2-Sliding Mode Trajectory Tracking Control and EKF Estimation for Quadrotors

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Abstract: This paper addresses the trajectory tracking problem for a quadrotor affected by external perturbations and inaccurate measures. A control strategy is proposed using time scale separation of the translational and rotational dynamics. A second order sliding mode controller is developed for the translational dynamic in order to deal with external perturbations while avoiding the undesired chattering effect. The rotational dynamics are controlled by a linear PD control. A data fusion algorithm using the Extended Kalman Filter (EKF) is proposed to estimate the position and velocity of the quadrotor, taking into account information from multiple commonly used sensors such as an Inertial Measurement Unit (IMU), vision systems and Global Positioning Systems (GPS). Simulations are carried out to test the proposed observer-controller scheme while introducing perturbations and inaccurate measures common in non expensive sensors.

1. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) have gained great relevance in the last years thanks to their huge potential in many civilian and military applications such as exploration, surveillance, search and rescue, in a cheap way and without risking human lives in dangerous situations. Particularly, four rotors rotorcrafts, also known as quadrotors, have received special interest since their rotor's configuration produces cancellation of the reactive torques, considerably simplifying their analysis and control. Also, they are suitable for vertical take off and landing, as well as hovering, making them a good choice for maneuvering in small spaces or perform high precision tasks.

Several stabilization and trajectory tracking control laws have been proposed for this kind of systems and validated through simulations and experiments in indoor applications, relying on a good measurement of the position and velocity. To cite some examples, [1] proposes an attitude stabilization control strategy for hover flight using nested saturations, while in [2] a sliding mode approach is used to accomplish position control of a quadrotor. In [3] a trajectory tracking control by means of a discrete time feedback linearization control scheme is proposed. A second order sliding mode algorithm for attitude control is achieved in [4].

However, big effort is still required to accomplish autonomous flight for outdoor applications, due to presence of external perturbations, especially the wind, and the lack of a good measurement for the position and velocity. Not expensive Global Positioning Systems (GPS) sensors can provide an estimation of the position and velocity, however, the errors, of 2m at best, and their low measurement rate of about 5Hz, are not suitable for precise applications and can interfere with the system stability, even more, GPS can easily lost its signal leaving the system without a position measurement. Another alternative widely studied are the optical flow sensors which use computer vision algorithms for estimating the motion velocity of a system, but they are noisy and sensibles for lighting changes. Data fusion algorithms are an interesting solution to this problem, they take information from multiple sensors, especially GPS, cameras and IMU, to improve the estimation of the position and velocity, for example, in [5] an observer-control scheme for quadrotors using the EKF is proposed and tested in real time indoor experiments, using data from an optic flow algorithm and an IMU (see also [6], [7], [8]).

In this work, it is considered the problem where a quadrotor is used in an exploration outdoor flight mission, where it has to track a desired position under difficult conditions, where the measurement error is big and external perturbations affect the system. An study is presented through simulations, about the effect of such errors on the controlled system behavior. For the control strategy, a time scale separation along with a second order sliding mode control for the translational dynamics and a linear PD controller for the rotational dynamics are used to accomplish the trajectory tracking. In order to improve the performance of the control scheme under bad mea-

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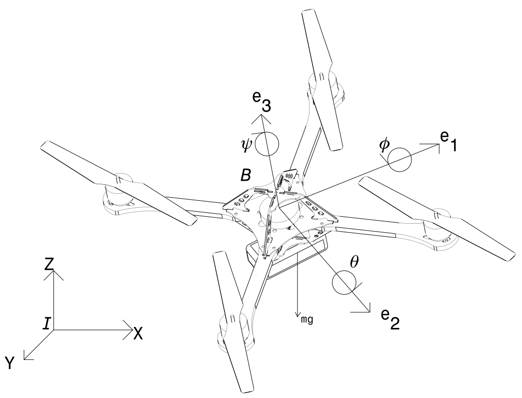

Fig. 1. Quadrotor in an inertial reference frame.

surements, an estimation algorithm is developed by using the EKF to fuse the data from multiple commonly used sensors, such as GPS, vision systems and IMU to measure position, velocity and orientation, respectively.

