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Bipolarity in argumentation graphs: Towards a better understanding

Claudette Cayrol, Marie-Christine Lagasquie-Schiex *

IRIT-UPS, Université de Toulouse, France

A B S T R A C T

Different abstract argumentation frameworks have been used for various applications within multi-agents systems. Among them, bipolar frameworks make use of both attack and support relations between arguments. However, there is no single interpretation of the support, and the handling of bipolarity cannot avoid a deeper analysis of the notion of support.

In this paper we consider three recent proposals for specializing the support relation in abstract argumentation: the deductive support, the necessary support and the evidential support. These proposals have been developed independently within different frameworks. We restate these proposals in a common setting, which enables us to undertake a comparative study of the modellings obtained for the three variants of the support. We highlight relationships and differences between these variants, namely a kind of duality between the deductive and the necessary interpretations of the support.

1. Introduction

Formal models of argumentation have recently received considerable interest across different AI communities, like defeasible reasoning and multi-agent systems [1–3]. Typical applications such as for instance negotiation [4] and practical reasoning [5] represent pieces of knowledge and opinions as arguments and reach some conclusion or decision on the basis of interacting arguments.

In formal argumentation, two types of approaches exist. The first one allows for the building of arguments [6]. The second one corresponds to abstract argumentation frameworks that model arguments as atomic entities, ignoring their internal structure and focusing on the interactions between arguments, or sets of arguments. In this case, several semantics can be defined that formalize different intuitions about which arguments to accept from a given framework.

The first abstract framework introduced by [7] limits the interactions to conflicts between arguments with the binary attack relation. Several specialized or extended versions of Dung's framework have been proposed (see for instance [8–13]). Among these extended versions, we are interested in the bipolar framework [14,15] which is capable of modelling a kind of positive interaction expressed by a support relation. [1] Positive interaction between arguments has been first introduced by [16,17]. In [14], the support relation is left general so that the bipolar framework keeps a high level of abstraction. The associated semantics are based on the combination of the attack relation with the support relation which results in new complex attack relations. However, introducing the notion of support between arguments within abstract frameworks has been a controversial issue and some counterintuitive results have been obtained, showing that the combination of both interactions cannot avoid a deeper analysis of the notion of support.

Moreover, there is no single interpretation of the support. Indeed, recently, a number of researchers proposed specialized variants of the support relation. Each specialization can be associated with an appropriate modelling using an appropriate

* Corresponding author.
E-mail addresses: ccayrol@irit.fr (C. Cayrol), lagasq@irit.fr (M.-C. Lagasquie-Schiex).

[1] [15] is a survey of the use of bipolarity in argumentation. It covers different steps of the argumentation process and is not restricted to bipolar interactions.

complex attack. However, these proposals have been developed quite independently, based on different intuitions and with different formalizations. In this paper we do not want to discuss all the criticisms which have been advanced, our purpose is rather to show that bipolar abstract frameworks provide a convenient way to model and discuss various kinds of support. In particular, we address a comparative study of these proposals, in a common setting. Moreover, it is essential to note that our goal *is not* to identify an approach that would be better than another one. We rather intend to explicit the differences between various kinds of support and to propose a common framework for handling each of them.

Section 2 presents a brief review of the classical and bipolar abstract argumentation frameworks. In Sections 3 to 6 we discuss three specializations of the notion of support and propose an appropriate modelling for each of them in the bipolar framework. Related works are discussed in Section 7. In Section 8 we conclude and give some perspectives for future work.

Note that this paper is an extended version of [18]. This extension consists in the introduction of new notions and new results (proofs are given in the appendix) and a deeper analysis of related works.

2. Background on abstract argumentation frameworks

2.1. Dung argumentation framework

Dung's seminal abstract framework consists of a set of arguments and one type of interaction between them, namely attack. What really means is the way arguments are in conflict.

Definition 1 (*Dung AF*)**.** A Dung's argumentation framework (AF, for short) is a pair $\langle \mathcal{A}, \mathcal{R} \rangle$ where $\mathcal{A}$ is a finite and non-empty set of arguments and $\mathcal{R}$ is a binary relation over $\mathcal{A}$ (a subset of $\mathcal{A} \times \mathcal{A}$), called the *attack relation*.

An argumentation framework can be represented by a directed graph, called the *interaction graph*, in which the nodes represent arguments and the edges are defined by the attack relation: $\forall a, b \in \mathcal{A}$, $a\mathcal{R}b$ is represented by $a \not\to b$.

Definition 2 (*Admissibility in AF*)**.** Given $\langle \mathcal{A}, \mathcal{R} \rangle$ and $S \subseteq \mathcal{A}$,

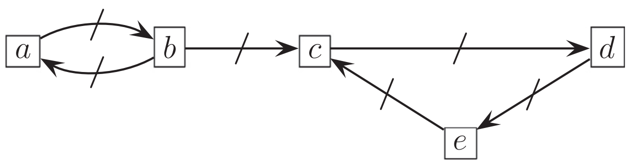
- S is *conflict-free* in $\langle \mathcal{A}, \mathcal{R} \rangle$ iff there are no arguments $a, b \in S$, such that $a\mathcal{R}b$.
- $a \in \mathcal{A}$ is *acceptable* in $\langle \mathcal{A}, \mathcal{R} \rangle$ with respect to S iff $\forall b \in \mathcal{A}$ such that $b\mathcal{R}a$, $\exists c \in S$ such that $c\mathcal{R}b$.
- S is *admissible* in $\langle \mathcal{A}, \mathcal{R} \rangle$ iff S is conflict-free and each argument in S is acceptable with respect to S.

Standard semantics introduced by Dung (preferred, stable, grounded) enable to characterize admissible sets of arguments that satisfy some form of optimality.

Definition 3 (*Extensions*)**.** Given $\langle \mathcal{A}, \mathcal{R} \rangle$ and $S \subseteq \mathcal{A}$,

- S is *a preferred extension* of $\langle \mathcal{A}, \mathcal{R} \rangle$ iff it is a maximal (with respect to $\subseteq$) admissible set.
- S is *a stable extension* of $\langle \mathcal{A}, \mathcal{R} \rangle$ iff it is conflict-free and for each $a \notin S$, there is $b \in S$ such that $b\mathcal{R}a$.
- S is *the grounded extension* of $\langle \mathcal{A}, \mathcal{R} \rangle$ iff it is the least (with respect to $\subseteq$) admissible set X such that each argument acceptable with respect to X belongs to X.

Example 1. Let AF be defined by $\mathcal{A} = \{a, b, c, d, e\}$ and $\mathcal{R}_{\text{att}} = \{(a, b), (b, a), (b, c), (c, d), (d, e), (e, c)\}$ and represented by the following graph. There are two preferred extensions ($\{a\}$ and $\{b, d\}$), one stable extension ($\{b, d\}$) and the grounded extension is the empty set.



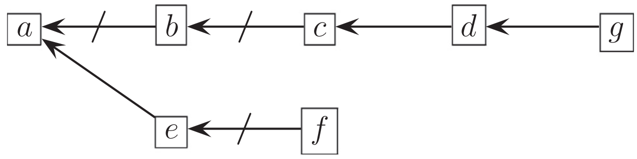
2.2. Bipolar argumentation framework

The abstract bipolar argumentation framework presented in [14,19] extends Dung's framework in order to take into account both negative interactions expressed by the attack relation and positive interactions expressed by a support relation (see [15] for a more general survey about bipolarity in argumentation).

Definition 4 (*BAF*)**.** A bipolar argumentation framework (BAF, for short) is a tuple $\langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ where $\mathcal{A}$ is a finite and non-empty set of arguments, $\mathcal{R}_{\text{att}}$ is a binary relation over $\mathcal{A}$ called the *attack relation* and $\mathcal{R}_{\text{sup}}$ is a binary relation over $\mathcal{A}$ called the *support relation*.

A BAF can still be represented by a directed graph $\mathcal{G}_b$ called the *bipolar interaction graph*, with two kinds of edges. Let a_i and $a_j \in \mathcal{A}$, $a_i \mathcal{R}_{\text{att}} a_j$ (resp. $a_i \mathcal{R}_{\text{sup}} a_j$) means that a_i attacks a_j (resp. a_i supports a_j) and it is represented by $a \not\to b$ (resp. by $a \to b$).

Example 2. For instance, in the following graph representing a BAF, there is a support from g to d and an attack from b to a



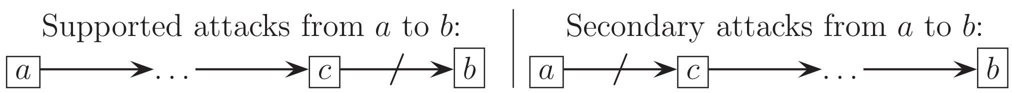
New kinds of attack emerge from the interaction between the direct attacks and the supports. These new attacks together with the direct attacks will be referred to as the *complex attacks* of the BAF. For instance, these complex attacks can be defined using the supported attack and the secondary attack which have been introduced in [19] (and previously in [14] with a different terminology):

Definition 5 ([19] *An example of complex attacks in a BAF*)**.** Let $\text{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$, *complex attacks in* BAF consist of the direct attack $\mathcal{R}_{\text{att}}$ and the supported and secondary attacks defined by:

- there is a *supported attack* from a to b iff there is a sequence $a_1 \mathcal{R}_1 \ldots \mathcal{R}_{n-1} a_n$, $n \geq 3$, with $a_1 = a$, $a_n = b$, $\forall i = 1 \ldots n-2$, $\mathcal{R}_i = \mathcal{R}_{\text{sup}}$ and $\mathcal{R}_{n-1} = \mathcal{R}_{\text{att}}$.
- There is a *secondary attack* from a to b iff there is a sequence $a_1 \mathcal{R}_1 \ldots \mathcal{R}_{n-1} a_n$, $n \geq 3$, with $a_1 = a$, $a_n = b$, $\mathcal{R}_1 = \mathcal{R}_{\text{att}}$ and $\forall i = 2 \ldots n-1$, $\mathcal{R}_i = \mathcal{R}_{\text{sup}}$.

The set of supported (resp. secondary) attacks will be denoted $\mathcal{R}_{\text{att}}^{\text{sup}}$ (resp. $\mathcal{R}_{\text{att}}^{\text{sec}}$).

So, according to the above definition, new kinds of attack, from a to b, can be considered in the following cases.



Note that the above definitions combine a direct attack with a sequence of direct supports, that is a direct or indirect support.

Notation 1. In the following, a `supports` b means that there is a sequence of direct supports from a to b.

Example 2 (Cont'd)**.** In this example, there is a supported attack from g (or d) to b and a secondary attack from f to a.

Acceptability semantics must be redefined for taking into account complex attacks. The first step in defining acceptability is the investigation of the notion of coherence for a set of arguments. The basic requirement is to avoid conflicts. That leads to extend the notion of conflict-freeness by replacing direct attacks by complex attacks. So, in the following, given a definition of complex attacks, we will talk about *conflict-freeness wrt*[2] *these complex attacks.*

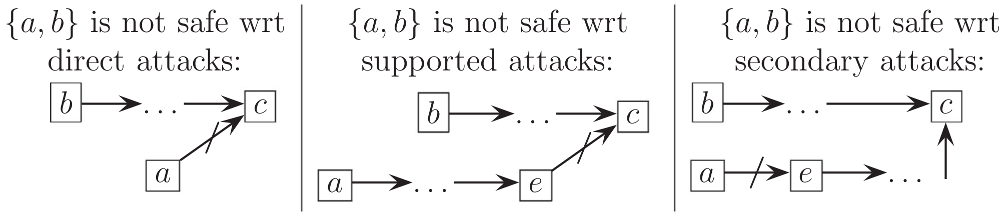
Moreover, the notion of coherence of a set of arguments can be still enforced by excluding sets of arguments which attack and support the same argument. This is a kind of external coherence reflected by the notion of safety [14]. So, as in the case of conflict-freeness, given a definition of complex attacks, we will talk about *safety wrt these complex attacks*.

