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Abstract

L'Hostis (2015) has demonstrated that triangle inequality which is one of the four mathematical properties of distances, and whose role is to ensure the optimal character of distances, reveals some key aspects of distances and of geographical spaces. We elaborate from this demonstration by investigating the optimality of distances in empirical approaches and in spatial theories.

The first part of paper explores the consequences of considering that the mathematical property of triangle inequality is always respected. Indeed, violations of triangle inequality are not observed in geographical spaces. The study of optimality of distances in empirical approaches confirms its role as a key property. The general *principle of least-effort* applies to most movements and spacings. But in addition, trajectories with lots of detours, as those of shoppers, runners and nomads, are optimal from a particular point of view. It is also the case for the theme of excess travel which is based on a disjunction between an optimum by a driver and an optimum by an external observer. Any movement, any spacing in cities or in geographical space in general exhibits some kind of optimality.

The place of distance in spatial theories reveals a contrasted position. In general, spatial theories do not place a great emphasis on distances. Several authors need to complement the idea of distance with an additional concept, like accessibility, in order to introduce a value for individuals and society. Conversely, for others including Brunet, distance embodies behaviours and intentions of actors. The discussion highlights the fact that optimum is relative to the actors involved. We observe that no consensus about the optimal nature of distances can be found in spatial theories, but strong arguments exist to support the idea.

Highlights

Violations of triangle inequality are not observed in geographical spaces.

Most movements and distances follow least-effort principle

Any movement, any spacing in geographical space exhibits some kind of optimality.

No consensus about the optimal nature of distances can be found in spatial theories

The demonstration is provided that all geographical distances are optimal

Keywords

Distance, geography, transport, optimality

Introduction

In the study of distances, the discussion of the linkages between mathematical properties and geographical or economical empirical observations has proven fruitful. In particular L'Hostis (2015) has demonstrated that triangle inequality which is one of the four mathematical properties of distances reveals some key aspects of distances and of geographical spaces. The role of triangle inequality is to ensure the optimal character of distances. L'Hostis's analysis of errors of interpretation of violations of triangle inequality leads to the statement that geographical distances are optimal and that detours and breaks contribute to this optimality. In this paper we elaborate from these elements by discussing the issue of the optimality of distances in the context of theoretical and applied transport geography. More broadly, our discussion on the optimality of distances contributes to highlighting the role of the concept of distance in relation with the movement and the spatial distribution of geographical objects.

It can be surprising to observe that, in the academic disciplines with a focus on spatiality, the scientific literature on distance itself is far from being as developed as the use of the concept (L'Hostis 2015). Indeed, the list of contributions focussing on distances (Deutsch et Isard 1961; Hall 1969; Gatrell 1983; Huriot, Smith, et Thisse 1989; Brunet 2009) is relatively short. Furthermore, for several researchers in the fields of spatial analysis, involving the domains of economics, geography and planning, distance is not considered as a central concept. Several arguments contribute to the marginalization of distance.

Firstly, in the *euclidianist* conception (Lévy 2008, 83), the distance is associated to the Euclidean straight line. The questioning of the straight line as a general model for routing between geographical locations feeds a suspicion about the capacity of distances to express the relative position of locations in space.

The second argument states that distance is too multifaceted to avoid ambiguity in scientific discourse. Its multiple forms makes it subject to uses too different one from another. Distance is the length of a route that can be measured with different units, but distance is also simply the idea of separation between locations, or the length of a linear infrastructure. In order to illustrate the argument of the multifaceted distance, we examine the role of distance in post-modern literature. The reflection of Hall (1969), that highlighted the differences of perception and the relativism of physical distances depending on culture, has fuelled and inspired the postmodernists (Steadman 1996). The analysis of the influence of distances on human behaviour has supported a relativism in urban studies and in urbanism in particular, in a general context of denunciation of the functionalistic modernism with Jacobs as a key contributor (Jacobs 1961). At the same time, a consensus places the idea of distance at the heart of geography. The first law of geography enunciated by Tobler that “everything is related to everything else, but near things are more related than distant things » (Tobler 1970, 236) demonstrates this role. Yet, this idea that everything is related to everything else creates a link between objects of geography and proposes a comprehensive vision that Sui considers contrary to postmodernist thinking, in the sense that it separates, distinguishes, relativises and rejects the idea that individuals and social groups are strongly linked to each other (Sui

2004, 271). These two pathways of thought are both inspired by the idea of distance and lead to opposite views according to the current paradigms. Distance, as a concept used in multiple, diverse and even conflicting analysis should be put aside.

