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A head-neck-system to walk without thinking
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Abstract

Most of the time, humans do not watch their steps when walking, especially on even grounds. They walk without thinking. How robust is this strategy? How rough can the terrain be to walk this way? Walking performance depends on the dynamical contribution of each part of the body. While it is well known that the head is stabilized during the motions of humans and animals, its contribution to walking equilibrium remains unexplored. To address the question we operate, in simulation, a simplified walking model. We show that the introduction of a stabilized head-neck system drastically improves the robustness to ground perturbations. It significantly influences the dynamics of walking and hence to the balance. Our study is a starting point of a wider research on the involvement of vertebral limbs in the mechanics and control of the human steady gait. It also studies the benefits of such simple control schemes capable to produce complex behaviors.

Introduction

Depending on the nature of the terrain, a walker’s foot placement requires more or less attention. On the top part of the pavement depicted by Figure 1, he/she has to anticipate what stones will next be used for stepping. On the other hand, walking on the pavement indicated in bottom part of the figure 1 does not require any anticipation of the foot placement: in this case we walk without thinking. While step watching clearly requires head and gaze control to see the stones, head control is apparently less important when walking on a flat surface. However neurophysiologists have observed that humans and animals stabilize their head when moving, even on flat surfaces. In former studies, we have shown that head stabilization facilitates the fusion of visual and vestibular information. It offers a consistent egocentric reference frame for motion perception and control in general [1] and for locomotion in particular [2,3]. Our recent results show that head stabilization improves the estimation of the vertical direction by a vestibular-like inertial sensor [4]. Here we contribute to elucidate the role played by the head to maintain balance when walking.

Our novel hypothesis is that the head-neck system plays an important role to balance the human steady gait trajectory, because it not only deals with the tilt estimation within the visuo-vestibular system and the neck position sense, but also it accounts for a substantial contribution to the dynamics of walking due to the head stabilization. Indeed, the head at the top of the body weighs around 8% of the total body mass. It then carries a significant part of the moment of inertia. Thus, head stabilization contributes to reduce the angular momentum of the body. Moreover, it may also be used to balance the walker by controlling reaction torques, similar to tightrope-walkers’ poles.

This study is the starting point of a wider research on the involvement of vertebral limbs in the mechanics and control of the human steady gait and the benefits of such simple control schemes capable to produce complex behaviors. The objective is also to improve diagnosis and therapy of unsteadiness and balance disorders, for example in the case of neck sensorimotor diseases [5,6]. Furthermore, our control models open new routes to design humanoid robots with more efficient and balanced biped locomotion.

We describe briefly hereafter the models and simulations we use to study the influence of the head-neck system on the dynamics of gait. Next, the results obtained with this setting are shown followed by a discussion on our interpretations and conclusions. The remaining of the paper consists of a detailed technical description of the materials and methods used to obtain our results.
Models

Based on mechanical concepts from passive robot walkers [7,8], we introduce a walking simulation scheme promoting the notion of ground texture as a key factor to study walking balance. Two simple walking mechanical models are then compared. These models include improvements to classical compass-like walkers, by adding torso, interleg actuation, spring-damper at the feet, and rough terrains. These additions are active research topics, but their effects and dynamics are beyond the scope of this paper. We refer then to [7,9–13] for dedicated studies.

In the first of our two models, the walker has a rigid neck and tends to stabilize the torso upright. In the second one the neck is modeled as a limb of two joints and the walker tends to maintain the head direction constant. Both walker models are inspired by the mechanical design of passive walking robots [7]. The energy efficiency of these robots, the low-frequency of their control and their natural limit-cycle dynamics are common characteristics with human locomotion [14,15].

Indeed, we do not aim at modeling perfectly the human gait. Up to now, only simple dynamical models allow to reproduce locomotion gaits [16]. Dynamical modeling of human walking is out of reach of all current simulators. In our study, we aim at reproducing a broad dynamics of an anthropomorphic walker in a steady limit-cycle gait, similarly to what is done in [17,18].

Figure 2 illustrates our model. It operates in the sagittal plane. It is made of five articulated rigid bodies: two bodies for the (knee-free) legs, one body for the torso, one for the neck and one for the head. Note that the neck is modeled as an articulated body and not as a simple joint. This setting reflects the property of the head-neck system to have two centers of rotation in the sagittal plane: one at the base of the neck and the other at ear level [19]. The mass distribution and the limb lengths are anthropometric (e.g., [20]).

The first walker (Model A) we consider, has a rigid neck, i.e. the torso, the neck and the head make a single rigid body. The second walker corresponds to the model of head stabilization (Model B): the neck joints are controlled to maintain a zero tilt for the head. Apart from the neck, both walkers have the same controls: the torso is actuated to be stabilized upright while a lightweight controller actuates the inter-leg angle. Finally a velocity driven foot impulsion is given just before the swing phase. The weakness of the control of the lower limbs make them sensitive to perturbations, and their dynamics can differ according to upper-body control.