Definition 6 ([14] *Safety in BAF*)**.** Let $\text{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. Let $\mathcal{R}_{\text{c-att}}$ be a set of complex attacks built from BAF. Consider $S \subseteq \mathcal{A}$, S is *safe* wrt $\mathcal{R}_{\text{c-att}}$ iff there are no arguments $a, b \in S$, and $c \in \mathcal{A}$ such that

- b `supports` c or $c \in S$ and
- there is a *complex attack* from a to c belonging to $\mathcal{R}_{\text{c-att}}$.

For instance, following the example of complex attacks given by Definition 5, the set $\{a, b\}$ can be considered as "incoherent" in each of the following cases:

[2] wrt: with respect to.



Another requirement has been considered in [14], which concerns only the support relation, namely the closure under $\mathcal{R}_{\text{sup}}$.

Definition 7 (*Closure in BAF*)**.** Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$, $S \subseteq \mathcal{A}$. S is *closed under* $\mathcal{R}_{\text{sup}}$ iff $\forall a \in S, \forall b \in \mathcal{A}$, if $a\mathcal{R}_{\text{sup}}b$ then $b \in S$.

So, following the same methodology as in Dung's framework, different acceptability semantics can be proposed in a bipolar argumentation framework, depending on the notion of attack (direct, supported, secondary, ...) and on the notion of coherence which are used (conflict-free, safe, closed under $\mathcal{R}_{\text{sup}}$).

3. Modelling various kinds of support

Handling support and attack at an abstract level has the advantage to keep genericity. An abstract bipolar framework is useful as an analytic tool for studying different notions of complex attacks, complex conflicts, and new semantics taking into account both kinds of interactions between arguments. However, the drawback is the lack of guidelines for choosing the appropriate definitions and semantics depending on the application. For instance, in Dung's framework, whatever the semantics, the acceptance of an argument which is not attacked is guaranteed. Is it always desirable in a bipolar framework? Two related questions are: Can arguments stand in an extension without being supported? Can arguments be used as attackers without being supported? It may depend on the interpretation of the support, as shown below.

In the following, we discuss three specialized variants of the support relation, which have been proposed recently: the deductive support, the necessary support and the evidential support. Let us first briefly give the underlying intuition, then some illustrative examples.

Deductive support [20] is intended to capture the following intuition: If $a\mathcal{R}_{\text{sup}}b$ then the acceptance of a implies the acceptance of b, and as a consequence the non-acceptance of b implies the non-acceptance of a.

Necessary support [21,22], is intended to capture the following intuition: If $a\mathcal{R}_{\text{sup}}b$ then the acceptance of a is necessary to get the acceptance of b, or equivalently the acceptance of b implies the acceptance of a.

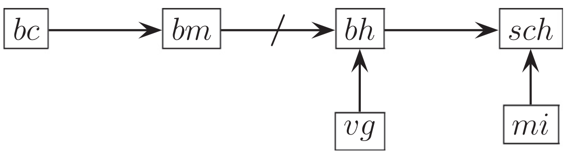
Evidential support [23,24] enables to distinguish between *prima-facie* and standard arguments. *Prima-facie* arguments do not require any support from other arguments to stand, while standard arguments must be supported by at least one *prima-facie* argument.

The following examples show that different interpretations of the support can be given strongly depending on the context, and that, according to the considered interpretation, some complex attacks need to be considered, while others are counterintuitive. It is important to note that these examples are not given here in order to express a preference over the different types of support. Their goal is only to illustrate the existing approaches.

Example 3. This example has been inspired from [21] (and also from a variant in [20]). Let us consider the following knowledge: Obtaining a Bachelor's degree with honors (*bh*) supports obtaining a scholarship (*sch*) and suppose that having at least one bad mark (*bm*) does not allow to obtain the honors (even if the average of marks normally allows it). One possible interpretation of the support is: obtaining a bachelor's degree is necessary for obtaining a scholarship. So, if we do not have a *bh* then we are sure that we do not have *sch*.

Now let us suppose that obtaining *sch* may be also fulfilled if the student justifies modest incomes (*mi*). A more appropriate interpretation of the support is a deductive one. In that case, a secondary attack from *bm* to *sch* would be counterintuitive. Moreover, it is known that making a blank copy (*bc*) supports having a very bad mark. With a deductive interpretation of that support, it makes sense to add a supported attack from *bc* to *bh*. Finally, we add the knowledge: having a very good mark for each test of the examination (*vg*) supports obtaining a Bachelor's degree with honors.

The whole example can be formalized in a BAF represented by the following graph:



Example 4 (*Example illustrating a necessary support*). Let us consider the following dialogue between three agents:

- Agent 1: The room is dark, so I will light up the lamp.
- Agent 2: But the electric meter does not work.
- Agent 1: Are you sure?
- Agent 3: The electrician has detected a failure.

This dialogue shows interactions between the positions *rd* (the room is dark), *ll* (the lamp will light up), *ew* (the electric meter works), and *fail* (there is a failure in the electric meter). These interactions can be formalized in a BAF represented by the following graph:

The intuitive interpretation of the support is a necessary one since the lamp cannot light up when the electric meter does not work. In that case, it makes sense to add a secondary attack from *fail* to *ll*.

The importance of the context clearly appears in the following example inspired by an example proposed in [20]:

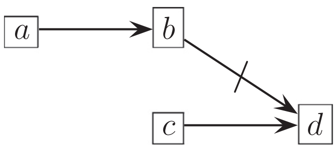
Example 5. Let us consider the following knowledge about football matches:

- if Liverpool wins last match then Liverpool wins Premier League,
- if Manchester does not win last match then Liverpool wins Premier League,
- if the best player of Liverpool is injured then Liverpool does not win last match.

The interactions between the positions *wlm* (Liverpool wins last match), *lpl* (Liverpool wins Premier League), *mnw* (Manchester does not win last match), and *bpi* (the best player of Liverpool is injured) can be formalized in a BAF represented by the following graph:

The intuitive interpretation of the support is a deductive one and not a necessary one. Indeed, Liverpool wins Premier League if Manchester does not win last match, even if Liverpool does not win last match. So adding a secondary attack from *bpi* to *lpl* is not the right modelling.

Example 6 (*Example for an evidential support*). Let us consider the BAF represented by the graph:



Assume first that the only *prima-facie* argument is *c*. So, *d* may stand, but neither *a* nor *b* is grounded in *prima-facie* arguments. As a consequence, the attack on *d* cannot be taken into account. So, *c* and *d* will be accepted.

Assume now that the *prima-facie* arguments are *a* and *c*. So, *b* and *d* may stand and the attack on *d* must be considered. In that case the accepted arguments are *a*, *b* and *c*. In order to reinstate *d*, an attack could be added either from *c* to *b* or from *c* to *a*. Indeed, an attack from *c* to *a* invalidates the attack on *d* by rendering *b* unsupported.

Finally, assume that the *prima-facie* arguments are *a*, *b* and *c*. The attack from *b* to *d* holds without the support by *a*. So an attack from *c* to *a* does not enable to reinstate *d*. There must be an attack from *c* to *b* for "saving" *d*.

We propose to restate various notions of support in the BAF framework. We will show that each specialized variant of the support can be associated with appropriate complex attacks. Then, we will be able to highlight links between these various notions of support.

We first discuss the deductive and necessary supports (Sections 4.1 and 4.2), and prove that these two specializations of the support are indeed dual. As a consequence, these two kinds of support can be handled simultaneously in a bipolar framework. Then in Section 5, we study a restricted version of evidential support and show that it can be viewed as a kind of weak necessary support.

4. A framework for a comparative study of deductive and necessary supports

4.1. Deductive supports

As explained above, a deductive support is intended to enforce the following constraint: If $b\mathcal{R}_{\text{sup}}c$ then the acceptance of b implies the acceptance of c, and as a consequence the non-acceptance of c implies the non-acceptance of b. Suppose now that $a\mathcal{R}_{\text{att}}c$. The acceptance of a implies the non-acceptance of c and so the non-acceptance of b. This strong constraint can be taken into account by introducing a new attack, called mediated attack.

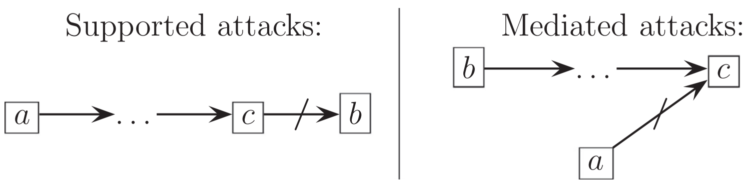
Definition 8 ([20] *Mediated attack*)**.** Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. There is a *mediated attack* from a to b iff there is a sequence $a_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}a_{n-1}$, and $a_n\mathcal{R}_{\text{att}}a_{n-1}$, $n \geq 3$, with $a_1 = b$, $a_n = a$.

The set of mediated attacks will be denoted $\mathcal{R}_{\text{att}}^{\text{med}}$.

Example 3 (Cont'd)**.** From $vg\mathcal{R}_{\text{sup}}bh$ and $bm\mathcal{R}_{\text{att}}bh$, the mediated attack $bm\mathcal{R}_{\text{att}}vg$ will be added.

Moreover, the deductive interpretation of the support justifies the introduction of supported attacks (cf Definition 5 and [19]). If $a\mathcal{R}_{\text{sup}}c$ and $c\mathcal{R}_{\text{att}}b$, the acceptance of a implies the acceptance of c and the acceptance of c implies the non-acceptance of b. So, the acceptance of a implies the non-acceptance of b.

So, with the deductive interpretation of the support, new kinds of attack, from a to b, can be considered in the following cases:



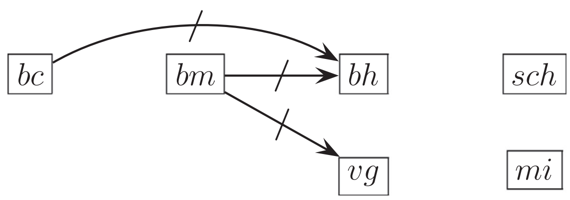
Notation 2. In the following, deductive support will be called *d-support* and the existence of a d-support between two arguments a and b will be denoted by a `d-supports` b.

Definition 9 (*Modelling deductive support*)**.** Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. The combination of the direct attacks and the d-supports results in the addition of supported attacks and mediated attacks.

As explained above, modelling deductive support in a BAF can be done in considering the associated Dung AF consisting of the same arguments and of the relation built from the direct attacks, the supported attacks and the mediated attacks:

Notation 3. Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports, the *associated Dung AF for the deductive support* is denoted by AF^D and defined by $\langle \mathcal{A}, \mathcal{R}_{\text{att}}^D \rangle$ with $\mathcal{R}_{\text{att}}^D = \mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{sup}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$.

Example 3 (Cont'd)**.** The following attacks are added: a supported attack from bc to bh and a mediated attack from bm to vg. Then support can be ignored, and we obtain the following AF^D:



AF^D has one preferred (and also stable and grounded) extension $\{bc, bm, sch, mi\}$.

The following results establish links between the different coherence requirements which can be defined, when modelling deductive support in a BAF.

Proposition 1. *Consider* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Given* $S \subseteq \mathcal{A}$*,*

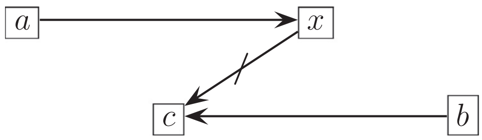
- *S is safe wrt $\mathcal{R}_{\text{att}}$ in BAF iff S is safe wrt $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ in BAF.*
- *S is safe wrt $\mathcal{R}_{\text{att}}$ in BAF iff S is conflict-free wrt $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ in BAF.*

Proposition 2. *Consider* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports and* AF^D *its associated Dung AF. Given* $S \subseteq \mathcal{A}$,

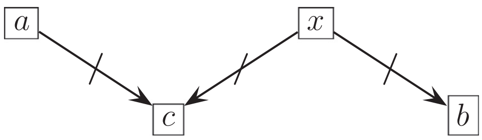
- *If* S *is conflict-free wrt* $\mathcal{R}_{\text{att}}$ *and closed under* $\mathcal{R}_{\text{sup}}$ *in* BAF, *then* S *is also conflict-free in* AF^D *(that is conflict-free wrt* $\mathcal{R}^D_{\text{att}}$*).*
- *If* S *is conflict-free wrt* $\mathcal{R}^D_{\text{att}}$ *and closed under* $\mathcal{R}_{\text{sup}}$ *in* BAF, *then* S *is also safe wrt* $\mathcal{R}^D_{\text{att}}$ *in* BAF.