A third argument, found in some geographic literature (Dumolard 2011), considers distance as an abstract notion independent from behaviours and hence lacking relevance in human geography or in urbanism. Distance would be meaningless.

Contrary to these three arguments, we pose the hypothesis that the introduction of the optimality of distances can contribute to clarifying the place of the concept of distance and to asserting its role in spatial disciplines. In order to test this hypothesis, we start by exploring the consequences of the respect of triangle inequality in understanding distances and space. We will then focus on the empirical approaches dealing with the optimality of distances before discussing the taking into account of this idea in spatial theories.

Triangle inequality respected : coherence of distance and space

It has been demonstrated that the non-respect of triangle inequality poses major difficulties for cartography (L'Hostis 1997, 116). This is due to the rupture of continuity of the geographical surface, which translates into topological ruptures in the sense of neighbourhoods: this is the problem of space inversion formalized by Bunge (1962, 171) and Tobler (1961, 106) and which abounds in geographical spaces (Gatrell 1983, 46). We will develop here the works on triangle inequality in the domains of economics and geography.

The economist, Tony Smith, attributes the demonstration of the fact that triangle inequality is the fundamental property of metrics (Smith 1989, 5) to the mathematician Fréchet (1906; 1918), who was the first to formalize the distance and its four properties. For Fréchet, the general form of the function indicating a measure of the separation between two points is a spread (*écart*) and becomes a distance only if it respects triangle inequality (Fréchet 1918, 55).

In the domain of spatial economics Smith has shown that any measure based on minimum paths respects triangle inequality (Smith 1989, 15). This means that developing spaces that violate triangle inequality implies creating linkages between locations that are not minimum paths; as an example of distance violating triangle inequality, Smith introduces the case of the discrimination distance between objects seen by a radar (Smith 1989, 7). In the same spirit, Gatrell also introduces a measure that violates triangle inequality, with a non-spatial index of dissimilarity (Gatrell 1983, 38), and Felsenstein discusses the possibility of non-metric spaces in the domain of separation of living species (Felsenstein 1986). In all these examples, the notion of path does not have the meaning we commonly admit in geography. As we see, this discussion takes us away from distances of transport and geography. This is a key observation because the layout of a transport network may include direct routes, close to the straight line, but sub-optimal. In this case, the layout of the network creates some confusion for the reader: the direct route outdated by the fast transport system remains a strong reference for the travel project. But such a situation does not lead to violations of triangle inequality (L'Hostis 2015).

In the majority of works dealing with international flows, triangle inequality is considered as respected by reference to the cross-border arbitrations (Lamure 1998; Eaton et Kortum 2002, 1745). The only exception found in the bibliographical work, for Behrens and alii., is that triangle inequality may be violated in the case of transaction costs not related to transport (2007). In this case, it would be possible for a matrix of general interaction costs between places, summing transport costs and transaction costs not related to transport, to generate a metric space which does not respect triangle inequality. Indeed, concerning international flows, transport costs are not the only ones involved to explain distances. Grouped under the terms of transaction costs, these costs involve various realities linked to geographical borders, including political, historical, or cultural aspects: a difference in language can cause a reduction of a half of economic exchanges between two countries (Rietveld et Vickermann 2004, 241). In particular, customs duties can represent a significant part of cost-distances between countries. Nevertheless, these costs have no link with distances in kilometres travelled by goods. The formation of these costs depends on a set of factors linked to choices in national or supra-national economic policy. In these conditions, it is possible, in theory, to imagine a violation of triangle inequality regarding these transaction costs. Let us consider three countries A, B as producers and C as an importer of goods. Customs duties are high for goods entering country C from country A, but much lower from country A to country B and from coutnry B to country C. In this case it is possible that the sum of customs duties AB and BC remains lower than the direct cost from A to C. The hypothesis of cross-border arbitration implies that this situation will not last, because the intermediate country will become a privileged entry point in C for goods produced in A. This means that the economic flow will seek to minimize the transaction costs by establishing a path with reduced costs. It would be a relevant economic policy for country C to reduce the custom duties of direct importation from A or to increase the fares from B in order to charge the goods originating from A through their direct or indirect path. In mathematical terms, this process means that the system will follow the property of triangle inequality, or will correct the violations of triangle inequality that cannot last in the international system.