Both models need to know the orientations of the upper-body segments in order to stabilize them. The model A has a unique orientation to be observed and controlled, since it has a rigid upper-body. The model B has three segments with different orientations. Nonetheless, if we know the orientation of one segment and the two joint-positions of the neck, we can reconstruct the two other orientations. So from sensing viewpoint, for both models, we place a tilt estimator in the head, while neck and torso inclinations are deduced from neck joint angles. This model of sensing is inspired by neurophysiology. Humans have an efficient orientation sensor in the head, including the vestibular systems. They have also a good accuracy in the estimation of neck joint angles, as the neck possesses a dense concentration of proprioceptors [21]. The estimation of the body segment directions is then obtained by a top-down combination of the neck joints with the head direction given by the vestibular system [1].

Detailed description of the models is presented in the Materials and Methods section.

Simulations

Head stabilization has two types of effects. The first one is mechanical: the head has a non-negligible mass contribution, and the motion of the head influences the dynamics of the body. The second one lies in attitude estimation: different motions generate different inputs on the tilt sensors and may lead to different magnitudes of estimation errors. For the purpose of isolating each effect, we study two sensor models for each walker model. The first one is an ideal tilt sensor, i.e. the head direction is perfectly...
known. The second one is a simplified model of the vestibular system as presented in [4]: it combines noisy measurements of an accelerometer, a gyrometer and an inclinometer. The ideal sensor makes the walking models equivalent in terms of sensing and it enables us to compare them only from a mechanical point-of-view. The vestibular model permits to reproduce the effects of head motion on the accuracy of tilt estimation, and to study their impact on gait balance.

Performance comparison is based on a metric called Mean First Passage Time (MFPT) [8]. MFPT is derived from the classical analysis of metastable systems [22]. This metric aims at providing an estimation of the distance a walker may travel safely. MFPT of walking systems is the average number of steps the walker makes before falling. However, in the context of walking simulation, MFPT estimation is highly time consuming. To overcome this bottleneck we had to introduce a new dedicated simulation algorithm based on limit-cycle properties [23].

Results

For noise-free environments and sensors, a well-controlled walker may walk indefinitely without falling. In other words, its MFPT is infinite. Perturbations, such as uneven terrains or noisy sensors, give rise to finite MFPTs, i.e. the walker necessarily falls at some stage. In this context, we introduce the notion of textured ground. A textured ground is a terrain for which the unevenness follows a probability law. The texture model consists in changing the slope of the ground at each step, following a centered Gaussian distribution. We can make the terrain more or less rough by changing the standard deviation (SD) of the Gaussian law.

We conducted simulations on four models resulting from the combination of the presence or absence of head stabilization with ideal or vestibular-like tilt sensor respectively. Each model was simulated on a set of different textures (from 0 to 0.1 rad of SD). For each model-texture combination, we computed the corresponding MFPT. The results of these simulations are described hereafter.

Noise-free sensor

In the case of an ideal attitude sensor, on flat terrain, and for both control models, it has not been possible to find an upper bound on MFPTs (see Fig. 3). This confirms the fact that, in the absence of perturbation, a well-controlled walker may walk indefinitely without falling. However,walker performances greatly differ as soon as a slight texture change appears. The phenomenon can be seen from the example of 0.01 rad standard deviation. In this case, MFPT of the rigid neck model is 23 steps, while head stabilization guarantees MFPT of more than 3 million steps! This performance improvement persists as the ground texture increases, even if the difference declines. This is purely due to mechanical effects, i.e. to the contribution of the head motion to the balance of the gait.

To better understand this effect, we compare the mechanical powers of A and B models during the period between two impacts of the limit cycle on level ground (Fig. 4-A). The two curves have almost the same integral, which means that they absorb a same amount of the energy between two feet impacts. The important feature is the difference in power consumption during the first 30% of the cycle. The head stabilization enables the walker to absorb less energy during the impact, while distributing the energy consumption over a longer period.

One of the consequences of this delay in energy absorption is on the dynamics of the swing leg. For both models, at the impact, a part of the momentum is transferred into a rotation motion around the new stance foot. But this transfer applies forces and moments on the upper-body. These forces tend to tilt the upper body which compensates this inclination using the torque actuator at the hip. Hip torques absorb an important amount of kinetic energy, and apply moments on the new swing leg which tend to slow its angular velocity. The head-neck system enables to delay a part of this absorption and let the swing leg have a greater initial velocity (Fig. 4-B). Later in the cycle, even if the same amount of energy...
is absorbed due to the delay, the walker has already reached a more stable configuration where the swing leg is already prepared for the next step.

Indeed, in the case of human gait, as for our simulated models, imbalance caused by uneven and textured ground result mostly in forward falls [24]. A forward fall happens when the swing foot reaches the ground too close to the stance foot, and cannot compensate for the destabilizing linear and angular momenta. When we increase the swing leg velocity in the beginning of the cycle, we enable the walker to reach earlier a balanced position with a forward leg better prepared for the next impact.

This effect suggests that the head-neck system is used to store the mechanical energy and to restore it at the appropriate phase of the walking cycle. These results give an insight into the role of the vertebral column in the management of gait mechanical energy. The whole upper-body inertia and elasticity would store energy and restore it at the right instant, similarly to [25] where the walking model which walks on level ground with zero-energy cost.

As this level we may conclude that head stabilization improves the dynamic balance of walking systems. We show next that the improvement is even better when considering a noisy gravity estimator.