From the above proposition it turns out that the notion of coherence enforced in a BAF (for instance, by using the closure under $\mathcal{R}_{\text{sup}}$) is stronger than conflict-freeness in the corresponding AF^D. Moreover, as shown by the following example, the comparison is strict, even in the case of maximal (for set-inclusion) coherent sets.

Example 7. Consider the following graph representing a BAF:



Among the sets which are conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$, the maximal ones are $\{a, x\}$ and $\{b, c\}$.
Let us consider AF^D represented by:



In AF^D, there is another maximal (for set inclusion) conflict-free set, the set $\{a, b\}$ which is not closed under $\mathcal{R}_{\text{sup}}$ in the corresponding BAF.

The closure requirement makes the notion of coherence in a BAF very strong. However, this requirement can be justified by the deductive interpretation of the support: Assume that $\mathcal{R}_{\text{sup}}$ only contains d-supports; it means that if $a\mathcal{R}_{\text{sup}}b$, the acceptance of a implies the acceptance of b; now, considering a sequence of supports $a_1\mathcal{R}_{\text{sup}}a_2\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}a_n$, the acceptance of a_1 implies the acceptance of a_2, which in turn implies the acceptance of a_3 …which implies the acceptance of a_n; so, by transitivity, the acceptance of a_1 implies the acceptance of a_n. Obviously, the condition of closure under $\mathcal{R}_{\text{sup}}$ enforces this property.

Going back to the interpretation of the deductive support, the following constraint hold on Example 7:

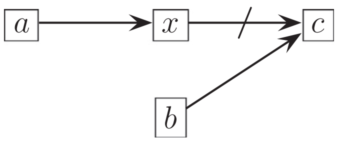
- The acceptance of a implies the acceptance of x,
- The acceptance of x implies the non-acceptance of c,
- The acceptance of b implies the acceptance of c.

It follows that the acceptance of a must imply the non-acceptance of b. However, this last constraint cannot be enforced in AF^D. This problem can be easily solved by considering not only direct attacks but also supported attacks in the definition of a mediated attack.

So, we propose to replace Definition 8 by the definition of a new "mediated" attack, called the super-mediated attack.

Definition 10 (*Super-mediated attacks*). Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. There is a *super-mediated attack* from a to b iff there is a direct attack or a supported attack from a to c, and a support from b to c.
The set of super-mediated attacks will be denoted $\mathcal{R}^{\text{s-med}}_{\text{att}}$.

For instance, there is a super-mediated attack from a to b in the following case:



Then, deductive support can be better taken into account by considering the associated Dung AF consisting of the same arguments and of the relation built from the direct attacks, the supported attacks and the super-mediated attacks:

Notation 4. Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports, the *complete associated Dung AF for the deductive support* is denoted by AF^{Dc} and defined by $\langle \mathcal{A}, \mathcal{R}^{Dc}_{\text{att}} \rangle$ with $\mathcal{R}^{Dc}_{\text{att}} = \mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}}$.

The following propositions show that the new AF^{Dc} enables to recover the closure under $\mathcal{R}_{\text{sup}}$.

Proposition 3. *Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. Given $S \subseteq \mathcal{A}$, if S is conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF, then S is also conflict-free in AF^{Dc}.*

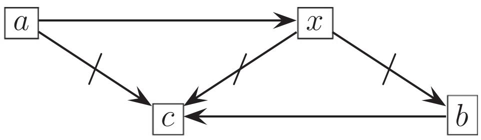
Proposition 4. *Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. Given $S \subseteq \mathcal{A}$, S is a $\subseteq$-maximal conflict-free set in AF^{Dc} iff S is $\subseteq$-maximal among the sets which are conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF.*

Example 7 (Cont'd)**.** In AF^{Dc}, the $\subseteq$-maximal conflict-free sets are $\{a, x\}$ and $\{b, c\}$.

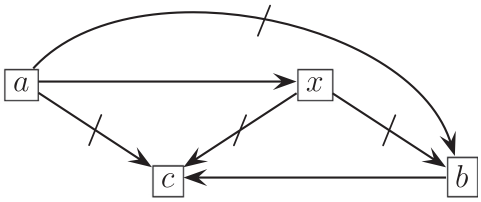
An alternative method for modelling all the attacks induced by the deductive support is to give an inductive definition for these new attacks. Let us first illustrate this method on Example 7:

Example 7 (Cont'd)**.** The new attacks could be obtained with two steps:

Step 1: we add the supported attack (a, c) and the mediated attack (x, b)



Step 2: From the support (a, x) and the new attack (x, b), we deduce a new kind of "supported" attack (a, b).



Note that the new attack from a to b can also be obtained as a new kind of "mediated" attack from the (supported) attack (a, c) and the support (b, c).

We propose an inductive definition of these new attacks, called *deductive complex attacks* (`d-attacks` for short), by combining the direct, supported and mediated attacks.

Definition 11 (*d-Attacks*)**.** Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. There exists a `d-attack` from a to b iff

- either $a\mathcal{R}_{\text{att}}b$, or $a\mathcal{R}^{\text{sup}}_{\text{att}}b$, or $a\mathcal{R}^{\text{med}}_{\text{att}}b$ (**Basic case**),
- or there exists an argument c such that a `supports` c and c `d-attacks` b (**Case 1**),
- or there exists an argument c such that a `d-attacks` c and b `supports` c (**Case 2**).

The set of `d-attacks` will be denoted $\mathcal{R}_{\text{d-att}}$.

It turns out that the set of `d-attacks` exactly corresponds to the attacks defined in AF^{Dc}, the complete associated Dung AF for the deductive support.

Proposition 5. *Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. $\mathcal{R}_{\text{d-att}} = \mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}}$. In other words, a `d-attacks` b iff $(a, b) \in \mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}}$.*

4.2. Necessary supports

Necessary support corresponds to the following interpretation: If $c\mathcal{R}_{\text{sup}}b$ then the acceptance of c is necessary to get the acceptance of b, or equivalently the acceptance of b implies the acceptance of c. Suppose now that $a\mathcal{R}_{\text{att}}c$. The acceptance of a implies the non-acceptance of c and so the non-acceptance of b. This constraint can be taken into account by introducing a new attack, which is exactly the secondary attack presented above (cf Definition 5 and [19]).

Note that this constraint has been considered in [21], where it was called extended attack.

Moreover, another kind of complex attack can be justified: If $c\mathcal{R}_{\text{sup}}a$ and $c\mathcal{R}_{\text{att}}b$, the acceptance of a implies the acceptance of c and the acceptance of c implies the non-acceptance of b. So, the acceptance of a implies the non-acceptance of b. This constraint relating a and b should be enforced by adding a new complex attack from a to b. Note that this complex attack was not considered in [21] but has been added in [22].

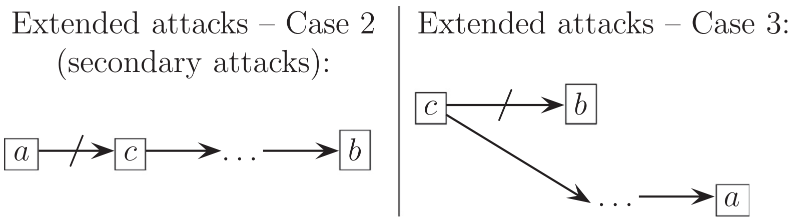
Let us recall the definition of extended attack proposed in [22] which enables to model necessary support in a BAF.

Definition 12 ([22] *Extended attack*). Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. There is an *extended attack* from a to b iff

1. either $a\mathcal{R}_{\text{att}}b$,
2. or there is a sequence $a_1\mathcal{R}_{\text{att}}a_2\mathcal{R}_{\text{sup}} \dots \mathcal{R}_{\text{sup}}a_n$, $n \geq 3$, with $a_1 = a$, $a_n = b$,
3. or there is a sequence $a_1\mathcal{R}_{\text{sup}} \dots \mathcal{R}_{\text{sup}}a_n$, and $a_1\mathcal{R}_{\text{att}}a_p$, $n \geq 2$, with $a_n = a$, $a_p = b$.

The set of the extended attacks will be denoted by $\mathcal{R}^{\text{ext}}_{\text{att}}$.

So, with the necessary interpretation of the support, new kinds of attack, from a to b, can be considered in the following cases:



Notation 5. In the following, necessary support will be called *n-support* and the existence of a n-support between two arguments a and b will be denoted by a `n-supports` b.

As in the deductive approach, it is possible to define the associated Dung AF:

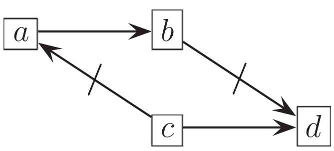
Notation 6. Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of n-supports, the *associated Dung AF for the necessary support* is denoted by AF^N and defined by $\langle \mathcal{A}, \mathcal{R}^N_{\text{att}} \rangle$ with $\mathcal{R}^N_{\text{att}} = \mathcal{R}^{\text{ext}}_{\text{att}}$.

Deductive support and necessary support have been introduced independently. However, they correspond to dual interpretations of the support in the following sense: a `n-supports` b is equivalent to b `d-supports` a. Besides, it is easy to see that the constructions of mediated attack and secondary attack are dual in the following sense: the mediated attacks obtained by combining the attack relation $\mathcal{R}_{\text{att}}$ and the support relation $\mathcal{R}_{\text{sup}}$ are exactly the secondary attacks obtained by combining the attack relation $\mathcal{R}_{\text{att}}$ and the support relation $\mathcal{R}^{-1}_{\text{sup}}$ which is the symmetric relation of $\mathcal{R}_{\text{sup}}$ ($\mathcal{R}^{-1}_{\text{sup}} = \{(b, a) | (a, b) \in \mathcal{R}_{\text{sup}}\}$). Moreover, the complex attacks which are missing in [21] and added in [22] as evoked previously can be recovered by considering the supported attacks built from $\mathcal{R}_{\text{att}}$ and $\mathcal{R}^{-1}_{\text{sup}}$.

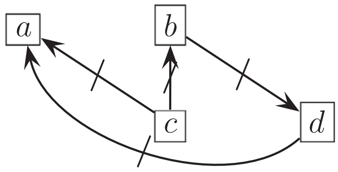
Consequently, the modelling by the addition of appropriate complex attacks satisfies this duality.

Proposition 6 (Modelling necessary support). *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of n-supports. The combination of the direct attacks and the n-supports can be handled by turning the n-supports into the dual d-supports and then adding the supported attacks and mediated attacks.*

Example 8. Consider BAF represented by:



Assume that the support relation has been given a necessary interpretation. That is a is necessary for b and c is necessary for d. It is equivalent to consider that there is a deductive support from b to a and also from d to c. Then, we add a supported attack from d to a and a mediated attack from c to b. The resulting AF^N is represented by:



It follows that $\{c, d\}$ is the only preferred (and also stable and grounded) extension.

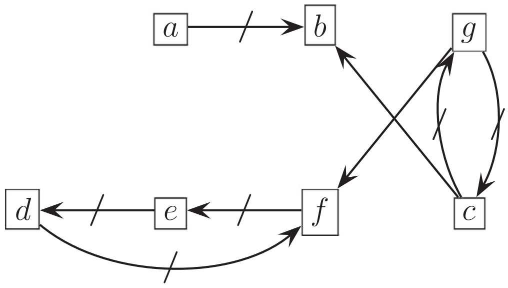
Due to the duality between necessary and deductive supports, the inductive process can be applied to the definition of extended attacks, leading to the *complete associated Dung AF for the necessary support*:

Notation 7. Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\mathsf{att}}, \mathcal{R}_{\mathsf{sup}} \rangle$ with $\mathcal{R}_{\mathsf{sup}}$ being a set of n-supports.

