the matrix $\left. \frac{\partial \Phi(x, \theta)}{\partial x} \right|_{\hat{x}}$ to form the vector $\alpha(\hat{x}) \in \mathbb{R}^{n_\alpha}$, where $n_\alpha \in \{1, \dots, n_x^2\}$. As a consequence, we can rewrite (5) as

$$\dot{x} = Ax + \Phi(\hat{x}, \theta) + \sigma(u, \theta) + \mathcal{A}(\alpha(\hat{x}))(x - \hat{x}).\quad (6)$$

with $\mathcal{A}(\alpha(\hat{x})) := A + \left. \frac{\partial \Phi(x, \theta)}{\partial x} \right|_{\hat{x}}$. Notice that each component α_i of α evolves in a compact set, *i.e.* $\alpha_i(\hat{x}) \in [\underline{\alpha}_i, \bar{\alpha}_i]$ for all $\hat{x} \in \mathbb{R}^{n_x}$, with known constants $\underline{\alpha}_i, \bar{\alpha}_i \in \mathbb{R}_{\geq 0}$, $i \in \{1, \dots, n_\alpha\}$. This comes from the fact that Φ is globally

Lipschitz. Hence, $\alpha(\hat{x})$ lies in the hyper-rectangle whose vertices are defined by

$$\mathcal{V} := \{(\omega_1, \dots, \omega_{n_\alpha}) \mid \omega_i \in \{\underline{\alpha}_i, \bar{\alpha}_i\}\}. \quad (7)$$

We consider the following observer candidate

$$\dot{\hat{x}} = A\hat{x} + \Phi(\hat{x}, \theta) + \sigma(u, \theta) + K(\alpha(\hat{x}))(y_f - C\hat{x}), \quad (8)$$

where $\hat{x} \in \mathbb{R}^{n_x}$ is the estimated state and $K(\alpha(\hat{x}))$ is the correction term to be designed. Our objective is to ensure that (8) is a local (exponential) observer for system (4) as defined below.

Definition 1: System (8) is a local observer of system (4) if there exists a neighborhood W of the origin of $\mathbb{R}^{n_x}$ such that for any initial condition x_0 and $\hat{x}_0$ of system (4) and (8) respectively, for any $u \in \mathcal{L}_\infty$, if $x_0 - \hat{x}_0 \in W$, then the corresponding solutions to (4) and (8) are such that $x(t) - \hat{x}(t)$ decays asymptotically to zero. We say that it is a local exponential observer to system (4) if $x(t) - \hat{x}(t)$ exponentially converges to zero. $\square$

We define the estimation error as $\tilde{x} := x - \hat{x}$. The dynamics of the estimation error system is, in view of (6) and (8),

$$\begin{aligned} \dot{\tilde{x}} &= \mathcal{A}(\alpha(\hat{x}))\tilde{x} - K(\alpha(\hat{x}))(y_f - C\hat{x}) \\ &= (\mathcal{A}(\alpha(\hat{x})) - K(\alpha(\hat{x}))C)\tilde{x}. \end{aligned} \quad (9)$$

We can interpret system (9) as a LPV system where $\alpha(\hat{x})$ plays the role of parameters. Thus, the design of the local observer (8) reduces to the computation of the parameter dependent gain $K(\alpha(\hat{x}))$ such that the origin is asymptotically or exponentially stable for system (9). Tractable conditions, expressed in terms of LMIs, can be obtained using quadratic Lyapunov functions [2]. In order to check the stability property of the LPV system (9) using a quadratic Lyapunov function $V(\tilde{x}) := \tilde{x}^T P \tilde{x}$ with P a real, symmetric, positive definite matrix, one has to solve the following parameter dependent Lyapunov inequality

$$A(\alpha)^T P + P A(\alpha) - C^T K(\alpha)^T P - P K(\alpha) C + \varepsilon \mathbb{I} \leq 0, \quad \forall \alpha \in \mathcal{V}, \quad (10)$$

