Semi-analytical stability model for interrupted milling processes

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SEMI-ANALYTICAL STABILITY MODEL FOR INTERRUPTED MILLING PROCESSES

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ABSTRACT

The stability diagrams are one of the most useful process planning tools to improve the productivity of the cutting processes when they are limited by chatter vibrations. There are many methods proposed in the literature to predict the stability of milling processes, but the main reference is the single frequency method [Altintas and Budak, 1995].

This work wants to be a tribute to the single frequency domain including the authors, remarking the positive properties, proposing an improvement to capture in a fast way the double period chatter, and maintaining the semi-analytical nature of the method.

Keywords: Stability, Milling, Chatter.

1. INTRODUCTION

The regenerative effect is the main origin of the chatter vibrations in machining operations [Tlusty and Polacek, 1957], [Tobias and Fishwick, 1958]. The pioneers introduced the concept of a directional factor, which is used to project the cutting forces to the mode direction and the vibration displacement to chip thickness. For milling processes, where these geometrical projections vary over time, Opitz proposed to calculate an average value of the coefficient during one period, and this way, he defined equivalent directional factors

[Opitz and Bernardi, 1970]. These models were used for years to obtain stability diagrams for milling.

However, these semi-analytical solutions were not accurate enough, and Minis and Yanushevsky formulated a new frequency domain numerical solution for milling processes [Minis and Yanushevsky, 1993]. They used Floquet's theorem and the Fourier series in their approach. Later on, Altintas and Budak [Altintas and Budak, 1995] developed the single frequency or zero-order solution method (ZOA), which gives a semi-analytical determination of stability limits. Nowadays, this method is the main reference in the field, and the performance of the different methods is always compared with it.

Three aspects can be addressed to understand the success of this method. First of all, the stability of the milling process is related to an eigenvalue problem, and compared with previous methods, it provides good results in continuous cutting even when the frequencies of two dominant modes are close to each other. In the other hand, it drives to a semi-analytical solution where the lobes can be obtained similarly to the method proposed by Tobias for turning [Tobias and Fishwick, 1958]. Therefore, practically, the stability diagram is obtained extremely fast. Finally, if the quality of the experimental FRFs is good enough with low noise levels, it is possible to introduce experimental frequency response functions (FRF) directly in the algorithm without any curve fitting.

The main drawbacks of single frequency approach are related to some inaccuracies in interrupted cutting due to double period chatter [Davies et al, 2000], [Bediaga et al, 2006] and mode interactions [Munoa et al, 2009], and finally the method has some difficulties to handle complex geometries [Dombovari et al, 2009], [Stepan et al, 2011].

The stability of interrupted cutting has been determined by alternative methods. Davies et al, [Davies et al, 2000] used a discrete map model for highly interrupted milling processes, where the time in cut is considered infinitesimal and modeled as an impact. Insperger and Stepan developed the semi-discretization technique for systems ruled by delayed differential equations [Insperger and Stepan, 2000]. Bayly et al, [Bayly et al, 2002] obtained similar results using temporal finite elements.

Double-period lobes can be calculated in the frequency domain by considering that chatter mechanism is composed of a dominant frequency and its harmonics spaced at positive and negative multiples of the tooth passing frequency. Multi frequency chatter stability models [Budak and Altintas, 1998], [Merdol and Altintas, 2004], [Zatarain et al, 2004], are able to predict double-period lobes that become more important as the milling becomes more intermittent.

The presence of closed shapes related to flip bifurcation instability processes were described by for the case of interrupted turning [Szalay and Stepan, 2003]. Finally, Zatarain related the dimensions of the lenticular island to the tool pitch of the helical end mills using multi frequency model [Zatarain et al, 2006].

The aim of this work is to show that the single frequency approach provides fast and accurate predictions in many industrial applications. Finally, an improvement is proposed to capture the double period chatter using a semi-analytical solution, and therefore, maintaining the main advantages of the method.

2. FREQUENCY DOMAIN MILLING STABILITY MODEL

The characteristic equation for milling stability analysis in frequency domain was developed by Budak and Altintas [Budak and Altintas, 1998]. Their approach was based on Cartesian displacements between the tool and the workpiece. In order to describe the regenerative effect, the dynamic cutting force is considered and the relation between the force and the vibrations is developed. For face milling operations, the cutting force can be given in the following form [Altintas, 2003]; [Munoa et al, 2005]:

$$\{F(t)\} = \left(\frac{K_t\, a_p\, Z}{2\,\pi\, \sin\kappa} \right) [A(t)]\{\Delta\, r(t)\}\,, \tag{1}$$

where

$$\{\Delta r(t)\} = \{r_{\mathrm{t}}(t) - r_{\mathrm{t}}(t-\tau)\} - \{r_{\mathrm{w}}(t) - r_{\mathrm{w}}(t-\tau)\}\,. \tag{2}$$