Let us now move from the domain of economics, where we find strong grounds to justify the respect of triangle inequality, to the theme of transport and geographical networks. For Ahmed and Miller in the literature on transport, the only metric property whose violation is "likely" is triangle inequality (Ahmed et Miller 2007). This prudence of the two authors is possibly due to the fact that their own study of time-space of Salt Lake City shows conversely a lack of violation of triangle inequality. In addition, their work mentions Tobler's conjecture according to which using only minimum paths permits avoiding any violation of triangle inequality (Tobler 1997).

More generally, the contributions on graphs, whether they be geographical or not, provide very relevant analysis for understanding the nature of geographical networks (Mathis 2007). Schilling, Rosing and ReVelle highlight that the two main characteristics that allow for distinguishing random networks from what they call "natural networks" (2000) are symmetry and the respect of triangle inequality. In this paper dealing with the development of location algorithms,, they show that the fact of imposing triangle inequality allows for the fastest convergence towards a solution. This constitutes a supplementary argument to support the importance of this property for geographic space.

In the domain of mental maps, a major part of the work consists in explaining the differences between mental and geographical space (Tobler 1976; Gatrell 1983, 130). Psychological analysis of the representation of space and distances reveal that for some subjects violations of triangle inequality can exist (Cadwallader 1979; Baird, Wagner, et

Noma 1982, 205). A part of these violations can be credited to the poor knowledge of space by individuals: our mobility routines leaves large parts of space in the unknown (Gatrell 1983, 132). Nevertheless, these gaps are not substantial (Moar et Bower 1983), which leads Baird, Wagner and Noma to follow the hypothesis that we could accept these deviations as errors of measure rather that affirming that the cognitive spaces are not metric (Baird, Wagner, et Noma 1982, 205; Golledge 1997, 238). According to this analysis the violations of triangle inequality are not supported by strong evidence in the domain of mental maps.

Tobler introduced several methods to build mathematical spaces from empirical data obtained from measures on the transport system (Tobler 1997). From the length of routes between cities in the mountainous western Colorado, he builds distances approximated by several methods derived from bidimensional regression. For Tobler, if the measures of separation of adjacent cities are minimum, then the distance that will be produced will respect triangle inequality (Tobler 1997). In his thesis, Tobler associates the violations of triangle inequality to spatial inversions by stating that “a place located two hours away cannot be closer than a place situated one hour away” (Tobler 1961, 120). In a geographical context, for Tobler, violations of triangle inequality are spatial aberrations. We can add that this principle applies also in a cost space.

In order to complete this investigation on triangle inequality, we will study the property of transitivity in networks. The mathematical models of network-distance respect transitivity as in Bae and Chwa (2005). These authors define the general mathematical properties of network-distances that superimpose themselves on the Euclidean distance measured on the sphere and that respect triangle inequality. Transitivity, which is ensured by triangle inequality, is a property of locations in geographical space which is not always observed in social space: even if individual A knows the individuals B and C well, it is possible that the latter have no direct link to each other. In the social network space, A is close to B and to C, but B and C are very far away from each other. In this case, the distance between B and C is greater than the sum of distances AB and AC, causing a violation of triangle inequality. It must be said that this situation depends on the way the distance is measured. Indeed, two types of measurement exists, whether we count the number of intermediate persons between two individuals, or we measure the frequency or the intensity of the contacts. When the measurement counts the number of intermediaries, as in the *small world* approaches (Watts 2003), transitivity is respected. Conversely, in the case where distance represents the inverse of the quantity of contacts, with no contact corresponding to an infinite measure and lots of contacts being equivalent to a short distance, many violations of triangle inequality are possible. As this measurement is more coherent with the idea of proximity between individuals, we can verify that social spaces violate triangle inequality, which underlines the fact that they are only partially determined by spatiality. In terms of spatial properties, social space cannot be considered as equivalent to a geographical or a transport space.