**Inertial sensor**

If we consider a walker’s head equipped with an on-board vestibular-like sensor, the head stabilization minimizes the error of gravity direction estimation [4]. We show here how head stabilization greatly improves the robustness to the estimation error when compared to trunk stabilization. To avoid a combination of mechanical and sensing effects, we compare separately the two sensor models for each walking model. The difference shown in Fig. 5 between the performance of the noise-free sensor walker and the inertial one is only due to a loss in the accuracy of the verticality estimation.

Compared to the noise-free sensor, the root-mean-square error of the tilt estimation is of $3.5^\circ$ for Model A and of $0.78^\circ$ for Model B. This confirms that head stabilization contributes to a better accuracy of the gravity estimation.

More interesting is the influence of this accuracy on gait balance. Small errors induce important differences in the performances of walking systems on rough terrain as shown in Figure 5: MFPTs for Model B is much greater than MFPTs for Model A and the property holds for any ground texture. Finally, MFPT on a horizontal surface is greater than $10^5$ when the head is stabilized, while it is less than 10 for Model A: we see clearly the importance of head stabilization, and how the difference with an ideal sensor becomes negligible on small textures.

**Discussion**

Our model-based approach of bipedal walking reveals the contribution of head-neck control to the gait dynamics. The effect of head stabilization is twofold. First, it impacts sensing. The dynamics of vestibular-like sensors relies on inertial and gravitational effects, which are driven by accelerations and rotations of the head. Stabilizing the head enables the walker to reduce excitation and to maintain the sensors in local dynamics for a finer posture-estimation.

Second, the effect of head stabilization is mechanical. Head stabilization is an heuristic answer to the question of taking advantage of the head mobility during walking. Indeed, while it is likely not the optimal control of the neck regarding balance, it is a very simple control that produces a complex behavior with significant benefits. It is worth to emphasize that the importance of this mechanical effect is also due to the weakness of lower limbs actuation, which makes them highly sensitive to external forces. Indeed, other stiffer controls can be found for which the mechanical contribution of the head stabilization is reduced. However, stiffer controllers implies increasing energy consumption and control frequency, which does not reflect the natural energy-efficiency of walking without thinking on low textures, where the walkers take maximum advantage of the natural passive dynamics of the body. On the contrary, stiff actuation and high
frequency control are required to guarantee balance for highly uneven and unpredictable environments, and in this case head stabilization is not expected to play such an important mechanical role.

These results fit with clinical observations on humans. The unsteadiness and the loss of balance resulting from head-neck system sensorimotor disturbances have been widely documented. Deficiency in neck-joint position sense, as well as its motor abilities have an important impact on balance, whether in chronic neck injuries [5, 6] or in one-time experimental induced impairment [26, 27]. It has even been suggested that the impairments in the neck somatosensory inputs and sensorimotor control are as important for balance as a lower-limb proprioception loss following a knee or an ankle injury [28]. Therefore, our study certainly opens new clinical perspectives in diagnosis and therapy.

Finally, other challenging issues remain to be addressed. Similarly to head stabilization and arms swinging, the vertebral limbs, by their mass, flexibility and control, surely have -yet unexplored- dynamical effects on steady gait. Furthermore, coming back to Figure 1, other questions arise. What is the critical value of the ground texture that imposes to watch one’s steps? How does the brain switch from a locomotion modality to another one when crossing the line separating the two pavements?

Materials and Methods

This material and methods section is structured as follows. We first discuss neuroscience and robotics models for human walking and we describe how passivity-based walkers gather many of its properties regarding dynamics and control. We describe the walkers models we study in terms of geometry and mass distribution. We then describe the control of the actuators of our models. We present the vestibular sensor model and the state observer we use to reconstruct the attitude of the walker. We switch then to a brief description of the method we introduced to estimate the balance performances of our walkers. We finish by describing briefly the simulation framework and the software environment we use in this study.

Passive-dynamics walkers

Neuroscience has showed a great will to understand the neurophysiological process of gait generation for humans. Several researchers suggest that locomotion patterns in vertebrates, including humans, are mainly generated within the spinal cord rather than in the brain. This generation is usually modeled by the combination of a rhythm generator called Central Pattern Generator (CPG) and reflexes in response to peripheral stimuli [29]. This provides the walker with an attractive limit-cycle behavior when no perturbations occur.

This model is not the basis of gait control for the vast majority of biped robots. For them, keeping balance reduces to respect a criterion related to a point on the ground, defined by the contact forces, called Zero Moment Point (ZMP). The criterion is to keep the ZMP strictly inside the surface beneath support limbs [30]. Several approaches showed then the possibility to control the ZMP position to enable safe locomotion [31]. However, the walking control using these approaches is usually energy inefficient, especially compared to humans. Moreover, the ZMP constraint needs to be respected at each instant, requiring high-frequency control. Frequencies which are far beyond the capabilities of the humans muscular system [14].