where $\varepsilon \in \mathbb{R}_{>0}$. Condition (10) is difficult to check in practice because there is an infinite combination of the parameters and it is nonlinear as it involves both P and $P K(\alpha(\hat{x}))$ as unknowns. A way to overcome this issue consists in rewriting the LPV approximate error system (9) in a polytopic form and to use a change of variables to tackle the nonlinear term $P K(\alpha(\hat{x}))$. Numerically tractable conditions are then derived in terms of LMIs by evaluating the condition (10) on the vertices of the polytope. To this end, the terms $\mathcal{A}(\alpha(\hat{x}))$ and $K(\alpha(\hat{x}))$ of (9) are rewritten as follows (as in [1])

$$\begin{aligned} \mathcal{A}(\alpha(\hat{x})) &= \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) \mathcal{A}^{[i]} \\ K(\alpha(\hat{x})) &= \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) K^{[i]}, \end{aligned} \quad (11)$$

where $\mathcal{A}^{[i]}$ and $K^{[i]}$ are the corners of the polytopes and $\mu_i(\alpha(\hat{x}))$ are interpolation functions. The terms $\mathcal{A}(\alpha(\hat{x}))$ and $K(\alpha(\hat{x}))$ are obtained by linear interpolation of each component $\alpha_i(\hat{x})$, $i \in \{1, \dots, n_\alpha\}$. As the vector $\alpha(\hat{x})$ belongs to a hyper-rectangle defined by the vertices set $\mathcal{V}$, then $\mathcal{A}(\alpha(\hat{x}))$ and $K(\alpha(\hat{x}))$ are respectively delimited by polytopes of $\mathbb{R}^{n_x \times n_x}$ and $\mathbb{R}^{n_x \times 1}$. These polytopes are defined by the sets of vertices

$$\begin{aligned} \mathbf{A} &:= \{\mathcal{A}^{[0]}, \dots, \mathcal{A}^{[2^{n_\alpha}-1]}\} \\ \mathbf{K} &:= \{K^{[0]}, \dots, K^{[2^{n_\alpha}-1]}\}. \end{aligned} \quad (12)$$

Each corner $\mathcal{A}^{[i]}$ of $\mathbf{A}$ and $K^{[i]}$ of $\mathbf{K}$ corresponds to a corner of $\mathcal{V}$. Let $b_{n_\alpha}^i \dots b_1^i$ be the binary representation of the index i with b_1^i the least significant bit and $b_{n_\alpha}^i$ the most significant bit. Then the parameter box corner corresponding to $\mathcal{A}^{[i]}$, $K^{[i]}$ is $(\tilde{\alpha}_1, \dots, \tilde{\alpha}_{n_\alpha})$ where

$$\tilde{\alpha}_j := \begin{cases} \underline{\alpha}_j & \text{when } b_j^i = 0 \\ \bar{\alpha}_j & \text{when } b_j^i = 1. \end{cases} \quad (13)$$

Furthermore, the interpolation functions are given by

$$\mu_i(\alpha(\hat{x})) := \prod_{j=1}^{n_\alpha} \frac{\gamma_{ij} \alpha_j(\hat{x}) + \beta_{ij}}{\underline{\alpha}_j - \bar{\alpha}_j} \quad (14)$$

with

$$\gamma_{ij} := \begin{cases} 1 & \text{when } b_j^i = 0 \\ -1 & \text{when } b_j^i = 1 \end{cases} \quad (15)$$

and

$$\beta_{ij} := \begin{cases} -\bar{\alpha}_j & \text{when } b_j^i = 0 \\ \underline{\alpha}_j & \text{when } b_j^i = 1. \end{cases} \quad (16)$$

It has to be noted that the following holds

$$\begin{aligned} 0 &\leq \mu_i(\alpha(\hat{x})) \leq 1 \quad \forall i \in \{0, \dots, 2^{n_\alpha}-1\} \quad \forall \hat{x} \in \mathbb{R}^{n_x}, \\ \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) &= 1. \end{aligned} \quad (17)$$

We now state the main result based on the use of a quadratic Lyapunov function that allows us to compute the local observer gains.