2. QUADROTOR DYNAMIC MODEL

The quadrotor can be represented as a rigid body in space with mass m and inertia matrix J, subject to gravitational and aerodynamic forces. Let us consider an inertial coordinate frame $I = \{X\ Y\ Z\}$, fixed to the ground and a body fixed coordinate frame, $B = \{e_1, e_2, e_3\}$ (see Fig. 1). Consider the vectors

$$\xi = [x\ y\ z]^T \tag{1}$$

$$\Phi = [\phi\ \theta\ \psi]^T \tag{2}$$

which stand for the position of the center of gravity, with respect to the inertial frame I, and the Euler angles roll, pitch and yaw, respectively. The motion equations are given by the Newton-Euler equations in the inertial frame I [9]

$$m\ddot{\xi} = TRe_3 - mge_3 \tag{3}$$

$$J\dot{\Omega} = -\Omega_x J\Omega + \Gamma \tag{4}$$

where $T \in \Re^+$ is the total thrust of the motors, g is the gravity constant and $\Gamma \in \Re^3$ is the control torque defined in the body fixed frame B. $R \in SO(3) : B \to I$ is the rotational matrix from the body frame to the inertial frame.

$\Omega = [p\ q\ r]^T$ represents the angular velocity in the body frame B. Ω_x stands for the skew symmetric matrix such that $\Omega_x v = \Omega x v$ is the vector cross product. The kinematic relation between the generalized velocities $\dot{\Phi} = (\dot{\phi}, \dot{\theta}, \dot{\psi})$ and the angular velocity Ω is given by [10]

$$\Omega = \begin{bmatrix} 1 & 0 & -s\theta \\ 0 & c\phi & c\theta s\phi \\ 0 & -s\phi & c\theta c\phi \end{bmatrix} \dot{\Phi} \tag{5}$$

3. CONTROL STRATEGY

Assuming that the closed loop dynamics of rotation is faster than the translational one, it is possible to separate the model in two independent subsystems [11]. The strategy consists on designing a controller for the translational

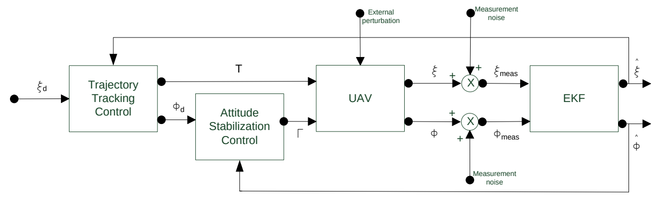

Fig. 2. Control scheme block diagram.

dynamics such that it guarantees the trajectory tracking by providing the desired orientation to an inner control loop. Due to its robustness property and in order to attenuate the chattering effect, a second order sliding mode control have been chosen [12] to deal with uncertainties and perturbations in the translational dynamics, while tracking a desired position. The attitude is controlled by a linear PD controller. Finally, in the next section, an estimation algorithm by means of an EKF is presented, it fuses information taken from multiple sensors to improve the state estimate that is used for the trajectory tracking and attitude stabilization controllers. The whole control scheme is presented in the block diagram at Fig. 2.

3.1 *Second Order Sliding Mode Trajectory Tracking Control*

Defining the desired position ξ_d and the position error $\bar{\xi} = \xi - \xi_d$, and substituting into (3) leads to

$$m\ddot{\bar{\xi}} = (TRe_3)_d - mge_3 - m\ddot{\xi}_d + w \tag{6}$$

where $w \in \Re^3$ is an external disturbance vector.

Let us consider the so called switching function with relative degree 2

$$\sigma = \bar{\xi} + k \int \bar{\xi}\, \mathrm{d}t \tag{7}$$

where $k \in \Re^{3x3}$ is a diagonal positive definite gain matrix. Then calculating the second time derivative

$$\ddot{\sigma} = \ddot{\bar{\xi}} + k\dot{\bar{\xi}} = (TRe_3)_d - mge_3 - m\ddot{\xi}_d + k\dot{\bar{\xi}} \tag{8}$$

Consider now $u = (TRe_3)_d$ to be the control input. Then using the Twisting Algorithm [12], the following discontinous controller is proposed

$$u = m(ge_3 + \ddot{\xi}_d - k_1\dot{\bar{\xi}} - r_1 Sgn(\sigma) - r_2 Sgn(\dot{\sigma})) \tag{9}$$

with

$$Sgn(x) = \begin{bmatrix} sgn(x_1) \\ sgn(x_2) \\ sgn(x_3) \end{bmatrix};\ sgn(x) = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases} \tag{10}$$

and $r_1,\ r_2 \in \Re$ are constant control parameters.