It must be observed that errors are more easily detected than correct conception. In this second group we can find for instance Lévy who, in order to criticize the Euclidean conception in cartography, takes as an illustration the path through B as faster than the direct route from A to C (Lévy 2008, 83). He provides here a correct analysis of the often wrongly interpreted (L’Hostis 2015) triangle inequality property. Another illustration is given by Miller and Wentz in their paper on the formalization of measurements in spatial analysis: they present the issue of the measure of length by discussing the properties of distances, but they never evoke geographical spaces with violations of triangle inequality (Miller et Wentz 2003).

In this discussion on the property of triangle inequality, we observe the emergence of the notion of optimum. We will now investigate in this direction.

Distance and the optimum : lessons from empirical approaches

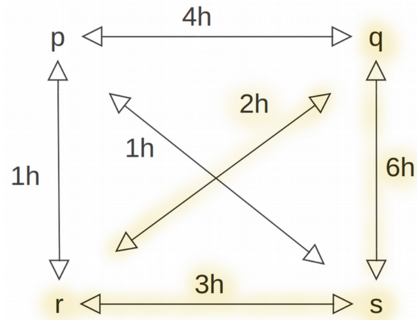

Figure 1. Four cities with non-optimal separation measures, violating triangle inequality (Haggett 2001, 341) underlined by the author

In this discussion we find two antagonistic ideas about the role that distances could play in the reflection on geographical space. On the one side, distance is considered as a neutral quantity, a quantity measured and devoid of predefined properties, as in the example given by Haggett (2001, 341) who proposes a non-optimal measure of distance. And on the other side some see space, and hence also distance, as a reality that is impossible to separate from its perception by individuals, to the extent of refusing to include it as a fundamental element in spatial disciplines (Chivallon 2008).

We defend here a third, which is that the idea that the distance of geography cannot be considered as a neutral quantity devoid of any mathematical property. Distances carries a set of geographical properties, in the same way as temperature or altitude, as geographical data, satisfy constraints, laws, and correspond to a spatial organization determined by geomorphological, historical, climatic or perceptive dimensions. If we consider the mathematical definition of distances, we must admit that triangle inequality is respected, which implies that distance is the minimum of the measures of separation between two locations (L'Hostis 1997; L'Hostis 2014). In the mathematical perspective, the minimum is a fundamental property of distances.

The optimum of distances is justified on solid theoretical grounds, but we will now discuss the optimum in the empirical approaches. We will consider observations in the social field, with a focus on movements in space. The analysis of mobilities, the modelling of transports, and, more generally, the empirical approaches shed light on the role of optimum of distances. From this analysis we will then discuss the way the optimum is integrated into spatial concepts.

When we observe realities in the social field, it can be admitted, according to Zipf's analysis (1949), that social phenomena follow the principle of *least-effort*. In his seminal book, Zipf provides a justification of least-effort in human behaviour by the need to minimize the amount of work necessary to perform any task. If the task requires a movement, then the individual will seek to minimize the total work needed; this is why distance and the main parameters of movement are involved in the minimization process. A task involving a movement and following the principles of least-effort, will have efforts dedicated to movement reduced to the minimum. Zipf's approach uses a corpus of psychology and economics (1949, 13), and provides analysis in the domain of geography. It is remarkable that the first example he chose to illustrate the principle of least-effort is the research of a minimum route between two cities linked by a road which is rectilinear on a flat terrain (Zipf 1949, 2). This path is then the shortest, the fastest and the easiest of all the possible routes. Zipf uses the notion of *least-work distance* (1949, 348) to identify the optimal route chosen by individuals and used for goods. In addition, all the distances he lists in his empirical geographical section (Zipf 1949, 374) are implicitly or explicitly minimal distances. From these elementary considerations, Zipf elaborates an economic theory for the access to and the processing of spatially distributed resources that leads to a law of the distribution and size of human settlements (Zipf 1949, 364-366). This law has been extensively used in the domains of geography. Nevertheless, in our approach, our focus is on the use of distance by Zipf, and essentially on the first steps of his theory leading to the rank-size rule of human settlements. In Zipf's approach, distance is one of the main parameters of the minimization of socio-spatial configurations: the routes used by individuals and the flows between settlements are minimum by the application of the principle of least-effort. To express it differently, the principle of least-effort can be read in itineraries and, more generally, in the choices of the parameters that contribute to the formation of geographical distances. The analysis of Zipf is hence deeply consistent with the hypothesis that distance implies an optimum.