Proposition 1: If there exist a real, symmetric, and positive definite matrix P , $R^{[i]}$ for $i \in \{1, \dots, 2^{n_\alpha}-1\}$, and $\varepsilon \in \mathbb{R}_{>0}$ such that the following LMIs are satisfied

$$\mathcal{A}^{[i]T} P + P \mathcal{A}^{[i]} - C^T R^{[i]T} - R^{[i]} C + \varepsilon \mathbb{I} \leq 0, \quad \forall i \in \{0, \dots, 2^{n_\alpha}-1\}, \quad (18)$$

then system (8) with $K(\alpha(\hat{x}))$ given by (11) and $K^{[i]} = P^{-1} R^{[i]}$ is a local exponential observer for system (4). $\square$

Proof. As mentioned above, to check the stability of the LPV system (9) using a quadratic Lyapunov function $V(\tilde{x}) = \tilde{x}^T P \tilde{x}$ with P a real, symmetric, and positive definite matrix, reduces to solve the parameter dependent Lyapunov inequality (10). Using the polytopic formulation, it suffices to check

that

$$\begin{aligned} & \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) \mathcal{A}^{[i]T} P + P \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) \mathcal{A}^{[i]} - C^T \\ & \times \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) K^{[i]T} P - P \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) K^{[i]} C + \varepsilon \mathbb{I} \leq 0, \\ & \forall i \in \{0, \dots, 2^{n_\alpha}-1\}, \forall \hat{x} \in \mathbb{R}^{n_x}, \end{aligned} \quad (19)$$

Using the change of variable $R^{[i]} = PK^{[i]}$, we obtain

$$\begin{aligned} & \sum_{i=0}^{2^{n_\alpha}-1} \mu_i(\alpha(\hat{x})) \left(\mathcal{A}^{[i]T} P + P \mathcal{A}^{[i]} - C^T R^{[i]T} - R^{[i]} C \right) + \varepsilon \mathbb{I} \leq 0, \\ & \forall i \in \{0, \dots, 2^{n_\alpha}-1\}, \forall \hat{x} \in \mathbb{R}^{n_x}, \end{aligned} \quad (20)$$

which can be verified by considering the vertices of the polytope, that is the LMIs (18). Since (18) is assumed to hold, the origin of system (9) is exponentially stable according to standard Lyapunov analysis. Now, let us consider the non-approximated estimation error system

$$\begin{aligned} \dot{\tilde{x}} &= (A - K(\alpha(\hat{x}))C)\tilde{x} + \Phi(x, \theta) - \Phi(\hat{x}, \theta) \\ &=: f(\tilde{x}, \hat{x}, \theta). \end{aligned} \quad (21)$$

By considering the dependence of f on $\hat{x}$ as a time-dependence, we write $f(\tilde{x}, \hat{x}, \theta) =: F(t, \tilde{x})$. We note that F is continuously differentiable since so is Φ . Furthermore, the Jacobian matrix of F at x is $\mathcal{A}(\alpha(x))$, which is bounded in view of (11) and (17) and Lipschitz, uniformly in t on $D = \{\tilde{x} \in \mathbb{R}^{n_\alpha} : |\tilde{x}| < r\}$ for any $r > 0$. Indeed, as α is smooth (since Φ is smooth), so are μ_i for any $i \in \{0, \dots, 2^{n_\alpha}-1\}$, $\mathcal{A}(\alpha(\hat{x}))$ and $K(\alpha(\hat{x}))$ (in view of (11) and (14)). Hence, $\mathcal{A}(\alpha(\hat{x}))$ is locally Lipschitz. In addition, we deduce from (11) and (17) that the Lipschitz constant and the bound of $\mathcal{A}(\alpha(\hat{x}))$ on D are uniform in $\hat{x}$ and thus in t . As a consequence, we apply Theorem 4.13 in [9] to conclude that the origin of system (21) is locally exponentially stable, which means that system (8) is a local exponential observer for system (4) according to Definition 1. $\square$