The closed loop dynamics is obtained substituting (9) in (8), it is

$$\ddot{\sigma} = -r_1 Sgn(\sigma) - r_2 Sgn(\dot{\sigma}) + \frac{1}{m}w \tag{11}$$

In order to analyze the stability of the closed loop system, let us consider the Lyapunov candidate function

$$V = r_1||\sigma||_1 + \frac{1}{2}\dot{\sigma}^T\dot{\sigma} \tag{12}$$

which is Lipschitz continuous. Differentiating (12) with respect to time leads to

$$\dot{V} = r_1 Sgn(\sigma)^T \dot{\sigma} + \ddot{\sigma}^T \dot{\sigma} \tag{13}$$

everywhere but on $\sigma = 0$ where V is not differentiable. By substituting the closed loop dynamics (11) in the above

$$\dot{V} = r_1 Sgn(\sigma)^T \dot{\sigma} + (-r_1 Sgn(\sigma) - r_2 Sgn(\dot{\sigma}) + \frac{1}{m} w)^T \dot{\sigma} \tag{14}$$

or, equivalently,

$$\dot{V} = -r_2 ||\dot{\sigma}||_1 + \frac{1}{m} w^T \dot{\sigma} \tag{15}$$

and thus

$$\dot{V} \le ||\dot{\sigma}||_1 (-r_2 + \frac{1}{m} ||w||) \tag{16}$$

if the external disturbance is bounded $||w|| \le ma$, for some $a > 0$, and choosing $r_2 > a$ leads to

$$\dot{V} \le 0 \tag{17}$$

The set S where $\dot{V} = 0$ is given by

$$S = \{(\sigma, \dot{\sigma}) | \dot{\sigma} = 0\} \tag{18}$$

on this set, the trajectories of the system take the form

$$\ddot{\sigma} = -r_1 Sgn(\sigma) - r_2 Sgn(0) + \frac{1}{m} w \tag{19}$$

where $Sgn(0) \in [-1, 1]$. An invariant set on S implies $\ddot{\sigma} = 0$, hence

$$0 = -r_1 Sgn(\sigma) - r_2 Sgn(0) + \frac{1}{m} w \tag{20}$$

selecting $r_1 > r_2 + a$ is enough to assure that the largest invariant set on S contains only the origin $\sigma = \dot{\sigma} = 0$. Using the extended version of the LaSalle invariance principle [13], all the trajectories of the system converge to the origin, and the system is stable. A formal proof for globally equiuniformly finite time stability for this kind of systems with the proposed gains conditions can be found in [14].

So the sliding mode control law (9) solves the trajectory tracking problem despite uncertainties and perturbations. It is important to notice that

$$R_d e_3 = \begin{bmatrix} R_{dx} \\ R_{dy} \\ R_{dz} \end{bmatrix} = \frac{(TRe_3)_d}{T_d} \tag{21}$$

with $T_d = \|(TRe_3)_d\|$. Hence, with a constant ψ_d, it is possible to write ϕ_d and θ_d explicitly as

$$\phi_d = arcsin\left(-\frac{R_{dy} - R_{dx} tan(\psi_d)}{sin(\psi_d) tan(\psi_d) + cos(\psi_d)}\right) \tag{22}$$

$$\theta_d = arcsin\left(\frac{R_{dx} - sin(\phi_d) sin(\psi_d)}{cos(\phi_d) cos(\psi_d)}\right) \tag{23}$$

For the attitude stabilization control, a proportional-derivative controller is proposed, that acts on the orientation error defined by $\bar{\Phi} = \Phi - \Phi_d$, this is

$$\Gamma = -k_{do} \dot{\bar{\Phi}} - k_{po} \bar{\Phi} \tag{24}$$

with the gain matrices $k_{do}, k_{po} \in \Re^{3x3}$ are diagonal positive definite.