On the contrary, passive-dynamic walking robots have remarkably close energetic efficiency to humans’, without complying to the ZMP balance criterion. The limit-cycle walking of a passive-dynamic robot is also considered as a suitable model for the human natural gait [32]. In addition, passive-dynamic walkers are commonly assumed to generate visually more human-like motions, even with the presence of geometrical or mechanical discrepancies between the robot and humans [7]. Accordingly, this similarity has attracted the interest of biomechanics in order to study the passivity properties of the human gait [15].
The walker models

The models A and B we simulate are the same planar 5 limbs walker (Fig 2). The limbs are: a head with a mass on the top, a neck and a torso with masses at the middle, and the legs, each of which with a mass at distance \( l \) from the hip. Each two successive limbs are attached with a rotational joint. The difference between the models A and B is that the model A has fixed neck, forming a straight rigid upper-boy (see Figure 6). Each leg is ended with a prismatic joint equipped with a spring-damper. We call toe, the bottom of the mobile part of the leg. The hip-toe length \( l_p \) at the rest position of the spring is denoted by \( l_{p,0} \). The masses distribution and limbs proportions are anthropometric [20] (see Table 1).

Control

During the simulations, the models are always walking, there is always at least a contact between a toe and the surface of the ground. If this condition is not satisfied, the robot is considered as fallen.

Except for toe off impulsion, the controllers for the robot are proportional-derivative (PD), each of them has two gain parameters. The gains for lower body are chosen to be lightweight, to simulate the low energy consumption of human’s steady walk. The upper body has to a bit stiffer to guarantee a successful vertical stabilization for the trunk and the head.

There are two ways for obtaining the values of the control gains. The first and the best one is to use optimization techniques to find out the best parameters of each model according to a balance criterion. However, our criterion, MFPT, is too long to compute and its optimization would take years of computation. The second method, to which we have to resort, is to take the values arbitrarily in the set of values guaranteeing stable steady gait on flat ground. If for several samples of these values, the stabilization phenomenon is still observed, that means that there is a strong probability that it is a general effect on the class of weakly actuated gaits. In our tests, among many reasonable gain values used, the studied effect remains visible as soon as the walkers are stable on flat surface.

The parameters we present hereafter are the values used to obtain the numerical outcomes described in Results section.

Toes. The exchange between the swing phase and the stance phase occurs at impacts. Impacts are considered inelastic and contacts are considered perfect with no slipping. The toe of the stance leg has a spring-damper dynamics. The contact force follows the direction of the stance leg and its magnitude has a proportional-derivative (PD) expression:

\[
f_t = -K_{toe,p}(l_p - l_{p,0}) - K_{toe,d} \dot{l}_p
\]

where \( K_{toe,p} = 5000 \) N/m is the elasticity of the spring and \( K_{toe,d} = 200 \) Ns/m is the damping factor. This force is applied only when it is positive because of the unilateral force constraint of the contact (the ground cannot pull the body).

When a leg is in a swing phase, its toe comes back instantly to the rest position \( l_{p,0} \) of the spring, and remains constant until the end of the swing phase. We denote then simply by \( l_p \) the length of the stance leg.

The walkers loose a part of their mechanical energy at each impact. They require then to be actively fed with an external source of energy. At the instant of take-off of the stance leg, a velocity controlled impulsion is applied to the ground to give propulsion to the robot (Figure 6). The required force for this impulsion is \( f_t \):

\[
f_t = h(\dot{l}_{p,r})
\]

where \( \dot{l}_{p,r} = 1 \text{ m.s}^{-1} \) is the desired velocity and \( h \) is the controller function. Most modern dynamic simulators provide implementations of velocity-driven motors. The dynamical simulator solves the problem of finding the exact force that has to be applied to the joint during one time-sample in order to reach the
desired velocity. This gives an automatic computation of \( h \) which has no closed form. We use it then to apply the force \( f_t \) during one time-step of simulation.

The idea of velocity-driven impulsion is to compensate for energy loss of foot impact. Moreover, by having a constant toe-off velocity at the beginning of each step, we ensure to start every step with comparable levels of energy and momentum. Therefore the impulsion force is smaller for high velocity stepping and bigger for low velocity ones, and this helps to reject a part of the deviations from the limit cycle.

In addition, the propulsion during human gait is of course a subject of debate, but several studies show the role of calf and ankle in the propulsion during the support phase [33,34], their contribution to swing leg initiation [35,36] and their involvement in the control of trunk dynamics [37]. We wanted then to reproduce a part of these effects with our simplified model of toe-propulsion.

**Interleg joint.** The inter-leg joint is controlled by a PD pure torque generator toward a reference angle:

\[
\tau_{\text{hip}} = -K_{\text{hip},p}(\theta - \theta_r) - K_{\text{hip},d}\dot{\theta}
\]  

where \( K_{\text{hip},p} = 1 \text{ N m/rad} \) is the proportional gain, \( \theta_r = 0.3 \text{ rad} \) is the reference angle and \( K_{\text{hip},d} = 0.15 \text{ N m s/rad} \) is the derivative gain. In order to preserve the natural dynamics of the legs the values of the gains are small.