Sociologists have introduced the term of *desire line* to describe the visible path generated by walkers in landscapes (Simmel 1997, 171). These desire lines reveal the tensions in the formation of distances, most often resulting from a compromise between least-effort and risks associated with road safety. It seems then legitimate to focus on the weight of the least-effort argument in pedestrian routes in general; to what degree is the least-effort trajectory favoured in general situations? Indeed, the desire lines are revealed under particular conditions, namely the existence of a surface which, as in the example of soil covered with grass, is able to record and make visible the flow of pedestrians.

Figure 2. The desire line revealing tensions in the formation of distances as a compromise between least-effort and risk associated with road safety (picture L'Hostis 2013)

Studies based on the analysis of video, such as the work of Moody and Melia on the shared space of Ashford in Great-Britain (Moody et Melia 2013), show that this situation of optimization of routes by pedestrians is far from isolated. On this space, almost half of the pedestrians routes favour a more rectilinear trajectory which is more dangerous than the detour through formal road crossings despite the fact that the shared space design principle was supposed to make them obsolete. The optimum and the relationship to the straight line influence these route choices: the desire line is the materialisation of a profound reality widely present in human behaviours.

Nevertheless, we can object that not every trajectory in geographical space seeks to minimize the effort. The list of

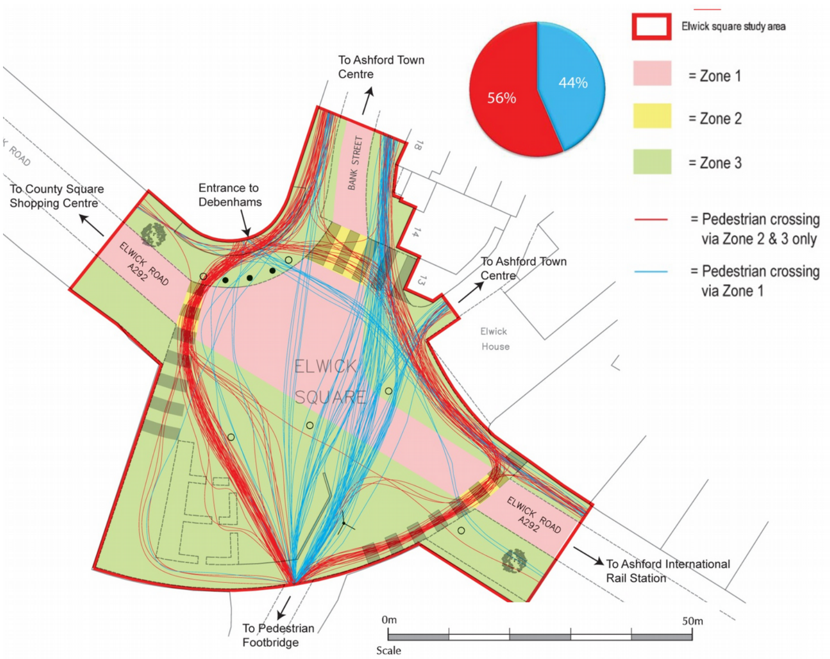

Figure 3. Optimisation in pedestrians routes in a shared space in Ashford (UK) (Moody et Melia 2013)