IV. APPLICATION TO A NEURAL MASS MODEL

We revisit the model of [8] which describes the electrophysiological activity of a cortical column composed of three interconnected populations of neurons (see Figure 1). The *firing rate* captures the average number of *action potentials* generated within the population of neurons per unit of time [5]. The conversion of the firing rate of the afferent populations to either excitatory or inhibitory postsynaptic membrane potential (EPSP) and (IPSP) is modeled, respectively, by using the linear transformation with an impulse response given by, as in [12], [18], [19],

$$h_e(\tau) = A_e \exp\{-a_e \tau\} \quad \tau \geq 0 \quad (22)$$

$$h_i(\tau) = A_i \exp\{-a_i \tau\} \quad \tau \geq 0, \quad (23)$$

with $h_e(\tau) = 0$ and $h_i(\tau) = 0$ for $\tau < 0$. Parameters $A_e > 0$ and $A_i > 0$ determine the maximum amplitude of the EPSP and IPSP, respectively, a_e and a_i represent the average time constants of the excitatory loop and inhibitory

loop respectively. Contrary to [8], we consider linear systems of order 1 in (22) and (23) (and not of order 2), however our results do apply to linear systems of higher order. This choice is justified by the fact that this simpler structure is still able to generate a wide range of output signals as studied in e.g., [12], [18], [19]. The mean membrane potential of a population is converted into the average firing rate of the population using the sigmoid function $S(v) = 2e_0/[1 + e^{r(v_0-v)}]$ for $v \in \mathbb{R}$, where $e_0 > 0$ determines the maximum firing rate of the neural population, $v_0 > 0$ is the postsynaptic membrane potential (PSP) for which a 50% firing rate is achieved, and r the steepness of S (see [8] for more details). Using the state space representation of (22), (23) and by following similar lines as in [8], the resulting model is

$$\begin{aligned} \dot{\bar{x}}_1 &= -a_e \bar{x}_1 + A_e S(\bar{x}_2 - \bar{x}_3) \\ \dot{\bar{x}}_2 &= -a_e \bar{x}_2 + A_e [u(t) + C_1 S(\bar{x}_1)] \\ \dot{\bar{x}}_3 &= -a_i \bar{x}_3 + A_i C_2 S(\bar{x}_1) \\ \bar{y} &= \bar{x}_2 - \bar{x}_3, \end{aligned} \quad (24)$$

where $\bar{x}_1$ is the membrane potential contribution from the pyramidal neurons to the excitatory and inhibitory interneurons, $\bar{x}_2$ is the membrane potential contribution from the pulse density u and the excitatory interneurons to the pyramidal neurons and $\bar{x}_3$ is the membrane potential contribution of the inhibitory interneurons to the pyramidal neurons. The neural populations are connected with connectivity strengths C_1 and C_2 which account for the total number of synapses established by interneurons onto the axons and dendrites of the neurons constituting the cortical column. The pulse density u is typically taken as a white noise function. For our simulations, the input u is a continuous white Gaussian noise of mean 90 and of variance 900. We apply the filter (2) to the output of system (24) and we obtain the system below

$$\begin{aligned} \dot{x}_1 &= -a_e x_1 + A_e S(x_2 - x_3) \\ \dot{x}_2 &= -a_e x_2 + A_e [u(t) + C_1 S(x_1)] \\ \dot{x}_3 &= -a_i x_3 + A_i C_2 S(x_1) \\ \dot{x}_4 &= -f_0 x_4 + x_2 - x_3 \\ y_f &= -f_0 x_4 + x_2 - x_3, \end{aligned} \quad (25)$$

which corresponds to (4) with

$$\begin{aligned} A &= \left[\begin{array}{ccc|c} -a_e & 0 & 0 & 0 \\ 0 & -a_e & 0 & 0 \\ 0 & 0 & -a_i & 0 \\ \hline 0 & 1 & -1 & -f_0 \end{array} \right] \\ C &= \left[\begin{array}{ccc|c} 0 & 1 & -1 & -f_0 \end{array} \right] \\ \sigma(u, \theta) &= (0, A_e u(t), 0, 0) \\ \Phi(x, \theta) &= (A_e S(x_2 - x_3), A_e C_1 S(x_1), A_i C_2 S(x_1), 0). \end{aligned} \quad (26)$$

We see in Figure 2, system (25) generates output signals with no offset after a transient time as desired. As in [3], we consider the parameter values presented in Table I. We envision an observer of the form (8) for system (25) and we

Parameter	Value
e_0	2.5
v_0	6
a_e	100
a_i	50
A_e	3.25
A_i	22
C	135
C_1	$0.8C$
C_2	$0.25C$
f_0	10

TABLE I: Parameter values.