A similar control strategy have already been tested in real time indoor experiments [15], where a good position and velocity measurement were available (a video can be watched at: https://www.youtube.com/watch?v=b7A-WgegIuY). To really explode the great potential of UAV in exploration, surveillance, search and rescue applications, it is desired to extend the results to outdoor experiments. However, serious problems arrive due to the lack of a good measurement of the position and velocity of the UAV.

4. ESTIMATION ALGORITHM

This section deals with the issue of estimate the position and velocity of the quadrotor by means of a data fusion algorithm using an EKF, taking information from multiple sensors whose measurements are inaccurate and noisy.

4.1 Extended Kalman Filter

The EKF is a well known, powerful and widely used estimation algorithm for the case of non linear systems of the form

$$\chi_{k+1} = f(\chi_k, u_k) + \omega_k \tag{25}$$

$$Z_k = g(\chi_k) + \nu_k \tag{26}$$

where χ is the state vector and Z is the measurement vector. The process and measurement noise ω, ν are supposed to be Gaussian with known covariance matrices Q and Ω, i.e.

$$\omega \sim N(0, Q) \tag{27}$$

$$\nu \sim N(0, \Theta) \tag{28}$$

Then, the a priori state estimates is given by

$$\hat{\chi}_{k+1|k} = f(\hat{\chi}_{k|k}, u_{k|k}) \tag{29}$$

and the estimated outputs is

$$Y_{k+1|k} = g(\chi_{k+1|k}) \tag{30}$$

The a priory error covariance can be calculated as

$$P_{k+1|k} = AP_{k|k}A^T + Q_k \tag{31}$$

where $A = [\frac{\delta f}{\delta \chi}(\hat{\chi}, u)]$. Now, the Kalman gain matrix can be determined

$$K_{k+1|k} = P_{k+1|k} G^T (G P_{k+1|k} G^T + \Theta_{k+1})^{-1} \tag{32}$$

with $G = [\frac{\delta g}{\delta \chi}(\hat{\chi})]$. After measuring the process, the a posteriori state estimate $\hat{\chi}$ and error covariance P_k are respectively

$$\hat{\chi}_{k+1|k+1} = \hat{\chi}_{k+1|k} + K_{k+1}(Z_{k+1} - Y_{k+1|k}) \tag{33}$$

$$P_{k+1|k+1} = (I - K_{k+1|k} G_{k+1}) P_{k+1|k} \tag{34}$$

4.2 Model Adaptation

Consider the state vector

$$\chi = [\xi \; \dot{\xi} \; \Phi \; \dot{\Phi}]^T \tag{35}$$

and the measurement vector

$$Z = [\xi_{GPS} \; \dot{\xi}_{OF} \; \Phi_{IMU} \; \dot{\Phi}_{IMU}]^T \tag{36}$$

it is, the position measured by a GPS, the velocity obtained by a vision system and an optic flow algorithm (an

extra height sensor is required for the z coordinate) and the orientation and angular speed given by an IMU.
The discrete time process dynamics, considering a sample time T_s small enough is given by

$$\dot{\chi}_{k+1} = \begin{bmatrix} \dot{\xi}_k T_s + \xi_k \\ (\frac{1}{m} T_k R_k e_3 - g e_3) T_s + \dot{\xi}_k \\ \dot{\Phi}_k T_s + \Phi_k \\ \Gamma_k T_s + \dot{\Phi}_k \end{bmatrix} \quad (37)$$

Since GPS sample frequency is slow (5Hz) with respect to the others sensors, and in order to deal with the GPS loss signal, the measured position is updated in the following form

$$\xi_{GPSk} = \xi_{GPSk-n} + n T_s \dot{\xi}_k \quad (38)$$

where n is the number of sample times T_s passed since the last valid GPS data.