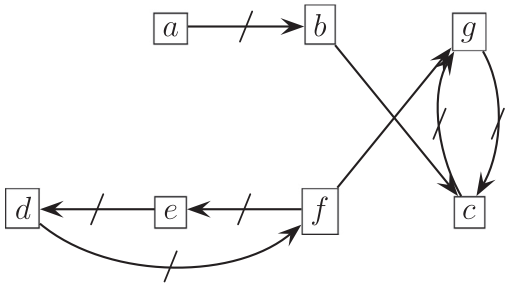
- BAF_{sym} denotes the bipolar framework defined by $\langle \mathcal{A}, \mathcal{R}_{\mathsf{att}}, \mathcal{R}^{-1}_{\mathsf{sup}} \rangle$.
- AF^{Dc}_{sym} denotes the complete associated Dung AF for BAF_{sym} (obtained using the direct attacks, the supported attacks and the super-mediated attacks issued from BAF_{sym}).
- And the *complete associated Dung AF for the necessary support* is denoted by AF^{Nc} and exactly corresponds to AF^{Dc}_{sym}.

The difference between AF^{N} and AF^{Nc} is illustrated by the following example:

Example 9. This example comes from [22]. Consider BAF represented by:



The corresponding BAF_{sym} ($\mathcal{R}^{-1}_{\mathsf{sup}}$ is used in place of $\mathcal{R}_{\mathsf{sup}}$) is represented by:



Then the associated Dung AF for the deductive support AF^{D}_{sym} of BAF_{sym} (which is exactly the associated Dung AF for the necessary support AF^{N} of BAF) is represented by:

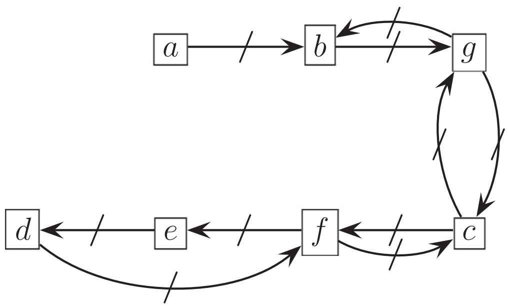


Table 1

Correspondences between abstract, deductive and necessary supports.

Abstract supports of [14,19]	Deductive supports of [20]	Necessary supports of [21,22]
Supported attack	Supported attack	Extended attack (case 3) with $\mathcal{R}_{\text{sup}}^{-1}$
Secondary attack	Mediated attack with $\mathcal{R}_{\text{sup}}^{-1}$	Extended attack (case 2)
	Mediated attack	Extended attack (case 2) with $\mathcal{R}_{\text{sup}}^{-1}$
S Safe wrt $\mathcal{R}_{\text{att}}$	No direct nor mediated	No extended attack (cases 1 and 2)
	Attack in S	with $\mathcal{R}_{\text{sup}}^{-1}$ in S
S Closed for $\mathcal{R}_{\text{sup}}$	S Closed for $\mathcal{R}_{\text{sup}}$	S Closed for $\mathcal{R}_{\text{sup}}^{-1}$
	AF^D	AF^N for $\mathcal{R}_{\text{sup}}^{-1}$
	AF^{Dc}	AF^{Nc} for $\mathcal{R}_{\text{sup}}^{-1}$
	`d-attacks`	`d-attacks` for $\mathcal{R}_{\text{sup}}^{-1}$

And the complete associated Dung AF for the deductive support AF_{sym}^{Dc} of BAF_{sym} (which is exactly the complete associated Dung AF for the necessary support AF^{Nc} of BAF) is represented by:

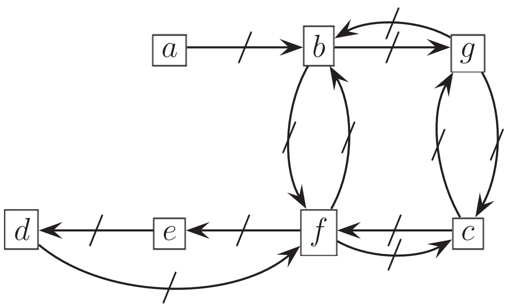


Table 1 gives a synthetic view of the correspondences between the three approaches (abstract, deductive and necessary).

4.3. Impact on self-attacking arguments

In the literature on the argumentation domain, it is very common to find some restrictions about self-attacking arguments. As taking into account deductive or necessary supports leads to introduce new complex attacks, it is interesting to characterize the cases where these attacks correspond to self-attacks. For deductive supports, the following proposition describes these cases:

Proposition 7. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Consider* $a \in \mathcal{A}$. *a is a self-attacking argument of* AF^{Dc} *iff*

- *either* $a\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that a* `supports` *b and* $b\,\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that a* `supports` *b and* $a\,\mathcal{R}_{\text{att}}b$,
- *or* $\exists b$ *and* $c \in \mathcal{A}$, *such that a* `supports` *c*, $c\,\mathcal{R}_{\text{att}}b$ *and a* `supports` *b*.

So if we need Dung AF without self attacking arguments, we have to restrict BAF.

A similar property is obtained for the necessary support using the duality between necessary and deductive supports:

Proposition 8. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of n-supports. Consider* $a \in \mathcal{A}$. *a is a self-attacking argument of* AF^{Nc} *iff*

- *either* $a\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that b* `supports` *a and* $b\,\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that b* `supports` *a and* $a\,\mathcal{R}_{\text{att}}b$,
- *or* $\exists b$ *and* $c \in \mathcal{A}$, *such that c* `supports` *a*, $c\,\mathcal{R}_{\text{att}}b$ *and b* `supports` *a*.

5. Evidential support

Evidential support [23,24] is intended to capture the notion of *support by evidence*: an argument cannot be accepted unless it is supported by evidence. Evidence is represented by a special argument, and the arguments which are directly supported by this special argument are called *prima-facie* arguments. Arguments can be accepted only if they are supported (directly or indirectly) by *prima-facie* arguments. Besides, only supported arguments can be used to attack other arguments.

In Oren's evidential argument framework, attacks and supports may be carried out by a set of arguments (and not only by a single argument). However, for the purpose of comparing different specializations of the notion of support, we will restrict the presentation of evidential support to the case where attacks and supports are carried out by single arguments. All the definitions that we give in the following are inspired by those given in [23,24].

Given $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$, we distinguish a subset $\mathcal{A}_e \subseteq \mathcal{A}$ of arguments which do not require any support to stand. These arguments will be called self-supported and correspond to the *prima-facie* arguments. We recall that in a BAF, a `supports` b means that there is a sequence of direct supports from a to b. So an evidential BAF can be defined as follows:

Definition 13 (*Evidential BAF (EAF)*)**.** An *evidential BAF (EAF)* is a tuple $\langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ where $\langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ is a BAF and $\mathcal{A}_e \subseteq \mathcal{A}$. $\mathcal{A}_e$ is called the set of *prima-facie* arguments.

So, evidential support (or *e-support* for short) can be defined as a particular case of the notion of (direct or indirect) support.

Definition 14 (*e-Supports*)**.** Let $\mathsf{EAF} = \langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$.

- a is *e-supported* iff either $a \in \mathcal{A}_e$ or there exists b such that b is e-supported and $b\mathcal{R}_{\text{sup}}a$.
- a is *e-supported by S* (or S `e-supports` a) iff either $a \in \mathcal{A}_e$ or there is an elementary sequence $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n\mathcal{R}_{\text{sup}}a$ such that $\{b_1 \ldots b_n\} \subseteq S$ and $b_1 \in \mathcal{A}_e$.
- S is *self-supporting* iff S `e-supports` each of its elements.

Example 8 (Cont'd)**.** Assume that $\mathcal{A}_e = \{a, c\}$. Then b is e-supported by $\{a\}$, d is e-supported by $\{c\}$. The sets $\{a, b\}$ and $\{c, d\}$ are self-supporting.

The combination of the direct attacks and the evidential support results in restrictions on the notion of attack and also on the notion of acceptability. The first idea is that only e-supported arguments may be used to make a direct attack on other arguments. This is formalized by the notion of e-supported attack.

Definition 15 (*e-Supported attack*)**.** Let $\mathsf{EAF} = \langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. S carries out an *e-supported attack* on a iff there exists $b \in S$ such that $b\mathcal{R}_{\text{att}}a$ and b is e-supported by S.

The second idea concerns reinstatement: If a is attacked by b, which is e-supported, a can be reinstated either by a direct attack on b or by an attack on c such that without c, b would be no longer e-supported. In order to enforce this idea, minimal (for set-inclusion) e-supported attacks have to be considered. We have:

Proposition 9. *Let* $\mathsf{EAF} = \langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. *X is a minimal e-supported attack on the argument a iff X is the set of arguments appearing in a minimal elementary sequence* $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n$ *such that* $b_1 \in \mathcal{A}_e$ *and* $b_n\mathcal{R}_{\text{att}}a$.

Note that a minimal e-supported attack on a given argument corresponds to a particular case of a supported attack as defined in Definition 5. In the case when $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n$ with $b_1 \in \mathcal{A}_e$ and $b_n\mathcal{R}_{\text{att}}a$, each b_i carries out a supported attack on a.

Now, following Oren's evidential argument framework, we propose a new definition for acceptability. There are two conditions on S, for a being acceptable wrt S. The first one is classical and concerns defence or reinstatement: S must invalidate each minimal e-supported attack on a (either by attacking the attacker of a or by rendering this attacker unsupported). The second condition requires that S `e-supports` a.

Definition 16 (*e-Acceptability*)**.** Let $\mathsf{EAF} = \langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. a is *e-acceptable* wrt S iff

- For each minimal e-supported attack X on a, there exists $b \in S$ and $x \in X$ such that $b\mathcal{R}_{\text{att}}x$ and
- a is e-supported by S.

Definition 17 (*e-Admissibility*)**.** Let $\mathsf{EAF} = \langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$. S is *e-admissible* iff

- Each element of S is e-acceptable wrt S and
- there are no arguments $a, b \in S$, such that $a\mathcal{R}_{\text{att}}b$.

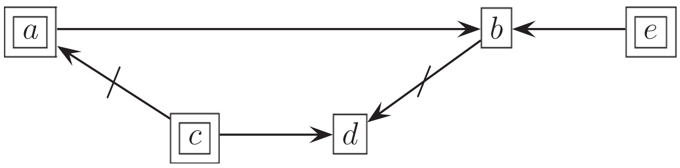
Example 8 (Cont'd). Assume that $\mathcal{A}_e = \{a, c\}$. There is only one minimal e-supported attack on d: $\{a, b\}$. As $c\mathcal{R}_{\mathtt{att}}a$ and d is e-supported by $\{c\}$, we have that d is e-acceptable wrt $\{c\}$. Then, $\{c, d\}$ is e-admissible. Note that there is no e-supported attack on b. However, b does not belong to any e-admissible set, because no e-admissible set `e-supports` b. Assume now that $\mathcal{A}_e = \{a, b, c\}$. $\{b\}$ is the only minimal e-supported attack on d. As no argument attacks b, no e-admissible set contains d. The only e-admissible set is $\{c, b\}$.

The above example enables us to highlight the relationship between the notion of evidential support and the notion of necessary support. It seems that evidential support can be viewed as a kind of *weak necessary support*, in the following sense: Assume that b is supported by a and c; with the necessary support interpretation, the acceptance of b implies the acceptance of a *and* the acceptance of c; with the evidential interpretation, if b is not self-supported, the acceptance of b implies the acceptance of a *or* the acceptance of c and, if b is self-supported, the acceptance of b implies no constraint on a and c.

The above comment suggests to consider the particular case when each argument is self-supported, that is $\mathcal{A}_e = \mathcal{A}$. In that case, X is a *minimal e-supported attack* on a iff X is reduced to one argument which directly attacks a. So, classical acceptability is recovered: a is e-acceptable wrt S iff a is acceptable wrt S in Dung's sense. And as each argument is self-supported, we also recover classical admissibility. That is to say that the support relation is ignored.