**Trunk.** On the other hand, we need to maintain the upper-body globally upright. There are two major approaches that enable passive walkers to control an upper-body. First, the bisector constrained walker, introduced by Wisse et al [38], is a compass with an upper limb that is constrained to be in the midway angle of the two legs, a successful real design was presented using chains, and has the advantage to be entirely passive. However, beside the fact that it is not an accurate model to human walking, its bisecting constraint introduces an instability, especially in the presence of a heavy upper body [39], which is the case for humans. The second method is to stabilize actively the upper body against the vertical, which better models the human gait [40]. The stabilization is achieved either by applying torque on both legs [41] or more commonly on the stance leg only [9]. We choose this last solution to not disturb the passive swing motion. The trunk torque is actuated by a pure torque generator, controlled by a PD that brings back the torque to vertical (\( \alpha = 0 \)):

\[
\tau_{\text{trunk}} = -K_{\text{trunk},p}\alpha - K_{\text{trunk},d}\dot{\alpha}
\]  

where \( \tau_{\text{trunk}} \) is the trunk-stance-leg torque, \( K_{\text{trunk},p} = 30 \text{ N m/rad} \) is the proportional gain and \( K_{\text{trunk},d} = 15 \text{ N m s/rad} \) is the derivative gain.

**Head stabilization.** For the model B, there are two other controllers, which are the neck stabilization and head stabilization, they are also controlled by PD pure torque generators. Their torques expressions are the following:

\[
\tau_{\text{neck}} = -K_{\text{neck},p}\gamma - K_{\text{neck},d}\dot{\gamma}
\]  

\[
\tau_{\text{head}} = -K_{\text{head},p}\beta - K_{\text{head},d}\dot{\beta}
\]  

where \( \tau_{\text{neck}} \) is the torque applied to the torso-neck joint, \( K_{\text{neck},p} = 5 \text{ N m/rad} \) is the neck proportional gain, \( K_{\text{neck},d} = 0.06 \text{ N m s/rad} \) is the neck derivative gain, \( \tau_{\text{head}} \) is the torque applied to the neck-head joint, \( K_{\text{head},p} = 15 \text{ N m/rad} \) is the head proportional gain and \( K_{\text{head},d} = 0.1 \text{ N m s/rad} \) is the head derivative gain.
Sensors

The walking models use two models of tilt sensors, both placed in the head. The first one is an ideal sensor, which gives perfect orientation $\beta$ and angular velocity $\dot{\beta}$ measurements. These are directly provided by the dynamic simulator. This sensor does not require any observation technique to retrieve the state of the walker.

The second model of sensors is a 2D version of the model of vestibular system described in [4]. It uses a set of inertial sensors giving noisy measurements that are provided to a Kalman filter in order to reconstruct the attitude of the head. This sensor will be described in depth hereinafter.

For both models, the orientation and the angular velocities of the other limbs $\alpha, \dot{\alpha}, \gamma, \dot{\gamma}$ are reconstructed by the use of articular angles and angular rates which are considered perfectly accurate. These estimations are used in the control of the upper body of the walker models.

The vestibular sensor

This sensor consists of:

1. An accelerometer, it provides noisy measurements of the acceleration and gravity in the head’s frame

\[
\begin{pmatrix}
\ddot{a}_x \\
\ddot{a}_z
\end{pmatrix} = \begin{pmatrix}
\ddot{p}_x \cos \beta - (\ddot{p}_z + g) \sin \beta \\
\ddot{p}_x \sin \beta + (\ddot{p}_z + g) \cos \beta
\end{pmatrix} + \begin{pmatrix}
w_1 \\
w_2
\end{pmatrix}
\]  

(7)

Where $(p_x, p_z)$ are the coordinates of the head in the global frame, $g$ is the standard earth’s gravity. $w_1$ and $w_2$ are the measurement noises.

2. A gyrometer which provides noisy estimations of the rotation of the head:

\[ r_h = \dot{\beta} + w_3 \]  

(8)

where $w_3$ is the measurement noise.

3. An inclinometer, which provides the inclination of the head with a slow dynamics, we denote this data by $\nu$, the sensor gives then a noisy estimation of $\nu$.

\[ i_h = \nu + w_4 \]  

(9)

where $w_4$ is the measurement noise.

The dynamics of the inclinometer is modeled by the dynamics of a damped pendulum of length $l_i = 2.5 \times 10^{-3}$ m, mass $m_i = 6 \times 10^{-4}$ kg and damping factor $K_{i,d} = 2.5 \times 10^{-10}$ N m s/rad (see Fig. 7).

Due to the small mass and dimensions of the pendulum, we consider the influence of the motion of the inclinometer on the walker’s dynamics as negligible. But the motion of the head influences the dynamics of the inclinometer. We describe the dynamics of $\mu = \nu + \beta$ the inclination of the pendulum in the global frame:

\[
\ddot{\mu}_i = \frac{-m_i l_i g \sin \mu - m_i l_i (\ddot{p}_x \cos \mu + \ddot{p}_z \sin \mu) - K_{i,d}(\dot{\mu} - \dot{\beta})}{I_i}
\]  

(10)

where $I_i = m_i l_i^2$ is the moment of inertia of the pendulum.