5. SIMULATIONS

Simulations were carried out to observe the behavior of the proposed observer-control scheme with external perturbations and inaccurate measurements from a GPS sensor. In order to introduce the errors from the GPS in a more realistic way, actual GPS data was collected under bad conditions for the GPS receiver, it is in an urban environment surrounded by buildings, then this real data was added to the simulated actual position of the quadrotor, considering also the rate of the GPS measures (5Hz) and including signal loss for twenty seconds every 100 seconds. Fig. 3 shows the described GPS measurement error, where the zero values every 100 seconds represent signal losses. A second sensor was considered to measure the velocity, with some white noise (Fig. 4 shows the noisy measured velocity), this can represent for example the measure from an optic flow sensor. Also, it was added an external perturbation ω like the one showed in Fig. 5 to test the robustness of the sliding mode controller.
It is desired to follow a spiral trajectory in the XY plane, centered in the origin, at a constant height of 3 meters.
The performance of the estimation algorithm can be studied through Fig. 6, where the estimation error is presented. It can be noticed that the estimator improve considerable the measured data and perfectly handles with the missing data while GPS signal is lost.
Fig. 7 shows the orientation of the helicopter. Position and tracking errors are presented in Figs. 8 and 9, showing good tracking results despite the measurement error and external perturbations. The control input u is shown in Fig. 10. Finally, Figs. 11 and 12 contain the real and desired position in the XY plane and in three dimensional space.

6. CONCLUSIONS AND FUTURE WORK

Simulations have shown promising results of the proposed control strategy for outdoor flight applications, despite the presence of perturbations and the errors introduced by sensors.
The presented estimation algorithm considerably improves the measured data and automatically deals with the loss data from sensors, helping to improve the performance of

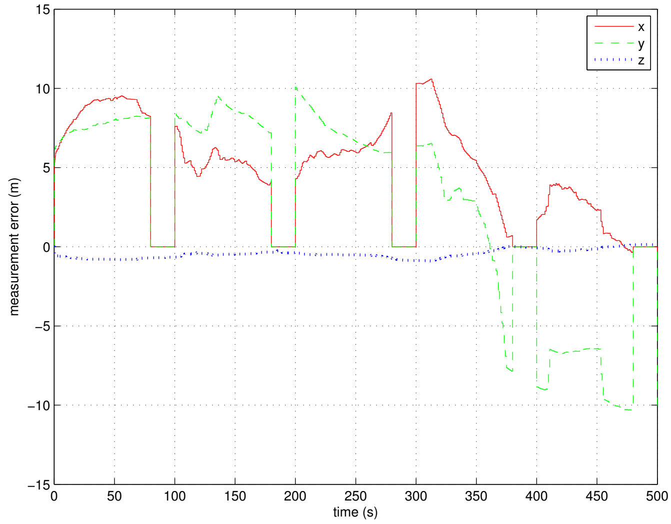

Fig. 3. GPS measurement error.

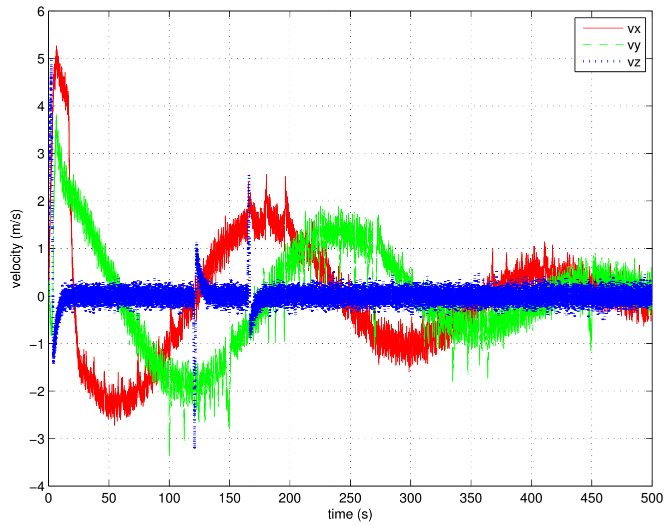

Fig. 4. Measured velocity.