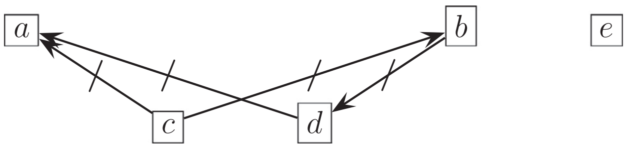
Another interesting case occurs when self-supported arguments are exactly those which do not have any support, that is $\mathcal{A}_e = \{a \in \mathcal{A}$ / there does not exist b such that $b\mathcal{R}_{\mathtt{sup}}a\}$. However, even in that particular case, evidential support cannot be modelled with necessary support, as shown by the following example.

Example 10. We complete Ex 8 by adding an argument e and a support from e to b:



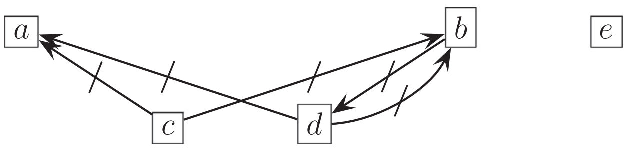
Assume that $\mathcal{A}_e = \{a, c, e\}$ (this is represented by a double box around the elements of $\mathcal{A}_e$). The only $\subseteq$-maximal e-admissible set is $\{c, e, b\}$. Indeed, d is not e-acceptable wrt $\{c\}$ since $\{e, b\}$ is a minimal e-supported attack on d and neither b nor e is attacked.

Now, if we handle the same graph with necessary supports, we first take $\mathcal{R}_{\mathtt{sup}}^{-1}$ and then add supported and mediated attacks. This results in adding an attack from d to a and an attack from c to b:



Taking into account these new attacks, the set is $\{c, b, e\}$ is no longer admissible (there is a conflict between c and b) and $\{c, d\}$ becomes admissible.

If we use the inductive definition for the complex attacks, the resulting AF^{Nc} is the following:



In this case, $\{d\}$ becomes admissible.

The above example shows that the notion of evidential support, even in the particular case of interactions between single arguments, cannot be reduced to strict necessary support (nor to deductive support). So, it is not possible to handle together in the same bipolar framework evidential support and necessary / deductive support. *A fortiori*, this remark is true when one considers that EAF are also able to handle attacks and supports by *sets of arguments*. However, the idea of considering attacks and supports between sets of arguments is related to the notion of *coalitions of arguments* and, in the following section, we show how coalitions can be defined using deductive and necessary supports.

6. Coalitions of supports

In this section, we consider only d-supports (if n-supports appear, they can be translated into d-supports without loss of generality).

Our idea is that coalitions of arguments can be used as *meta arguments* and our purpose is to turn a BAF into a *meta Dung AF* so that the usual Dung's semantics may be applied. A first attempt has been done in [19]. However the proposed meta argumentation system presented some important drawbacks (the main reason is that no interpretation was given to the support).

In the current paper, we show how to fix these drawbacks using a new definition of meta Dung AF which will enable to establish a one-to-one correspondence between extensions of the meta framework and those of AF^{Dc}.

Intuitively, each argument a gives rise to a coalition that contains all the arguments supported (directly or indirectly) by a. A coalition attacks another one if the former contains at least one argument that attacks (with $\mathcal{R}_{\text{att}}$) an argument of the second one.

Definition 18 (*d-Coalition*)**.** Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. Let $a \in \mathcal{A}$, the *d-coalition* [3] associated with the argument a is defined by: $C(a) = \{a\} \cup \{b \text{ s.t. } a \texttt{ supports } b\}$.

Note that $C(a)$ corresponds to the set of nodes that are reachable from a by support edges in the directed graph $\mathcal{G}_b$.

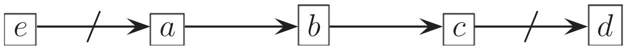
Formally, we define a meta argumentation framework corresponding to a BAF in the following way: For each argument $a \in \mathcal{A}$, the d-coalition $C(a)$ is the meta argument associated with a.

Definition 19 (*Meta framework*)**.** Let $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ with $\mathcal{R}_{\text{sup}}$ being a set of d-supports. The Dung meta argumentation framework corresponding to BAF is $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$, where

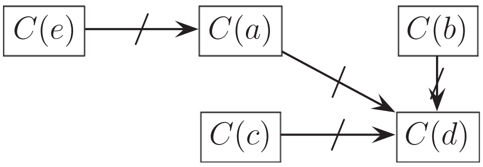
- $\mathcal{A}^{\mathcal{C}}$ denotes the set of all the meta arguments obtained from $\mathcal{A}$ ($\mathcal{A}^{\mathcal{C}} = \{C(a), a \in \mathcal{A}\}$)
- $\mathcal{R}^{\mathcal{C}}$ is an attack relation defined by: $C(a)\mathcal{R}^{\mathcal{C}}C(b)$ iff there exists $x \in C(a)$ and $y \in C(b)$ such that $x\mathcal{R}_{\text{att}}y$.

The following examples illustrate this definition.

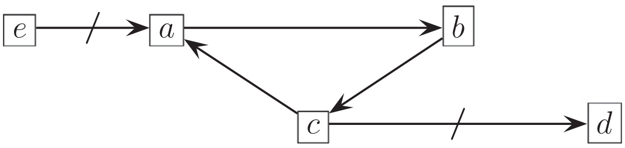
Example 11. Consider BAF represented by:



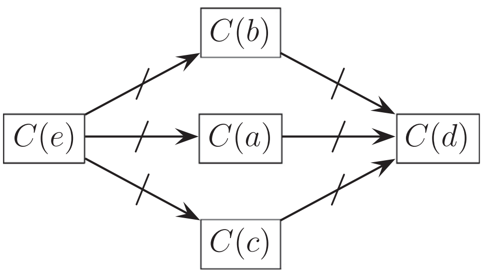
We have $C(a) = \{a, b, c\}$, $C(b) = \{b, c\}$, $C(c) = \{c\}$, $C(d) = \{d\}$, $C(e) = \{e\}$. So the corresponding Dung meta argumentation framework is represented by:



Example 12. Consider BAF represented by:



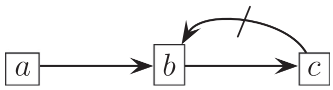
We have $C(a) = \{a, b, c\}$, $C(b) = \{a, b, c\}$, $C(c) = \{a, b, c\}$, $C(d) = \{d\}$, $C(e) = \{e\}$. So the corresponding Dung meta argumentation framework is represented by:



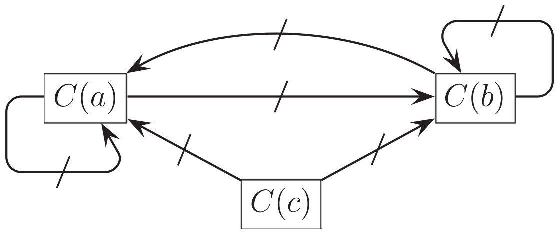
[3] d-Coalition means "deductive coalition".

Note that three distinct meta arguments correspond to the same d-coalition of arguments.

Example 13. Consider BAF represented by:



We have $C(a) = \{a, b, c\}$, $C(b) = \{b, c\}$, $C(c) = \{c\}$. So the corresponding Dung meta argumentation is represented by:



Note that we obtain self-attacking arguments.

The attacks in the meta framework can be characterized in terms of attacks in AF^{Dc}.

Proposition 10. *Let* BAF $= \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Let* $a, b \in \mathcal{A}$. $C(a)\mathcal{R}^{\mathcal{C}}C(b)$ *iff there is an attack from* a *to* b *in* AF^{Dc}.

As a direct consequence, we have:

Proposition 11. *Let* BAF $= \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Let* $S = \{a_1, a_2, \ldots, a_n\} \subseteq \mathcal{A}$. S *is conflict-free in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is conflict-free in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.

The usual Dung's semantics can then be applied on the meta framework. The main result is that there is a one-to-one correspondence between extensions of the meta framework and those of AF^{Dc}.

Proposition 12. *Let* BAF $= \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Given* $S \subseteq \mathcal{A}$, $S = \{a_1, a_2, \ldots, a_n\}$.

1. *S is a $\subseteq$-maximal conflict-free set in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is a $\subseteq$-maximal conflict-free set in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.
2. *S is admissible in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is admissible in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.
3. *S is stable in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is stable in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.

Applying the previous propositions on Example 11 shows that the set $\{e, b, c\}$ is stable in AF^{Dc} and $\subseteq$-maximal among the admissible sets of AF^{Dc}. And in Example 12, the set $\{e, d\}$ is stable in AF^{Dc} and $\subseteq$-maximal among the admissible sets of AF^{Dc}.

7. Related works

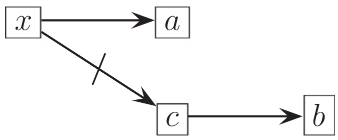
Related works can be partitioned into two parts; the first part is related to the notion of meta argumentation and the second part concerns a more general framework.

7.1. About meta argumentation

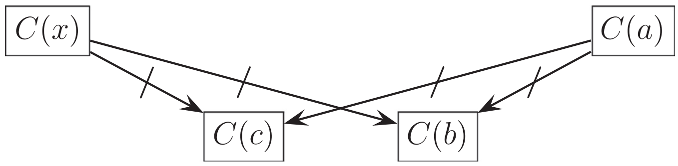
Our approach for meta argumentation is close to the approach described in [25] using necessary support. In this work, the meta argument associated with an argument a, called cluster, contains a and all the arguments that are directly necessary for a. Turning the necessary supports into the dual deductive supports, the cluster will contain a and all the arguments that directly d-support a. In contrast, the d-coalition contains a and all the arguments that support a, directly or not.

Nevertheless, in [25], the binary relation which encodes the necessary support is assumed irreflexive and transitive. The transitive nature of the necessary support enables to recover indirect support in the clusters. So, clusters are similar to d-coalitions. However, the irreflexive nature of the necessary support excludes some of the d-coalitions. In the particular case when the support relation is irreflexive and transitive, using the results presented in Section 6 and the duality between deductive and necessary support, it can be proved that the meta framework based on clusters enables to encode the attacks in AF^{Nc}, and not only the extended attacks defined in Definition 12.

Example 14. Consider BAF using n-supports and represented by:



The clusters are $C(a) = \{a, x\}, C(b) = \{b, c\}, C(c) = \{c\}, C(x) = \{x\}$.



Note that there is a meta attack from $C(a)$ to $C(b)$ whereas there is no extended attack from a to b.

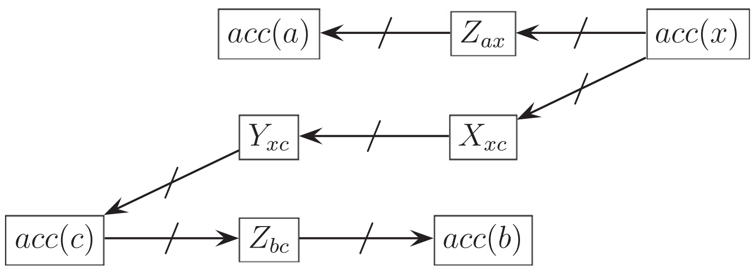
Using deductive support, [20] described a meta argumentation framework in which meta arguments are auxiliary arguments representing pairs of interacting arguments. More precisely,

- A direct attack from x to c is encoded by a path of length 3 in the meta framework: $acc(x)\mathcal{R}_{\mathsf{att}}X_{xc}\mathcal{R}_{\mathsf{att}}Y_{xc}\mathcal{R}_{\mathsf{att}}acc(c)$;
- A support from b to c is encoded by a path of length 2 in the meta framework: $acc(c)\mathcal{R}_{\mathsf{att}}Z_{bc}\mathcal{R}_{\mathsf{att}}acc(b)$;

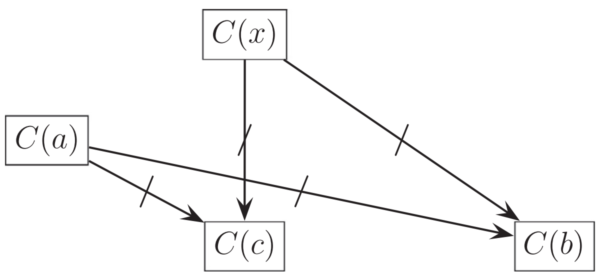
  where X_{xc} (resp. Y_{xc}, Z_{bc}) is read as "the attack from x to c is not active" (resp. "the attack from x to c is active", "b does not support c") and $acc(x)$ is read as "x is acceptable".