The accelerometers and the gyrometer represent the phasic system of the vestibular system, which detects mostly peaks in acceleration and jerk. For this reason, we give them higher noise levels but fast dynamics. The inclinometer, with a lower-level noise, reflects the linear dynamics of the tonic system at
lower-frequencies [42]. Therefore, we make sure that the measurement noise vector \( w = (w_1 \ w_2 \ w_3 \ w_4)^T \) follows a multivariate centered Gaussian white noise, with the following covariance matrix:

\[
R = \begin{pmatrix}
2.5 \times 10^{-3} & 0 & 0 & 0 \\
0 & 2.5 \times 10^{-3} & 0 & 0 \\
0 & 0 & 9 \times 10^{-2} & 0 \\
0 & 0 & 0 & 3 \times 10^{-4}
\end{pmatrix}
\] (11)

Other modeled dynamics

In order to use these measurements to reconstruct the head orientation \( \beta \), we need to have a model of the dynamics of \( \beta \). The torque on the head \( \tau_{\text{head}} \) is known, because it is provided by the on-board stabilization controller. The orientation dynamics is described by the following differential equation:

\[
\ddot{\beta} = \frac{\tau_{\text{head}} + gm_h(c_x \sin \beta + c_y \cos \beta)}{I_{\text{head}}} + v_1
\] (12)

where \( I_{\text{head}} = m_h l_h^2 \) is the moment of inertia of the head, \( v_1 \) is the process noise and \((c_x, c_y)\) are the coordinates of the head’s center of mass (CoM) in the head’s reference frame which is located at the head-neck joint. This model of \( \beta \) is wrong, and this is not how we simulate the dynamics of the head. This model is just used to predict the head dynamics by using only given data, state variables and known inputs. The error between this model and the real dynamics are integrated to the process noise \( v_1 \).

Finally, we don’t know the dynamics of the linear accelerations of the head \((\ddot{p}_x, \ddot{p}_z)\). We make then the simplest assumption by considering that this acceleration is zero. The process noises \( v_2 \) and \( v_3 \) represent the committed modeling error:

\[
\begin{align*}
\ddot{p}_x &= v_2 \\
\ddot{p}_z &= v_3
\end{align*}
\] (13, 14)

Kalman filtering for observing the orientation

We use an extended Kalman filter to exploit the signal vector \( y = (a_1 \ a_2 \ r_h \ \dot{i}_h)^T \) and the dynamics models to reconstruct the inclination of the head \( \beta \). Therefore, we define the sensor state vector \( s = (\beta \ \dot{\beta} \ \mu \ \dot{\mu} \ \ddot{p}_x \ \ddot{p}_z) \).

The state’s dynamics and the measurements are time-discretized:

\[
\begin{align*}
s_k &= f(s_{k-1}, \tau_{\text{head},k-1}) + v_k \\
y_k &= g(s_{k-1}) + w_k
\end{align*}
\] (15, 16)

where \( k \) is the index of the current time-sample, \( v_k \) is the process noise and \( f \) and \( g \) are the functions describing the state dynamics and the measurements, respectively.

The modeling errors of the function \( f \) are represented by process errors. The process error vector \( v_k = (0 \ v_{1,k} \ 0 \ 0 \ v_{2,k} \ v_{3,k})^T \) follows also a multivariate centered Gaussian white noise, with the following covariance matrix:

\[
Q = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.125 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \times 10^4 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \times 10^4 & 0
\end{pmatrix}
\] (17)
Once these definitions given, we can describe the observer. The extended Kalman filter enables to reconstruct the state $s$ by minimizing the quadratic error. The estimation of $s$ is denoted $\hat{s}$ and its update equations are:

\[
\hat{s}_k^- = f(\hat{s}_{k-1}, \tau_{head,k-1}) \tag{18}
\]
\[
P_k^- = F_k P_{k-1} F_k^T + Q \tag{19}
\]
\[
K_k = P_k^- G_k T (G_k P_k^- G_k^T + R)^{-1} \tag{20}
\]
\[
\hat{s}_k = \hat{s}_k^- + K_k (y_k - g(\hat{s}_k)) \tag{21}
\]
\[
P_k = (I - K_k G) P_k^- \tag{22}
\]

where $\hat{s}_k^-$ is the prediction of the $k$-th state, $K_k$ is the Kalman gain, $P_k^-$ and $P_k$ are the error covariance matrices of $\hat{s}_k^-$ and $\hat{s}_k$ respectively.

We choose to set the initial value of the state to $s_0 = 0$ and the initial error covariance matrix to:

\[
P_0 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 100 & 0 \\
0 & 0 & 0 & 0 & 0 & 100
\end{pmatrix} \tag{23}
\]

**MFPT estimation**

MFPT of a walking system, is the mean number of steps it makes before to fall. This balance metric has been introduced by Byl and Tedrake [8]. However, the proposed methods are either to use Monte-Carlo sampling, or to discretize all the state-space and to use Markov chain model. The first method is time-consuming especially for rare falls. The second method can only be used for low-dimensional walking systems and cannot be extended to complex walkers.

For this reason, we propose a method which relies on the presence of an attractive limit-cycle [23]. This method can be used for complex systems and takes much lower computation time than Monte-Carlo sampling.

The dynamics of walking systems is cyclic. At each step, there is an impact and a swing phase. If we know the pre-impact state of the walker, all the state evolution is predictable during this state until the following impact. This property is particularly interesting on textured ground, because the next impact itself is not predictable when the slope is uncertain. Therefore, it is enough to have the sequence of pre-impact states to be able to reconstruct all the dynamics of the walking system. The pre-impact states constitute a manifold called Poincaré section and the intersection of the dynamics with this manifold is called a first recurrence map or a Poincaré map.