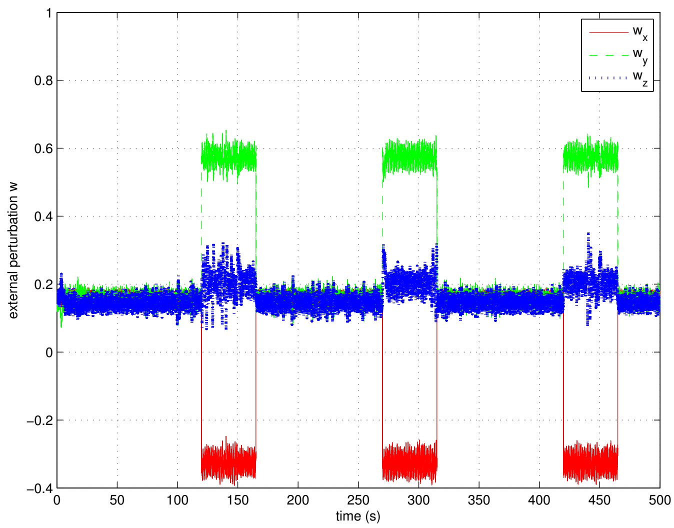

Fig. 5. External perturbation w.

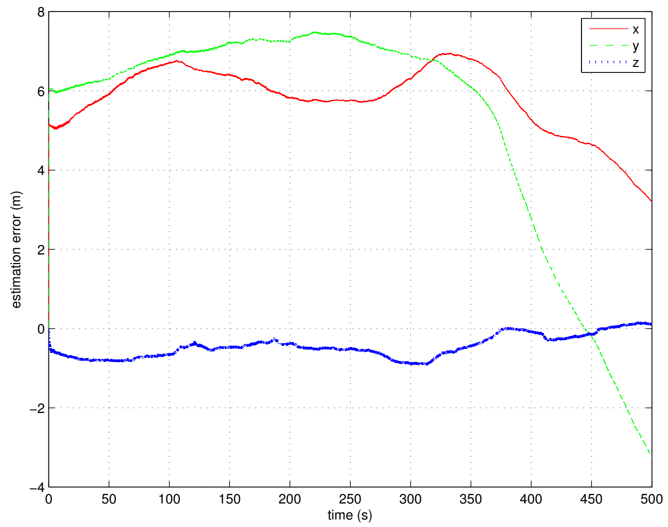

Fig. 6. Estimation error.

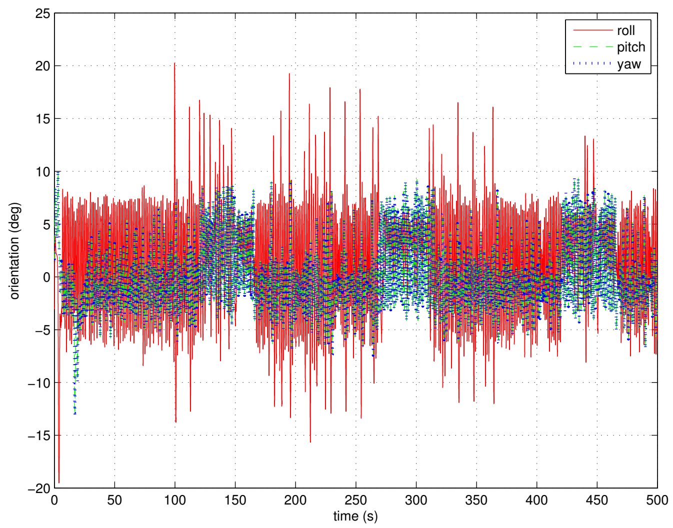

Fig. 7. Orientation.

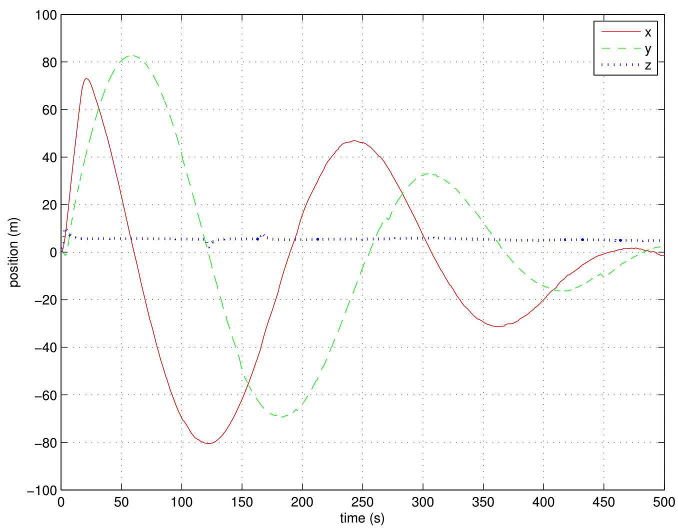

Fig. 8. Position.