The above approach enables to encode a mediated attack, but does not enable to encode a supported attack (see the following example):

Example 7 (Cont'd)**.** In this example the meta argumentation framework proposed by [20] is represented by:



And the Dung meta argumentation framework obtained with our proposition (see Section 6) is represented by:



With both approaches, we obtain the same ⊆-maximal admissible set $\{a, x\}$ in the original framework. Nevertheless, an essential difference between both approaches concerns the conflict-free sets: following the meta argumentation framework proposed by [20], the set $\{a, c\}$ is conflict-free that is not the case in our approach.

Moreover, the reading of the auxiliary arguments is not intuitive. From the reading giving by Boella, there should be a symmetric attack between X_{xc} and Y_{xc}. As for the encoding of a deductive support from an argument b to an argument c, it seems strange to create an attack from "c is accepted" to "b does not support c".

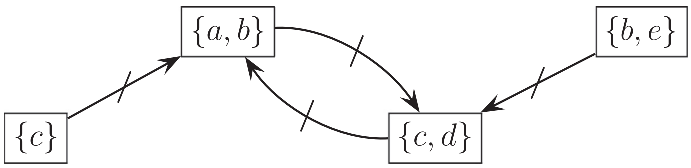
The last approach proposing a meta argumentation framework is given by [24]. In order to compare with our approach, we still consider the particular case where attacks and supports are carried out by single arguments. The meta arguments represent groups of arguments and are built from the notion of self-supporting path, as follows:

- Each ⊆-maximal self supporting path of $\langle \mathcal{A}, \mathcal{R}_{\text{sup}} \rangle$ is a meta argument.
- For each $x \in \mathcal{A}$ such that x is directly attacked, each ⊆-maximal self supporting path of $\langle \mathcal{A} \setminus \{x\}, \mathcal{R}_{\text{sup}} \rangle$ is a meta argument.

There is a meta attack from S_1 to S_2 iff there exist $a \in S_1$ and $b \in S_2$ such that $a\mathcal{R}_{\text{att}}b$.

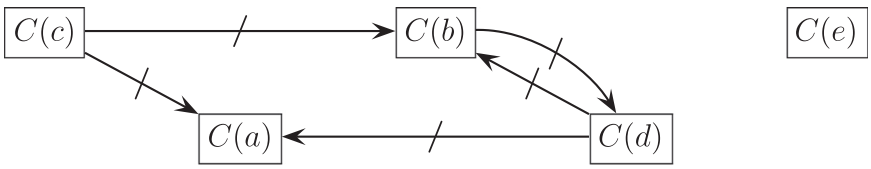
The meta argumentation framework of [24] captures the e-admissibility in the sense that $\{S_1, S_2, \ldots, S_p\}$ is admissible in the meta framework iff the set of arguments $S_1 \cup S_2 \ldots \cup S_p$ is e-admissible in the evidential argumentation framework. Let us consider again Example 10.

Example 10 (Cont'd). Assuming that $\mathcal{A}_e = \{a, c, e\}$, the meta argumentation framework proposed by [24] is represented by:



It follows that $\{\{e, b\}, \{c\}\}$ is admissible in the meta framework. Note that there are two distinct meta arguments that contain b, reflecting the notion of weak necessary support: e is necessary for b or a is necessary for b.
In contrast, with the necessary support, turned into deductive supports, we obtain the d-coalitions $C(a) = \{a\}$, $C(b) = \{b, a, e\}$, $C(c) = \{c\}$, $C(d) = \{d, c\}$, $C(e) = \{e\}$. Note that there is only one d-coalition containing b.

So the Dung meta argumentation framework obtained with our approach is represented by:



It follows that $\{C(c), C(d), C(e)\}$ is admissible in the meta framework, and so $\{c, d, e\}$ is admissible in AF^{Dc}. In our approach, b belongs to only one d-coalition. So, there is only one attack against the meta argument $C(d) = \{c, d\}$, for which there exists a counter attack ($C(d)$ attacks $C(b)$). In contrast, in the meta argumentation framework proposed by [24], there are two attacks against the meta argument $\{c, d\}$: one attack is by the meta argument $\{a, b\}$, for which there exists a counter attack; the other attack is by the meta argument $\{b, e\}$, for which there is no counter attack.

7.2. Other works

Another interesting related work has been proposed in the more general setting of Abstract Dialectical Framework (ADF for short) [26].

This framework allows to represent a variety of dependencies between nodes in an interaction graph.

In a BAF, there are two kinds of edges, one for the support and one for the attack. In contrast, there is only one kind of edge in an ADF. An edge between a and b represents a dependency between a and b. The kind of dependency is specified by associating an acceptance condition with each node of the graph. The acceptance condition of s specifies how the status of s depends on the status of the parents of s, and gives the exact conditions under which s is accepted.

Acceptance conditions are much more flexible than the conditions described above for deductive, necessary or evidential support. For instance, if c depends on a and b, the following constraint can be taken into account: c is accepted if and only if exactly one of $\{a, b\}$ is accepted.

Formally, an ADF is a directed graph whose nodes represent arguments which can be accepted or not. For each node s, the set of its parents in the graph is denoted by $par(s)$. An acceptance condition of s, denoted by C_s, is a function that assigns to each subset R of $par(s)$ one of the values *in*, *out*. $C_s(R) = in$ means that if the nodes in R are accepted and those in $par(s) \setminus R$ are not accepted, then s is accepted. So, the *exact conditions* under which s is accepted are given by the subsets $R \subseteq par(s)$ such that $C_s(R) = in$.

Note that if s has no parent in the graph, then s is accepted if and only if $C_s(\varnothing) = in$. Moreover, as explained in [26], if each edge represents an attack, $C_s(R) = in$ iff $R = \varnothing$.

In the following, we show that the ADF model does not always enable to capture exactly the notion of deductive support. We consider three different examples. In each case, starting from a BAF considered as a dependency graph, we write the possible acceptance conditions and try to determine whether some of them may correspond to deductive or necessary support.

Example 15. Consider the BAF represented by:

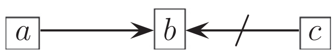
We are interested in the acceptance condition C_b.
Considering $par(b) = \{a\}$, there exist four possible cases for defining C_b:

	Sets R s.t. $C_b(R) = in$	Conditions under which b is accepted
	None	No R s.t. $C_b(R) = in$; so there is no condition under which b is accepted (b cannot be accepted whatever the status of a)
C_b^0	$\varnothing$	$\exists$ one R s.t. $C_b(R) = in$; so there is only one condition under which b is accepted
C_b^1	$\{a\}$	$\exists$ one R s.t. $C_b(R) = in$; so there is only one condition under which b is accepted
C_b^2	$\varnothing, \{a\}$	$\exists$ two R s.t. $C_b(R) = in$; so there are two different conditions under which b is accepted

The second case exactly corresponds to an attack from a to b ($C_b(R) = in$ iff $R = \varnothing$). Since, in our example, there is no attack from a to b, we do not use it. Thus, in order to characterize the support from a to b, it only remains two possible acceptance conditions for b, C_b^1 and C_b^2.

- $C_b^1(\{a\}) = in$ means *b is accepted if a is accepted*. And as C_b^1 specifies only one condition under which b is accepted, we also have the equivalence *b is accepted if and only if a is accepted*. So, C_b^1 models a support which is both deductive and necessary.
- $C_b^2(\varnothing) = in$ means *b is accepted if a is not accepted*.
  $C_b^2(\{a\}) = in$ means *b is accepted if a is accepted*.
  So b will be accepted whatever the status of a. Thus the notion of support is not captured.

Example 16. We complete the above example by adding an attack from c to b. So, we consider the BAF represented by:



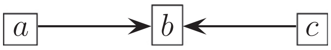
We are interested in the acceptance condition C_b. As there is an attack from c to b, if c is accepted, then b cannot be accepted. This constraint can be expressed by the following constraint on C_b: If $C_b(R) = in$ then $c \notin R$. So only two subsets of $par(b)$ may be *in*: $\varnothing$ and $\{a\}$. And we obtain the same discussion as in the above example. As there is no attack from a to b, we cannot have $C_b(R) = in$ iff $R = \varnothing$. So, there are two possible acceptance conditions for b, C_b^1 and C_b^2 defined by:

$$C_b^1(R) = in \text{ iff } R = \{a\} \text{ and } C_b^2(\varnothing) = in, C_b^2(\{a\}) = in.$$

- $C_b^1(\{a\}) = in$, means *b is accepted if a is accepted and c is not accepted*. And as C_b^1 specifies only one condition under which b is accepted, we also have the equivalence *b is accepted if and only if a is accepted and c is not accepted*. It follows that if b is accepted, then a is accepted. So, C_b^1 enables to model a necessary support from a to b (and not the deductive support since the condition *if a is accepted then b is accepted* does not hold).
- With C_b^2, b is accepted iff c is not accepted, whatever the status of a. So none notion of support is captured.

This example shows that a deductive support cannot always be captured in the ADF model.

Example 17. Consider the BAF represented by:



We are interested in the acceptance condition C_b, and among all the possible functions we discuss C_b^1, C_b^2, C_b^3 defined by:

$$C_b^1(\{a\}) = in,\ C_b^1(\{c\}) = in,\ C_b^1(\{a, c\}) = out,\ C_b^1(\varnothing) = out;$$
$$C_b^2(\{a\}) = in,\ C_b^2(\{c\}) = in,\ C_b^2(\{a, c\}) = in,\ C_b^2(\varnothing) = out;$$
$$C_b^3(\{a, c\}) = in,\ C_b^3(\{a\}) = out,\ C_b^3(\{c\}) = out,\ C_b^3(\varnothing) = out.$$

So, we have:

- With C_b^1, b is accepted if and only if exactly one of $\{a, c\}$ is accepted.
- With C_b^2, b is accepted iff a is accepted *or* c is accepted. So C_b^2 enables to model a support which is both deductive ("if" part) and weak necessary ("only if" part).
- With C_b^3, b is accepted if and only if a is accepted *and* b is accepted. So C_b^3 enables to model a support which is both necessary ("only if" part) and weak deductive ("if" part).

This example shows that a purely deductive (resp. purely necessary) support cannot always be captured in the ADF model.

8. Conclusions and future works

In this paper, we have considered three recent proposals for specializing the support relation in abstract argumentation: the deductive support, the necessary support and the evidential support. These proposals have been developed independently within different frameworks and with appropriate modellings, based on different intuitions.

We have restated these proposals in a common setting, the bipolar argumentation framework. Basically, the idea is to keep the original arguments, to add complex attacks defined by the combination of the original attack and the support, and to modify the classical notions of acceptability. We have proposed a comparative study of the modellings obtained for the considered variants of the support, which has enabled us to highlight relationships and differences between these variants. Namely, we have shown a kind of duality between the deductive and the necessary interpretations of support, which results in a duality in the modelling by complex attacks. In contrast, the evidential interpretation is quite different and cannot be captured with deductive or necessary supports.

So, the abstract bipolar argumentation framework is a suitable tool for handling applications where deductive *as well as* necessary supports are expressed. By cons, it is no longer the case as soon as evidential supports also appear in the same applications.

Evidential support has been captured by a meta argumentation framework, which instantiates Dung's framework with meta arguments. Following the same line, we have also proposed a meta argumentation framework taking into account the deductive/necessary supports and preserving some semantics. This proposition allows for new understandings of the differences between the variants of support.

This paper addresses how various notions of support can be handled in abstract argumentation and so it is a first step towards a better understanding of the notion of support in argumentation.

The next step is to discuss how these different notions of support can be built from the internal structure of the arguments (see [27]). In particular, it would be interesting to study how these three types of support (deductive, necessary, evidential) can capture different types of reasoning when arguments are built from pieces of knowledge.