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The state vector of the robot is: $\xi = (\theta, \dot{\theta}, \phi, \alpha, \dot{\alpha}, \gamma, \dot{\gamma}, \beta, \dot{\beta}, l_p, \dot{l}_p)$. The limit cycle has then one intersection with the Poincaré section in a given state $\xi_l$ (see Table 2 for the values of $\xi_l$ for our models, note that the displayed precision is necessary as the dynamics is sensitive to perturbations in the value of the limit-cycle). However, the walking dynamics stops intersecting the Poincaré section when the walker falls. For simplifications, we gather all the fall-states into one symbol $\xi_f$ (see Fig 8). The number of steps is then the number of times the walker’s state intersects the Poincaré section before to reach $\xi_f$.

The estimation of MFPT proceeds as follows:

1. use sampling technique, each sample starts with initial state $\xi_l$, simulate the dynamics, and stops either when the walker comes back to the limit cycle $\xi_l$ (success) or falls and reaches $\xi_f$ (failure).

2. for each sample, record the number of steps, and the “success” or “failure” answer.
3. obtain estimations of \( p_f \), the probability of a sample to fail, \( n_f \) the mean number of steps of a failure and \( n_l \) the mean number of steps of a success.

4. compute \( r = \frac{1-p_f}{p_f} \) the mean number of returns to the limit cycle before to fall.

5. MFPT is then estimated by \( n = r n_l + n_f \).

A “success” sample is a simulation where the walker comes back to the limit cycle. However, the probability that any walker’s state is exactly equal to \( \xi_l \) is zero. This means that the walker almost never actually reaches the limit cycle state. Thus, instead of checking equality with \( \xi_l \) in order to detect the return to the limit-cycle, we check if the current state is inside a given neighborhood \( \Xi_l \) of \( \xi_l \).

Consequently, before to start estimating MFPT, our method requires two data: the limit-cycle state \( \xi_l \) and a suitable neighborhood \( \Xi_l \). To obtain them, we make a rough sampling of 8000 states in the state-space of the walker, and we simulate the walking motion, on flat ground, until it falls or it converges to the limit-cycle. If a simulated motion does not fall, we assume that an accurate convergence to the limit cycle happens after 400 steps. In a perfect environment and simulation, the walker’s state is supposed to converge to a unique limit value \( \xi_l \). But due to numerical rounding and to simulation errors, the states have still multiple values even after the convergence. So for each viable sample, we record the states between the 400th and the 500th step. This gives a set of \( \xi_i \) of recorded states after the convergence. This set of states describes a small area in the state space within which the dynamics of the walker’s state changes: it is no longer attracted to a limit-state \( \xi_l \), but it generates a local chaotic behavior. We call this area the limit kernel. This area is attractive, in the sense that, on flat ground, when the state is outside of it, the dynamics will make it converge to the limit kernel, and in case the walker doesn’t fall, the state will enter the area in a finite number of steps with probability 1. Moreover, if the state is inside the limit kernel, it will never go out, in the absence of disturbances. The limit kernel is then the suitable definition for our neighborhood \( \Xi_l \).

Even in the absence of a unique value of the limit-cycle state, the simulation still requires an estimation of \( \xi_l \), because it is the initial state of the walker when estimating MFPT. We approximate \( \xi_l \) by the mean value of the set of \( \xi_i \) (see Table 2).

The limit kernel \( \Xi_l \) is approximated by a multidimensional ellipsoid such that:

\[
\xi \in X_l \iff (\xi - \bar{\xi}_i)^T C (\xi - \bar{\xi}_i) < d_0
\]  

where \( C = (\text{cov}(\{\xi_i\}))^+ \) is the semi positive definite matrix defined by the Moore-Penrose pseudoinverse of the covariance matrix of the set \( \{\xi_i\} \), and \( d_0 \) is a threshold defining the size of \( \Xi_l \). For our case, we take \( d_0 = 1000 \).

More details and discussions of this approach, comparisons with existing methods and generalizations to cases of bifurcations and chaos are available in [23].

**Simulations**

Simulations are performed in a tailored c++ framework for simulating passivity based walkers. The resolution of the dynamics integration is achieved using the dynamic simulator Open Dynamics Engine (ODE). This dynamics engine has the advantage to solve the dynamics of mechanical systems subject to constraints, such as unilateral contacts and joint constraints. It provides also ideal controllers for velocity referenced linear and angular motors, which are necessary for the generation of the impulsion at the toe take-off of our models.

Since the walker has no knees, the swing leg always touches the ground when both legs are aligned. This contact is undesirable and the simulation has then to take care of this aspect by ignoring the contacts of the swing leg until the inter-leg angles exceeds a threshold, the collision detection is then activated to catch the actual impact with the ground. A fall is spotted by detecting a contact between the floor and
a non-support limb. The ground slope is changed after the toe-off to simulate the texture of the ground. This texture is equivalent to having a polygonal ground (see Figure 1).

The time-discretization is of 1 KHz, but a special simulation time-sample is set to 0.1 ms at the double support instant in order to increase the physical realism of the impacts. This framework simulates over 25 steps by second. This enables a fast exploration of the possible dynamics of the walker on textured ground, and makes computation-time accessible for the estimation of MFPT when using our proposed method.