those cases is long: the route of a run course, a trekking route, a tourist trip, the apparently erratic trajectory of a missile, the stroll of the shopper or the wandering of the nomad. All these movements rarely minimize the time spent or the number of kilometres travelled, or, more generally, the parameters of the effort needed for the movement. But all of them optimize a different parameter: the pleasure taken from the deambulation for the *flâneur (Tester 1994)* or the tourist, the achievement of an effort by the runner, the maximisation of the number of opportunities to find a resource for the nomad or the shopper, the minimisation of the risk of interception for the missile. All these cases depart from least-effort, if we consider for instance the extra effort needed by the missile to perform its misleading trajectory, but they do not invalidate the hypothesis of the optimisation. All these trajectories are optimal.

In order to further illustrate the optimization process in the present mobility, we consider the notion of time-budgets. The debate on the constancy of time-budgets and the increase in distances travelled shows that individuals optimize their time by maintaining their personal time-budget devoted to mobility and adjusting the distances travelled (Banister 2011). we can see this phenomenon as a process of optimization of distances that involves individuals on a daily basis.

In the domains of traffic engineering and network economics, many models are developed out of analogies with physics, especially fluid dynamics (Leurent, Chandakas, et Poulhès 2011; Ma et Lebacque 2012; Costeseque et Lebacque 2013). In this domain too, optimum is an essential principle with explicit references to the Fermat principle according to which light propagates itself locally following a route which minimizes the duration of the trip.

In the domain of the analysis of mobilities again, the theme of *excess travel* emerged from doubts concerning the widely admitted economic principle of transport as a derived demand, i.e. the fact that transport has no justification for itself and only exists for the sake of the activity it will allow at destination (King et Mast 1987; Mokhtarian 1998; Mokhtarian et Salomon 2001; Kanaroglou, Higgins, et Chowdhury 2015). Excess travel involves trips that are longer than routes considered optimal by computation (King et Mast 1987), and also trips realized by pure pleasure of moving (Mokhtarian et Salomon 2001). But this literature highlights the gap between itineraries chosen by car drivers and routes considered optimal through external measurement. This idea does not question the existence of an optimum for individuals, who can decide with full or partial awareness of an objective situation of movement. The reflection is more oriented towards a disjunction between an optimum perceived by individuals and an optimum established by external observers. The theme of the excess travel does not threaten the thesis of distance as an optimum.

Many authors have expressed the hypothesis of the existence of a need for separation between home and work-place (Lynch et Rodwin 1958; Lynch 1981, 194). The people interviewed do not want, in their vast majority, to do away with their time-budget devoted to transport justifying their choice by a need to separate home and work. This corresponds to a desire to separate functions and activities in order to separate private life and professional life (Belton 2009; Pradel et Chevallier 2010). This hypothesis of separation does not question the idea of the optimum; indeed the need for distance corresponds to a form of spatial optimization.

The principle of least-effort is now used in the modelling of pedestrian mobility (Guy et al. 2010), or for the analysis of the street network at the agglomeration level (Masucci et al. 2009). We have discussed, with the desire line, the consequences of the fact of neglecting the least-effort principle in urban design: users may develop their own network and ignore the itineraries that the designers had foreseen for them. In the study of mobility, the effort and its minimization are used as a basic idea for their understanding (Kellerman 2012, 27), but this idea does not lead to significant analysis, and many other factors are highlighted. In geography, for many authors the mobility choices address a human desire for a minimization of distances (Wolpert 1964; Gatrell 1983; Golledge 1997). Even if he is not an omniscient rational being, the individual develops a rational intention (Wolpert 1964, 558).

Empirical analysis shows how important the optimality of distances is for movement and spatial distribution. But do the spatial theories embody this aspect? And if yes, how?