More generally, it would be interesting to extend the discussion to the case when attacks and supports can be carried out by sets of arguments, as in the evidential argumentation framework.

Another interesting topic for further research is the representation of defeasible support in bipolar frameworks. A promising proposal has been given in [20]. As interactions between arguments are represented by auxiliary arguments in the meta argumentation framework, an attack from an argument to a support can be easily represented by an attack in the meta framework. A direct representation of defeasible support in a BAF or in the meta framework based on coalitions must be investigated.

As regards meta argumentation, the study of coalitions could be continued in connection with recent works about weighted argumentation frameworks (see [28,10,11,29,30]). The following questions appear to be relevant: How can weights on the arguments and/or weights on the supports be combined into weights on coalitions? Can the cardinality be taken into account for weighting a coalition? How can weighted coalitions be handled in the meta framework?

Acknowledgements

We wish to thank the reviewers for their very constructive work. Thanks to their suggestions, this paper has been greatly improved.

Appendix A. Proofs

Proposition 1. *Consider* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Given* $S \subseteq \mathcal{A}$*,*

- *S is safe wrt* $\mathcal{R}_{\text{att}}$ *in* BAF *iff S is safe wrt* $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ *in* BAF*.*
- *S is safe wrt* $\mathcal{R}_{\text{att}}$ *in* BAF *iff S is conflict-free wrt* $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ *in* BAF*.*

Proof.

- For the first result, it is sufficient to prove that if S is safe wrt $\mathcal{R}_{\text{att}}$ in BAF, then S is also safe wrt $\mathcal{R}_{\text{att}}^{\text{med}}$ in BAF. Assume that S is not safe wrt $\mathcal{R}_{\text{att}}^{\text{med}}$ in BAF. Then there exists $a, b \in S$, and $c \in \mathcal{A}$ such that (b `supports` c or $c \in S$) and there is a mediated attack from a to c. So, there is a sequence $c_1(= c)\mathcal{R}_{\text{sup}} \dots \mathcal{R}_{\text{sup}} c_p$, and $a\mathcal{R}_{\text{att}} c_p$, $p \geq 2$, and either $c \in S$ or b `supports` c with $b \in S$.
  If $c \in S$, we obtain a contradiction with the assumption that S is safe wrt $\mathcal{R}_{\text{att}}$ in BAF.
  If $b \in S$ and b `supports` c, there exists a sequence $b_1(= b)\mathcal{R}_{\text{sup}} \dots \mathcal{R}_{\text{sup}}\ b_{n-1}\ \mathcal{R}_{\text{sup}}\ b_n(= c)$, $n \geq 2$. By concatenating the two sequences, we obtain a sequence of supports from b to c_p, and so a contradiction with the fact that S is safe wrt $\mathcal{R}_{\text{att}}$ in BAF.
- Due to the above result, if S is safe wrt $\mathcal{R}_{\text{att}}$ in BAF, then S is safe wrt $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ in BAF, and so by definition of safety, S is conflict-free wrt $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ in BAF. Conversely, let S be conflict-free wrt $\mathcal{R}_{\text{att}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$. Assume that S is not safe wrt $\mathcal{R}_{\text{att}}$. Then there exists $a, b \in S$, and $c \in \mathcal{A}$ such that (b `supports` c or $c \in S$) and there is a direct attack from a to c.
  If $c \in S$, we obtain a contradiction with the assumption that S is conflict-free wrt $\mathcal{R}_{\text{att}}$ in BAF.
  If $b \in S$ and b `supports` c, we obtain exactly a mediated attack from a to b and so a contradiction with the fact that S is conflict-free wrt $\mathcal{R}_{\text{att}}^{\text{med}}$ in BAF. □

Proposition 2. *Consider* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports and* AF^D *its associated Dung AF. Given* $S \subseteq \mathcal{A}$*,*

- *If S is conflict-free wrt* $\mathcal{R}_{\text{att}}$ *and closed under* $\mathcal{R}_{\text{sup}}$ *in* BAF*, then S is also conflict-free in* AF^D *(that is conflict-free wrt* $\mathcal{R}_{\text{att}}^D$*).*
- *If S is conflict-free wrt* $\mathcal{R}_{\text{att}}^D$ *and closed under* $\mathcal{R}_{\text{sup}}$ *in* BAF*, then S is also safe wrt* $\mathcal{R}_{\text{att}}^D$ *in* BAF*.*

Proof.

- For the first result, it is sufficient to prove that "if S is conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$, then S is conflict-free wrt $\mathcal{R}_{\text{att}}^{\text{sup}} \cup \mathcal{R}_{\text{att}}^{\text{med}}$ in BAF". This proof is made by a *reduction ad absurdum*.
  Assume that there are arguments $a, b \in S$ with a supported attack from a to b. There is a sequence $a_1(= a)\mathcal{R}_{\text{sup}} \dots \mathcal{R}_{\text{sup}}$ $a_{n-1}\mathcal{R}_{\text{att}} b$, with $n \geq 3$. As S is closed under $\mathcal{R}_{\text{sup}}$, we have $a_{n-1} \in S$. So there is a direct attack between two elements of S, which contradicts the assumption that S is conflict-free wrt $\mathcal{R}_{\text{att}}$.
  Now assume that there are arguments $a, b \in S$ with a mediated attack from a to b. There is a sequence $b_1(= b)\mathcal{R}_{\text{sup}} \dots \mathcal{R}_{\text{sup}}$ b_p, and $a\mathcal{R}_{\text{att}} b_p$, $p \geq 2$. As S is closed under $\mathcal{R}_{\text{sup}}$, we have $b_p \in S$. So there is a direct attack between two elements of S, which contradicts the assumption that S is conflict-free wrt $\mathcal{R}_{\text{att}}$.
- The second result follows from the following remark: In Definition 6, if S is closed under $\mathcal{R}_{\text{sup}}$, the condition (b `supports` c or $c \in S$) reduces to $c \in S$. So, if S is closed under $\mathcal{R}_{\text{sup}}$, S is safe wrt a complex attack is equivalent to S is conflict-free wrt that complex attack. □

Proposition 3. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Given* $S \subseteq \mathcal{A}$*, if S is conflict-free wrt* $\mathcal{R}_{\text{att}}$ *and closed under* $\mathcal{R}_{\text{sup}}$ *in* BAF*, then S is also conflict-free in* AF^{Dc}*.*

Proof. Let S be conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF. Due to Proposition 2, we know that S is conflict-free in AF^D (that is conflict-free wrt $\mathcal{R}_{\text{att}}^D$).
So, we have to prove that there is no super-mediated attack between two elements of S. Assume that it is not the case, and that there exists a super-mediated attack from a to b in S. Then, there exists $c \in \mathcal{A}$ such that $a\mathcal{R}_{\text{att}}^{\text{sup}} c$ and $b\mathcal{R}_{\text{sup}} c$. The supported attack from a to c is composed of a support from a to an argument d and an attack from d to c. As S is closed under $\mathcal{R}_{\text{sup}}$ in BAF, d belongs to S, so there is in fact a mediated attack from d, element of S, to b, which contradicts the fact that S is conflict-free wrt $\mathcal{R}_{\text{att}}^D$. □

Proposition 4. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Given* $S \subseteq \mathcal{A}$*, S is a* $\subseteq$*-maximal conflict-free set in* AF^{Dc} *iff S is* $\subseteq$*-maximal among the sets which are conflict-free wrt* $\mathcal{R}_{\text{att}}$ *and closed under* $\mathcal{R}_{\text{sup}}$ *in* BAF*.*

Proof.

- ($\Rightarrow$)-Part: Let S be a $\subseteq$-maximal conflict-free set in AF^{Dc}. So, S is conflict-free wrt $\mathcal{R}_{\text{att}}$ in BAF. Assume that S is not closed under $\mathcal{R}_{\text{sup}}$ in BAF. Then, there exist $a \in S$ and $b \notin S$ with $a\mathcal{R}_{\text{sup}}b$. As S is $\subseteq$-maximal conflict-free in AF^{Dc}, $S \cup \{b\}$ is not conflict-free in AF^{Dc}. So there exists $c \in S$ such that either $b\mathcal{R}^{Dc}_{\text{att}}c$ or $c\mathcal{R}^{Dc}_{\text{att}}b$. In each case, it is possible to build an attack between a and c, which contradicts the fact that S is conflict-free set in AF^{Dc}. Let us enumerate the different cases which may be encountered.
  - If $b\mathcal{R}_{\text{att}}c$, as $a\mathcal{R}_{\text{sup}}b$, we obtain a supported attack from a to c.
  - If $b\mathcal{R}^{\text{sup}}_{\text{att}}c$, as $a\mathcal{R}_{\text{sup}}b$, we also obtain a supported attack from a to c.
  - If $b\mathcal{R}^{\text{s-med}}_{\text{att}}c$, as $a\mathcal{R}_{\text{sup}}b$, we obtain a super-mediated attack from a to c.
  - If $c\mathcal{R}_{\text{att}}b$, as $a\mathcal{R}_{\text{sup}}b$, we obtain a mediated attack from c to a.
  - If $c\mathcal{R}^{\text{sup}}_{\text{att}}b$, as $a\mathcal{R}_{\text{sup}}b$, we obtain a super-mediated attack from c to a.
  - If $c\mathcal{R}^{\text{s-med}}_{\text{att}}b$, as $a\mathcal{R}_{\text{sup}}b$, we also obtain a super-mediated attack from c to a.

  So, we have proved that S is conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF. It remains to prove that S is $\subseteq$-maximal. Assume that it is not the case. Then, there exists $S' \subseteq \mathcal{A}$, such that S is strictly included in S' and S' is conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF. Due to Proposition 2, it holds that S' is conflict-free in AF^{D} (that is conflict-free wrt $\mathcal{R}^{D}_{\text{att}}$). As S is strictly included in S', there exists $b \in S'$ such that $b \notin S$. So, since S is conflict-free wrt $\mathcal{R}^{Dc}_{\text{att}}$ and S' is conflict-free wrt $\mathcal{R}^{D}_{\text{att}}$, there is an attack between b and an element a of S and this attack must be a super-mediated one (as S' is conflict-free wrt $\mathcal{R}^{D}_{\text{att}}$, this attack can be neither a direct attack, nor a supported attack, nor a simple mediated attack). Two cases may occur.
  - If the attack is from a to b, there exists a supported attack from a to an argument d and a support from b to d. The supported attack from a to d is composed of a support from a to an argument e and an attack from e to d. As S is closed under $\mathcal{R}_{\text{sup}}$ in BAF, e belongs to S and so to S'. So, there is in fact a mediated attack from e, element of S', to b, which contradicts the fact that S' is conflict-free wrt $\mathcal{R}^{D}_{\text{att}}$.
  - If the attack is from b to a, there exists a supported attack from b to an argument d and a support from a to d. As S is closed under $\mathcal{R}_{\text{sup}}$ in BAF, d belongs to S and so to S'. So, there is a supported attack from b to an element of S', which contradicts the fact that S' is conflict-free wrt $\mathcal{R}^{D}_{\text{att}}$.
- ($\Leftarrow$)-Part: Let S be a subset of $\mathcal{A}$, $\subseteq$-maximal among the sets which are conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF. Due to Proposition 3, we know that S is conflict-free in AF^{Dc}.
  It remains to prove that S is $\subseteq$-maximal among the conflict-free sets in AF^{Dc}. Assume that it is not the case. Then, there exists $S' \subseteq \mathcal{A}$, such that S is strictly included in S' and S' is conflict-free in AF^{Dc}. As $\mathcal{A}$ is finite, we can assume that S' is $\subseteq$-maximal. So, from the ($\Rightarrow$)-part of the proof, we know that S' is conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF. That is in contradiction with S being $\subseteq$-maximal among the sets which are conflict-free wrt $\mathcal{R}_{\text{att}}$ and closed under $\mathcal{R}_{\text{sup}}$ in BAF. $\square$

Proposition 5. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports.* $\mathcal{R}_{\text{d-att}} = \mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}}$*. In other words, a*`d-attacks`*b iff* $(a, b) \in \mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}}$*.*

Proof.