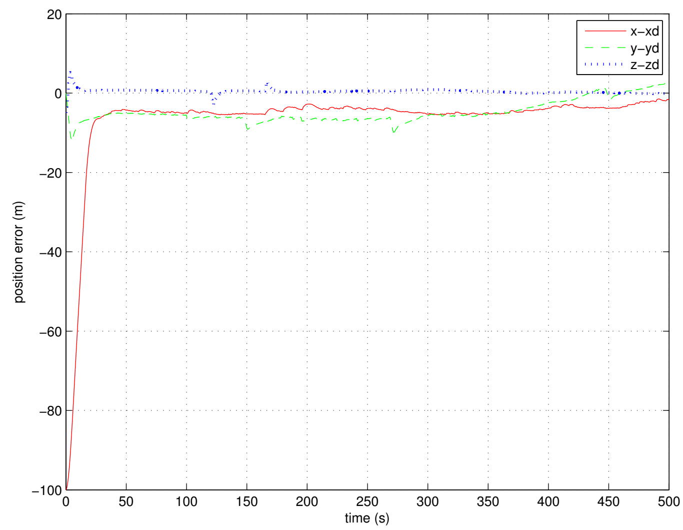

Fig. 9. Tracking error.

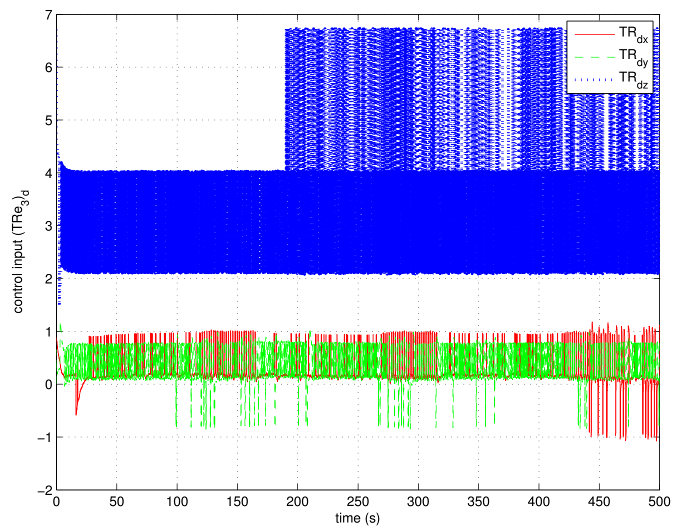

Fig. 10. Control input u.

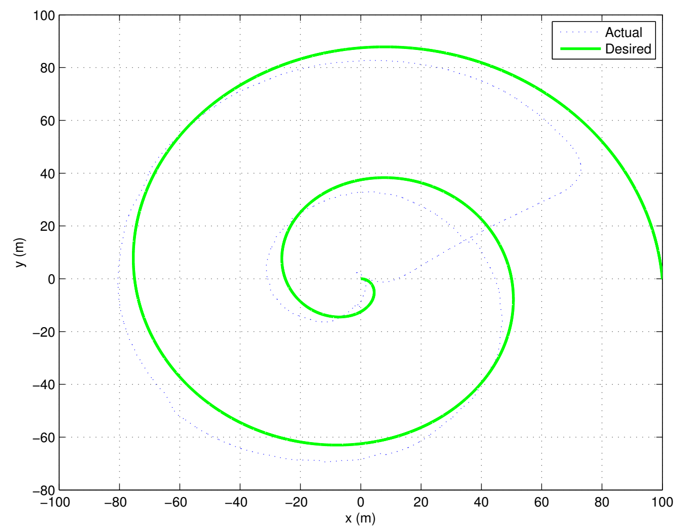

Fig. 11. XY plane.