Distance optimum in spatial theories

The idea of the optimum, despite its fundamental character, is mostly missing in the definitions of distance provided by the geographic literature. For instance Pumain (2009) details the mathematical properties of distance but omits to associate distance to the minimum measurement of separation. The notion of minimum is involved only in the application to the geographic domain of the mathematical concept. For Pumain the notion of field, defined as a geographical space influenced by a force located in a pole, is interpreted as “society taking into account the laws of physics” (Pumain 2009, 43), in the sense of saving energy, resources and hence of “minimising an expense that influences social activities in space” (Pumain 2009, 43). In the same rationale, for Gatrell the minimum character of distances is admitted but only implicitly (Gatrell 1983). Similarly, Dumolard, in a work on distance and accessibility (Dumolard 2011, 192), introduces optimality only when he develops the notion of accessibility, and not in the section on distances. For Dumolard accessibility is a specification of the notion of distance, in the sense that it implies subjects on the move, transport means, perception of a route, a temporal dimension and an anticipation (Dumolard 2011, 193). The specialists of mobility share this view when they see accessibility as linking people and places (Kellerman 2012, 15).

The definition of distance in Brunet's book on the concepts of geography does not precise the four mathematical properties. In addition, distance is not seen as a minimum measure. More recently Brunet (2009), privileging a literary approach of the concept, introduces the mathematical forms but neither the properties nor the idea of the minimum.

For economists in general, distance does not contain the idea of optimum, and the concept of distance has to be specified in economic terms in order to establish a relevant economic approach. This view is not shared by several specialists of spatial economics: in an encyclopedia of spatial economics, the chapter on space and distance (Huriot, Perreur, et Derognat 1994) refers to the notion of *minimum cost distance* (Huriot, Smith, et Thisse 1989) as a general concept admitted by spatial economists. Considering spatiality as a basic idea for economic analysis implies the recognition of optimum as a key character in the understanding of distance between locations.

Mentioning the existence of non-optimality in distances is in fact rare in scientific literature. And these occurrences can sometimes be rephrased. Some ethologists refer to a non-optimal distance for species facing fast modifications of their environment and that are not able to adapt their behaviour (Fahrig 2007). Woodland birds habitat becoming more and more sparse causes an increase of the length of routes between two groves which increases the mortality risk for birds. In this study, non-optimality is caused by a lack of information or an inability to make use of a new information: individuals choose their trips ignoring an information that could be valuable for them. We find here the disjunction already highlighted concerning the debate on excess travel, between the optimum of an individual and the optimum of an external observer. The trips remain optimal from the point of view of moving birds, but the high mortality rate due to changes in their environment generates some non-optimality at the level of the group. Fahrig draws attention on the use of minimum path algorithms for estimating animals movements (Fahrig 2007, 1010), because they imply that individuals have the ability to optimize their trips, which is not always the case. In order to grasp the situation, it is necessary to identify the right criteria for optimising the trip, which also depend on available information when choices are made.

Extending theoretical developments in regional science on the behaviour of individuals (Isard et Dacey 1962a; Isard et Dacey 1962b), the reference text of Olsson and Gale on behaviour and spatial theories (1968) allows for further discussing optimality and non-optimality of decisions. Olsson and Gale introduce the notion of *subjective distance* (1968, 231) to address the fact that individuals make their optimal choice with their subjectivity; the optimum of a person can be challenged by another individual or by an external observer having access to more information. But optimum remains key in establishing choices. In this view, non-optimality comes from a lack of information, from subjective filters that distort information, or from choices of factors and their relative weight in decision processes, and is only perceptible by an external observer.