- It is easy to prove that $\mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}} \subseteq \mathcal{R}_{\text{d-att}}$. If $a\mathcal{R}_{\text{att}}b$, or $a\mathcal{R}^{\text{sup}}_{\text{att}}b$, or $a\mathcal{R}^{\text{med}}_{\text{att}}b$, then by definition (Basic case), a `d-attacks` b. If there is a super-mediated attack from a to b, composed of a supported attack from a to c (so, a `d-attacks` c – Basic case), and a support from b to c, then, by definition (Case 2), there is a `d-attack` from a to b.
- Conversely, we have to prove that $\mathcal{R}_{\text{d-att}} \subseteq \mathcal{R}_{\text{att}} \cup \mathcal{R}^{\text{sup}}_{\text{att}} \cup \mathcal{R}^{\text{s-med}}_{\text{att}}$. We give a proof by structural induction. Let (a, b) such that a `d-attacks` b.
  Basic case: either $a\mathcal{R}_{\text{att}}b$, or $a\mathcal{R}^{\text{sup}}_{\text{att}}b$, or $a\mathcal{R}^{\text{med}}_{\text{att}}b$. So, $a\mathcal{R}^{Dc}_{\text{att}}b$.
  Case 1: there exists an argument c such that a `supports` c and c `d-attacks` b. Assuming that $c\mathcal{R}^{Dc}_{\text{att}}b$, we have to prove that $a\mathcal{R}^{Dc}_{\text{att}}b$. As $c\mathcal{R}^{Dc}_{\text{att}}b$, we have either $c\mathcal{R}_{\text{att}}b$, or $c\mathcal{R}^{\text{sup}}_{\text{att}}b$, or $c\mathcal{R}^{\text{s-med}}_{\text{att}}b$. If $c\mathcal{R}_{\text{att}}b$ or $c\mathcal{R}^{\text{sup}}_{\text{att}}b$, as a `supports` c, we obtain a supported attack from a to b. If $c\mathcal{R}^{\text{s-med}}_{\text{att}}b$, as a `supports` c, we obtain a super-mediated attack from a to b. So, in each case, $a\mathcal{R}^{Dc}_{\text{att}}b$.

Case 2: there exists an argument c such that a d-attacks c and b supports c. Assuming that $a\mathcal{R}_{\text{att}}^{Dc}c$, we have to prove that $a\mathcal{R}_{\text{att}}^{Dc}b$. As $a\mathcal{R}_{\text{att}}^{Dc}c$, we have either $a\mathcal{R}_{\text{att}}c$, or $a\mathcal{R}_{\text{att}}^{\text{sup}}c$, or $a\mathcal{R}_{\text{att}}^{\text{s-med}}c$. If $a\mathcal{R}_{\text{att}}c$, as b supports c, we obtain a mediated attack from a to b. If $a\mathcal{R}_{\text{att}}^{\text{sup}}c$, or $a\mathcal{R}_{\text{att}}^{\text{s-med}}c$, as b supports c, we obtain a super-mediated attack from a to b. So, in each case, $a\mathcal{R}_{\text{att}}^{Dc}b$. □

Proposition 6. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of n-supports. The combination of the direct attacks and the n-supports can be handled by turning the n-supports into the dual d-supports and then adding the supported attacks and mediated attacks.*

Proof. Given $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$, let us consider $\mathsf{BAF}_{sym} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}}^{-1} \rangle$ [4] and its associated Dung AF for the deductive support AF_{sym}^{D}. So, using the duality between n-supports and d-supports, we have $\mathsf{AF}^{N} = \mathsf{AF}_{sym}^{D}$. □

Proposition 7. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Consider* $a \in \mathcal{A}$*.* a *is a self-attacking argument of* AF^{Dc} *iff*

- *either* $a\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that* a supports b *and* $b\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that* a supports b *and* $a\mathcal{R}_{\text{att}}b$,
- *or* $\exists b$ *and* $c \in \mathcal{A}$, *such that* a supports c, $c\mathcal{R}_{\text{att}}b$ *and* a supports b.

Proof. The proposition is an obvious consequence of definitions and propositions concerning the deductive support. □

Proposition 8. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of n-supports. Consider* $a \in \mathcal{A}$*.* a *is a self-attacking argument of* AF^{Nc} *iff*

- *either* $a\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that* b supports a *and* $b\mathcal{R}_{\text{att}}a$,
- *or* $\exists b \in \mathcal{A}$, *such that* b supports a *and* $a\mathcal{R}_{\text{att}}b$,
- *or* $\exists b$ *and* $c \in \mathcal{A}$, *such that* c supports a, $c\mathcal{R}_{\text{att}}b$ *and* b supports a.

Proof. The proposition is an obvious consequence of Proposition 7 following the duality between deductive and necessary supports. □

Proposition 9. *Let* $\mathsf{EAF} = \langle \mathcal{A}, \mathcal{A}_e, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$*.* X *is a minimal e-supported attack on the argument* a *iff* X *is the set of arguments appearing in a minimal elementary sequence* $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n$ *such that* $b_1 \in \mathcal{A}_e$ *and* $b_n\mathcal{R}_{\text{att}}a$.

Proof. We first notice that if $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n$ is an elementary sequence such that $b_1 \in \mathcal{A}_e$ and $b_n\mathcal{R}_{\text{att}}a$, the set $X = \{b_1, \ldots, b_{n-1}, b_n\}$ is an e-supported attack on a (this follows from the definition of e-supported attack). Furthermore, assuming that the sequence $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n$ is minimal exactly means that there is no elementary sequence $c_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}c_p$ such that $c_1 \in \mathcal{A}_e$, $c_p\mathcal{R}_{\text{att}}a$ and $\{c_1 \ldots c_p\} \subseteq \{b_1 \ldots b_n\}$.

- ($\Rightarrow$)-Part: Let X be a minimal e-supported attack on the argument a. By definition, there exists $b \in X$ such that $b\mathcal{R}_{\text{att}}a$ and b is e-supported by X. So, either $b \in \mathcal{A}_e$ or there is an elementary sequence $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_{n-1}\mathcal{R}_{\text{sup}}b$ such that $\{b_1 \ldots b_{n-1}\} \subseteq X$ and $b_1 \in \mathcal{A}_e$. Let Y be a subset of $\mathcal{A}$ defined as follows: If $b \in \mathcal{A}_e$, then $Y = \{b\}$, else $Y = \{b_1 \ldots b_{n-1}, b\}$. Obviously, Y e-supports b, so Y is an e-supported attack on a. Moreover $Y \subseteq X$. As X be a minimal e-supported attack on the argument a, we conclude that $Y = X$.

  It remains to prove the minimality of the sequence. Assume that there exists an elementary sequence $c_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}c_p$ such that $c_1 \in \mathcal{A}_e$, $c_p\mathcal{R}_{\text{att}}a$ and $\{c_1 \ldots c_p\} \subset X$. Due to the preliminary remarks given in the proof, $Z = \{c_1 \ldots c_p\}$ is an e-supported attack of a. As $Z \subset X$, there is a contradiction with the fact that X be a minimal e-supported attack on the argument a.
- ($\Leftarrow$)-Part: Let X be the set of arguments appearing in a minimal elementary sequence $b_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}b_n$ such that $b_1 \in \mathcal{A}_e$ and $b_n\mathcal{R}_{\text{att}}a$. From the preliminary remarks given in the proof, X is an e-supported attack on a. It remains to prove the minimality. Assume that there exists Z such that $Z \subset X$ and Z is an e-supported attack on a. From the ($\Rightarrow$)-part of the proof, we can build an elementary sequence $c_1\mathcal{R}_{\text{sup}} \ldots \mathcal{R}_{\text{sup}}c_p$ such that $c_1 \in \mathcal{A}_e$, $c_p\mathcal{R}_{\text{att}}a$ and $\{c_1 \ldots c_p\} \subseteq Z$. As $Z \subset X$, we have $\{c_1 \ldots c_p\} \subset X$, which contradicts the fact that X is the set of arguments appearing in a minimal elementary sequence. □

[4] The subscript "sym" means that the support relation taken into account is symmetrical to the initial support relation.

Proposition 10. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Let* $a, b \in \mathcal{A}$. $C(a)\mathcal{R}^{\mathcal{C}}C(b)$ *iff there is an attack from a to b in* AF^{Dc}.

Proof.

- ($\Rightarrow$)-Part: $C(a)\mathcal{R}^{\mathcal{C}}C(b)$ means that there exist $x \in C(a)$ and $y \in C(b)$ such that $x\mathcal{R}_{\text{att}}y$. Due to the definition of a d-coalition, four cases may be encountered.
  - $x = a$, $y = b$ and $x\mathcal{R}_{\text{att}}y$: there is a direct attack from a to b.
  - $x = a$, there is a sequence of supports from b to y and $x\mathcal{R}_{\text{att}}y$: there is a mediated attack from a to b
  - $y = b$, there is a sequence of supports from a to x and $x\mathcal{R}_{\text{att}}y$: there is a supported attack from a to b
  - there is a sequence of supports from a to x, a sequence of supports from b to y and $x\mathcal{R}_{\text{att}}y$: there is a super-mediated attack from a to b

  In each case, we find an attack from a to b in AF^{Dc}.
- ($\Leftarrow$)-Part: Assume that there is an attack from a to b in AF^{Dc}. Due to the definition of AF^{Dc}, three cases may be encountered.
  - there is a direct attack from a to b: as $a \in C(a)$ and $b \in C(b)$, we have $C(a)\mathcal{R}^{\mathcal{C}}C(b)$
  - there is a supported attack from a to b: it means that there is a sequence of supports from a to an argument c and a direct attack from c to b. So we have $c \in C(a)$, c attacks b and $b \in C(b)$ and then $C(a)\mathcal{R}^{\mathcal{C}}C(b)$
  - there is a super-mediated attack from a to b: it means that there is a sequence of supports from b to an argument c and a direct or supported attack from a to c. So we have $c \in C(b)$ and c is attacked by an argument of $C(a)$, and then $C(a)\mathcal{R}^{\mathcal{C}}C(b)$. □

Proposition 12. *Let* $\mathsf{BAF} = \langle \mathcal{A}, \mathcal{R}_{\text{att}}, \mathcal{R}_{\text{sup}} \rangle$ *with* $\mathcal{R}_{\text{sup}}$ *being a set of d-supports. Given* $S \subseteq \mathcal{A}$, $S = \{a_1, a_2, \ldots, a_n\}$.

1. *S is a* $\subseteq$*-maximal conflict-free set in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is a* $\subseteq$*-maximal conflict-free set in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.
2. *S is admissible in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is admissible in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.
3. *S is stable in* AF^{Dc} *iff* $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ *is stable in* $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$.

Proof.

1. Follows directly from the above proposition, since each argument corresponds to exactly one meta argument
2. Assume that $S = \{a_1, a_2, \ldots, a_n\}$ is admissible in AF^{Dc}. So S is conflict-free in AF^{Dc}. We know that $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ is also conflict-free in $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$. Now assume that $C(a_i)$ is attacked by a meta argument $C(b)$. Due to the previous results, we know that b attacks a_i in AF^{Dc}. As S is admissible in AF^{Dc}, there exists $a_j \in S$ such that a_j attacks b in AF^{Dc}. So, $C(a_j)$ attacks $C(b)$ in $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$. We have proved that $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ is admissible in $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$. The proof for the converse is analogous.
3. Assume that $S = \{a_1, a_2, \ldots, a_n\}$ is stable in AF^{Dc}. So S is conflict-free in AF^{Dc}. We know that $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ is also conflict-free in $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$. Assume that $C(b)$ is a meta argument not belonging to $\{C(a_1), C(a_2), \ldots, C(a_n)\}$. So $b \notin S$. As S is stable, there exists $a_i \in S$ such that a_i attacks b in AF^{Dc}. So, $C(a_i)$ attacks $C(b)$ in $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$. We have proved that $\{C(a_1), C(a_2), \ldots, C(a_n)\}$ is stable in $\langle \mathcal{A}^{\mathcal{C}}, \mathcal{R}^{\mathcal{C}} \rangle$. The proof for the converse is analogous. □

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