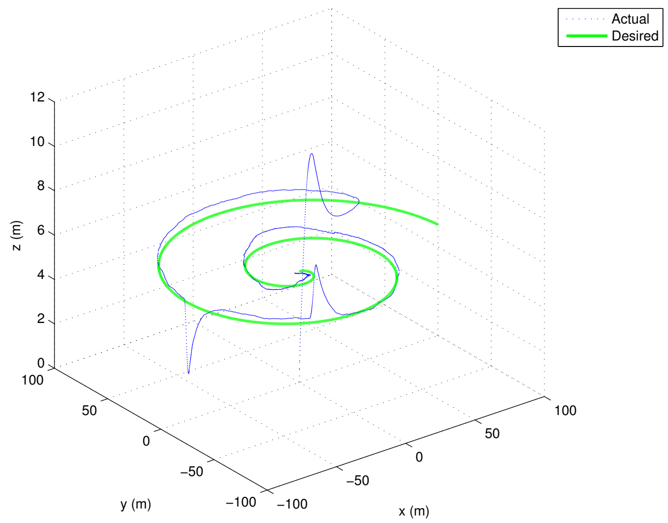

Fig. 12. 3d.

the overall control scheme.
Future work includes to implement the proposed strategy on a quadrotor equipped with a GPS sensor, for real time outdoor experiments.

REFERENCES

[1] P. Castillo, R. Lozano, & A. Dzul. *Modelling and control of mini-flying machines.* Springer-Verlag, Londres, 2005.

[2] R. Xu & Ü. Özgüner. Sliding mode control of a quadrotor helicopter. *En Proc. 45th IEEE Conference on Decision & Control*, 2006.

[3] M. Guisser, H. Medromi, J. Saadi & H. Ifassiouen. Stabilization and trajectory tracking in discrete-time of an autonomous four rotor mini-rotorcraft. *In Proceedings of the 2th International Symposium on Communications, Control and Signal Processing*, 2006.

[4] L. Derafa, L. Fridman, A. Benallegue & Ouldali. Super Twisting Control Algorithm for the Four Rotors Helicopter Attitude Tracking Problem. *In 11th International Workshop on Variable Structure Systems*, Mexico City, Mexico, 2010.

[5] L. E. Munoz, P. Castillo & P. Garcia. Observer-control scheme for autonomous navigation: Flight tests validation in a quadrotor vehicle. *In International Conference on Unmanned Aircraft Systems (ICUAS)*, Atlanta, 2013.

[6] M. George & S. Sukkarieh. Tightly coupled INS/GPS with bias estimation for UAV applications. *In Proceedings of the 2005 Australian Conferences on Robotics and Automation.*, 2005.

[7] W. Ding, J. Wang & A. Almagbile. Adaptive filter design for UAV navigation with GPS/INS/optical flow integration. *International Conference on Electrical and Control Engineering*, 2010.

[8] J. Wang, M. Garratt, A. Lambert, J. Jianguo, S. Han & D. Sinclair. Integration of GPS/INS/vision sensors to navigate unmanned aerial vehicles. *In The International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences. Vol. XXXVII. Part B1.* Beijing 2008.

[9] F. Kendoul, D. Lara, I. Fantoni & R. Lozano. Nonlinear control for systems with bounded inputs: real-time embedded control applied to UAVs. *In Proceedings of the 45th IEEE Conference on Decision and Control*, 2006.

[10] J. Roskam. *A*irplane Flight Dynamics and Automatic Flight Controls, Part I, Roskam Aviation and Engineering Corporation, ch. 2, pp 25-31. USA, 1982.

[11] S. Bertrand, N. Gunard, T. Hamel, H. Piet-Lahanier & L. Eck. A hierachical controller for miniature VTOL UAVs: Desing and stability analysis using singular perturbation theory. In *Control Engineering Practice*, 19, 1099-1108. 2011.

[12] Y. Shtessel, C. Edwards, L. Fridman & A. Levant. *S*liding Mode Control and Observation, Birkhauser, 2010.

[13] D. Shevitz and B. Paden, Lyapunov stability theory of nonsmooth systems. *In IEEE Trans. Automat. Control* 39, pp. 1910-1914, 1994.

[14] Y. Orlov, Finite time stability and robust control synthesis of uncertain switched systems. *In SIAM Journal of Control Optimization 43* 1253-1271, 2005.

[15] D. A. Mercado-Ravell, R. Castro-Linares, R. Lozano-Leal. "Quadrotors Flight Formation Control Using a Leader-Follower Approach". *In European Control Conference*, Zurich, Switzerland, 2013.