obtain an approximate estimation error system (9) with

$$\mathcal{A}(\alpha(\hat{x})) = \begin{bmatrix} -a_e & A_e \alpha_1(\hat{x}) & -A_e \alpha_1(\hat{x}) & 0 \\ A_e C_1 \alpha_2(\hat{x}) & -a_e & 0 & 0 \\ A_i C_2 \alpha_2(\hat{x}) & 0 & -a_i & 0 \\ 0 & 1 & -1 & -f_0 \end{bmatrix}, \quad (27)$$

where

$$\alpha(\hat{x}) = (\alpha_1(\hat{x}), \alpha_2(\hat{x})),$$

and

$$\alpha_1(\hat{x}) = \left. \frac{\partial S(x_2 - x_3)}{\partial x_2} \right|_{\hat{x}_2, \hat{x}_3}, \quad \alpha_2(\hat{x}) = \left. \frac{\partial S(x_1)}{\partial x_1} \right|_{\hat{x}_1}.$$

Each α_i evolves in $[0, \frac{1}{2}e_0 r]$, with $\frac{1}{2}e_0 r$ being the Lipschitz constant of the sigmoid function. The particular structure of Φ leads to a vector α of dimension $n_\alpha = 2$. Hence, the matrix $\mathcal{A}(\alpha(\hat{x}))$ lives in a hyper-rectangle with 2^2 vertices. We compute a gain $K^{[i]}$ for each vertex $\mathcal{A}^{[i]}$ by solving LMIs (18) as explained in Section III. The gain values are

$$K^{[0]} = \begin{bmatrix} -1, 35 & -1, 43 & -1, 16 & -0, 57 \end{bmatrix}$$

$$K^{[1]} = \begin{bmatrix} -0, 16 & -1, 63 & -2, 13 & 0, 17 \end{bmatrix}$$

$$K^{[2]} = \begin{bmatrix} -1, 34 & -1, 45 & -1, 34 & -0, 57 \end{bmatrix}$$

$$K^{[3]} = \begin{bmatrix} -0, 14 & -1, 64 & -2, 30 & 0, 17 \end{bmatrix}.$$

Figure 3 presents simulations results for the initial conditions $x(0) = (0.14, 19, 12.5, 7)$ and $\hat{x}(0) = (0.24, 19.4, 12.9, 7.4)$. We see that the estimation error decays to zero as expected. Figure 4 depicts the variations of $\alpha_1(\hat{x})$ and $\alpha_2(\hat{x})$. We note that these variables, which we consider as parameters in the design, are indeed time-varying.

It has to be noticed that we could not design a global observer using [4] as the required LMI does not hold for the considered set of parameters. Nevertheless, we have varied the values of A_e , A_i and C within the set $\Theta := [2.6, 9.75] \times [17.6, 110] \times [108, 675]$ to identify feasible values for the results in [4]. Those parameters are known to play an important role in the dynamics of (24) (see [8], [17]) and are expected to lie Θ . We have sampled the set Θ with a step of 0.5 for A_e , 5 for A_i and 30 for C to this purpose. We have tested the LMI in [4] for these values. Figure 6

Fig. 3: Simulation results: true state (red), estimated state (blue).

Fig. 4: Evolution of: $\alpha_1(\hat{x})$ (red dashed line), $\alpha_2(\hat{x})$ (blue line).

shows the feasible set of model parameters. We have also plotted the feasible set of parameters for our design, see Figure 5. We see that the range of parameters where LMIs (18) is verified is wider than the one for which the LMI in [4] holds. More precisely, LMI (18) is verified for 90% of the tested parameters values, while the LMI in [4] holds for 49% of the values.

Fig. 5: Model parameters for which (18) is verified.

V. CONCLUSIONS

We have presented a local observer design for a class of neural mass models which relies on LPV techniques. The observer gains are nonlinear in the state estimates and are constructed by solving LMIs. The results apply to neural mass models for which the outputs are centered at the origin as with real EEG recordings.

A drawback of local observers is that they need to be initialized sufficiently close to the true system state to converge. We will present in future work a hybrid estimation scheme that relies on the results of this study to ensure the global convergence of the state estimation error.

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Fig. 6: Model parameters for which the LMI in [4] is verified.