The geographer Bunge has enunciated the principle of proximity in geography by observing that objects that interact tend to locate as close to each other as possible (Bunge 1962). This principle of proximity is seen by Olsson and Gale (1968) as a spatial translation of Zipf's principle of least-effort (1949). As we have seen earlier, optimality is considered by several geographers as linked to the concept of accessibility, involving the passage from an abstract measure, distance, to a concrete measure linked to an intended or a realized trip. Conversely, for several other authors, distance is considered as intrinsically linked to an intention; this is the case of Brunet who links distance to the project of transaction of actors (Brunet 2009, 16). For Brunet, distance makes its full sense only in the context of actors with a project of transaction. Distance is then associated to the idea of movement, to its vectors i.e. way and vehicles, and also to effort (Brunet 2009, 16). Brunet does not introduce an intermediary concept that would add a human dimension to an abstract measure; he does not feel this need since for him distance is at the heart of geography: “distance does not exist without the places of which it is the interval” (Brunet 2009, 16). The observation of ways and relays provide another argument for linking distance and optimality. The spatial distribution of rest stops, relays, road tolls, service stations, pumping stations and radio relays depends on the idea that “any movement implies an energy expenditure, and this energy has to be periodically reloaded” (Brunet 2009, 16). Ironically, the reasoning of Brunet of the demonstration of optimality of distances uses the same illustration of relays as the opposite erroneous (L’Hostis 2015) reasoning of Huriot, Smith and Thisse (1989) who see the ubiquity of rests-stops as an evidence of violation of triangle inequality, which ensures optimality of distance. The same example is used with diametrically opposite conclusions: considering the formulation of a distance function (L’Hostis 2015) that addresses the issue posed by Huriot, Thisse and Smith, we support Brunet's analyse. As we see, there is presently no consensus about the optimal nature of distances in spatial theories, but strong arguments exist to support the idea.

The ultimate aspect to discuss about optimality is the idea that it must be credited to an actor. In all the examples developed so far, it is always the actor on the move that seeks optimisation. This optimisation follows a wide range of criteria as we have seen: effort, pleasure, chances of survival, etc. In addition, optimality of distance can be read in the choices of route infrastructure selection, and in the choices of the organisation of transport systems. Optimum is relative since the optimum of a transport operator is not always coincident with the optimum of a traveller. Each actor makes his choices according to his own interest: for instance concerning aerial transport, the operator will seek to maximize the load of his vehicles, while the traveller will seek to minimize costs with higher waiting time, or with more connections as trade-offs. In this case, the multiple individual optimisation processes will lead to an overall optimisation. But this relativity of optimums can also lead to disjunctions: a set of individual optimisation processes does not necessarily form a social optimum if we consider the externalities associated with an extensive car use. We join here the approach of Martouzet (2002) who employs the plural of *rationalities* to address the plurality of decisions in the domain of spatial planning: for him several rationalities exist and the plural form expresses ideas of multiplicity, non-unicity, actor-dependency that lies behind the principle of rationality.

Conclusion

Starting from the observation of the gap between the broad use of distance in spatial disciplines literature and the limited coverage of the concept, we have hypothesized that a reflection on optimality could clarify its role and function. The first step is to explore the consequences of considering that the mathematical property of triangle inequality is always respected. Indeed, violations of triangle inequality are not observed in geographical spaces. This reflection leads us to focus on optimality which is the character that triangle inequality ensures.

The study of optimality of distances in empirical approaches confirms its role as a key property. The general *principle of least-effort* applies to most movements and spacings. But in addition, trajectories with lots of detours, as those of shoppers, runners and nomads, are optimal from a particular point of view. It is also the case for the theme of excess travel which is based on a disjunction between an optimum by a driver and an optimum by an external observer. More broadly than the cases found in literature, which even if revealing remain partial in covering all phenomena, our rationale is supported by the fact that for distances, optimality is a key character. Any movement, any spacing in cities or in geographical space in general exhibits some kind of optimality.

The place of distance in spatial theories reveals a contrasted position. In general, spatial theories do not place a great emphasis on distances. Several authors need to complement the idea of distance with an additional concept, like accessibility, in order to introduce a value for individuals and society. Conversely, for others including Brunet, distance embodies behaviours and intentions of actors. The discussion highlights the fact that optimum is relative to the actors involved. Optimality is not unequivocal: each actor establishes his optimal distances according to information available. Despite the complexity involved by actors preferences and choices, this reinforces the idea that distances can be considered as a key concept in human geography and in spatial economics. Furthermore, there is presently no consensus about the optimal nature of distances in spatial theories, but strong arguments exist to support the idea, and we have exposed them in this contribution.

The optimum of distances is rather counter-intuitively supported by the existence of detours and breaks in trajectories and movements. From the key observation of optimality of distance many implications can be developed with direct implications for transport geography, urbanism and spatial planning. This indicates orientations for further work on distances.

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