Acknowledgments

References


Figure Legends

Figure 1. Pavement in Roma: two textured grounds. In the bottom part, we walk without thinking, in the upper part, one has to watch his/her step.

Figure 2. A representation of the models we simulate. The A model is the same structure subject to the constraints $\alpha = \beta = \gamma$. The B model has stabilized neck joints. The rough terrain is modeled with a slope change at each step.
Figure 3. **Mean number of steps with an ideal orientation sensor.** Mean number of steps of the walker models equipped with an ideal orientation sensor on different textures of the ground. By texture we mean the standard deviation of the ground slope. MFPTs are displayed in logarithmic scale. For higher ground roughness, MRPT of both models drops such that they need to change their walking control: watching their step becomes necessary.

Figure 4. **Mechanical power and leg velocity over the walking limit cycle.** A- The mechanical power during phases of limit-cycle on flat ground. From 0% to 30% of the cycle, the distribution of the mechanical energy consumption is different, the head stabilization offers a lower absorbtion following the impact. B- The angular velocity of the swing leg during the limit-cycle. The influence of the mechanical power difference in the first 5% is visible, the swing leg has a better initial velocity until 30% of the cycle.

Figure 5. **Mean number of steps with ideal and vestibular sensors.** MFPTs of both models with inertial sensor are compared to MFPTs with ideal one. We see in B that when we stabilize the head, we have the same performances with and without ideal sensor against relatively small perturbations. If we don’t stabilize the head (in A), the estimation errors still degrade the MFPT even for high roughness of the ground.

Figure 6. **The models’ actuations.** On the left, we see the two models of upper-body (for model A and model B). On the right, a scheme of the spring-damper at impact and the impulsion at take-off.

Figure 7. **The inclinometer model with a damped pendulum.** The slow dynamics of this sensor delays the sensor’s response and makes it more accurate when stabilized.

Figure 8. **Representation of poincaré map for limit-cycle walking system.** The solid line represents the dynamics of the state. The green loop represents the attractive limit cycle. The red ball represent fall-state. The blue plane represent the Poincaré section. The dynamics intersect the Poincaré section in a sequence of states.

<table>
<thead>
<tr>
<th></th>
<th>Length (m)</th>
<th>Mass (Kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>$l_h=0.09$</td>
<td>$m_h=0.4$</td>
</tr>
<tr>
<td>Neck</td>
<td>$l_n=0.07$</td>
<td>$m_n=0.1$</td>
</tr>
<tr>
<td>Torso</td>
<td>$l_t=0.75$</td>
<td>$m_t=4.5$</td>
</tr>
<tr>
<td>Leg</td>
<td>$l_p,0=1$</td>
<td>$m_l=1.5$</td>
</tr>
<tr>
<td>Leg CoM</td>
<td>$l_l=0.40$</td>
<td></td>
</tr>
</tbody>
</table>

The proportions of segments lengths and masses are anthropometric.
Table 2. The value of the limit-cycle state for each model.

<table>
<thead>
<tr>
<th></th>
<th>Rigid neck (model A)</th>
<th>Head stabilization (model B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$ (rad)</td>
<td>$6.2299180819971001 \times 10^{-1}$</td>
<td>$5.1725140512546985 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\dot{\theta}$ (rad/s)</td>
<td>$-2.6420926894326163 \times 10^{-1}$</td>
<td>$1.0569297195519001$</td>
</tr>
<tr>
<td>$\dot{\eta}$ (rad/s)</td>
<td>$1.5756657271195875$</td>
<td>$1.3167098310857720$</td>
</tr>
<tr>
<td>$\phi$ (rad)</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$\alpha$ (rad)</td>
<td>$-2.8762875858822364 \times 10^{-2}$</td>
<td>$-2.8922093483844728 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\dot{\alpha}$ (rad/s)</td>
<td>$-2.833396497756952 \times 10^{-1}$</td>
<td>$-2.7748692131065689 \times 10^{-1}$</td>
</tr>
<tr>
<td>$\gamma$ (rad)</td>
<td>$-2.8762875858822364 \times 10^{-2}$</td>
<td>$-2.600560458444371 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\dot{\gamma}$ (rad/s)</td>
<td>$-2.833396497756952 \times 10^{-1}$</td>
<td>$-6.5999316473546024 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\beta$ (rad)</td>
<td>$-2.8762875858822364 \times 10^{-2}$</td>
<td>$-4.486308262077859 \times 10^{-3}$</td>
</tr>
<tr>
<td>$\dot{\beta}$ (rad/s)</td>
<td>$-2.833396497756952 \times 10^{-1}$</td>
<td>$-1.3160142438761284 \times 10^{-2}$</td>
</tr>
<tr>
<td>$l_p$ (m)</td>
<td>$-1.1240788365238969 \times 10^{-2}$</td>
<td>$-1.2773555160365727 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\dot{l_p}$ (m/s)</td>
<td>$1.531068267403354 \times 10^{-2}$</td>
<td>$-1.2773555160365727 \times 10^{-2}$</td>
</tr>
</tbody>
</table>

The precision of this estimation is necessary as the step dynamic is very sensitive to its initial state.