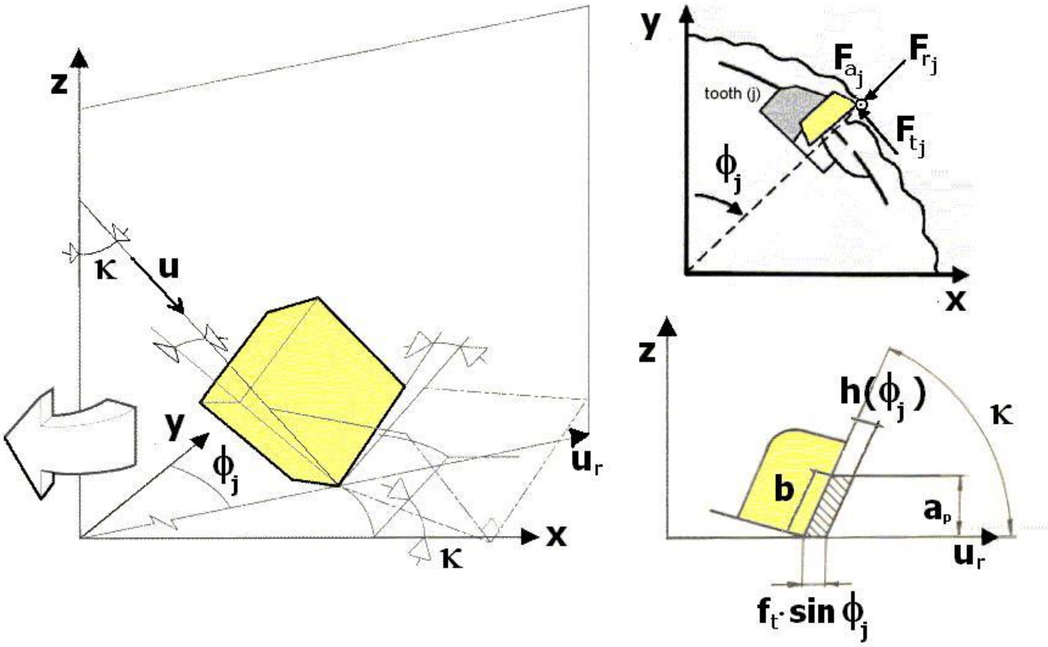


Figure 1. Cutting geometry.

Here, $\{\Delta r(t)\}$ is the relative regenerative vibration between the tool (t) and the workpiece (w), a_p is the depth of cut, K_t is the tangential cutting coefficient, τ is the tooth passing period, Z is the number of flutes, κ is the lead angle and [A(t)] is the Cartesian directional factor matrix. The directional factor matrix concentrates the projection of the cutting force onto the mode direction and the projection of the vibration onto the chip thickness. The Cartesian approach leads to the use of the corresponding directional coefficient matrix [A(t)]. The exact expression of the directional coefficient matrix can be found in the literature [Altintas, 2003]; [Munoa, 2007].

In stable stationary milling, the dynamic cutting force is periodic at tooth passing period τ. In case of unstable stationary cutting, an additional dominant frequency (ω_c) arises close to one of the essential natural frequencies combined with some modulations $\omega_{c,k}= \omega_c+k\Omega$ related to the tooth passing frequency $\Omega = 2\pi/\tau$. The next formulation can be considered for the modal milling force and vibration:

$$\{F(t)\} = \sum_{k=-\infty}^{\infty} \{F_k\} \mathrm{e}^{\mathrm{j}(\omega_c + k\Omega)t} \tag{3}$$

and $\{r(t)\} = \sum_{r=-\infty}^{\infty}\{r_k\}\,\mathrm{e}^{\mathrm{j}(\omega_c + r\,\Omega)t}$. (4)

Considering this frequency pattern and operating, the regenerative term can be rewritten as

$$\{\Delta r\} = (1 - \mathrm{e}^{-\mathrm{j}\,\omega_c\,\tau})\{r\}. \tag{5}$$

The Cartesian directional matrix is also time periodic and, consequently, a discrete Fourier development is possible, thus

$$[A(t)] = \sum_{l=-\infty}^{\infty}[A_l]\,\mathrm{e}^{\mathrm{j}\,l\,\Omega\,t} . \tag{6}$$

Taking into account all the different developments at (3) and (4) operating with the different harmonics as a product of (1), (5) and (6), the next expression is obtained

$$\sum_{k=-\infty}^{\infty}\{F_k\}\,\mathrm{e}^{\mathrm{j}(\omega_c + k\,\Omega)t} = \left(\frac{K_t\,b\,Z}{2\,\pi\,\sin\kappa}\right)(1 - \mathrm{e}^{-\mathrm{j}\,\omega_c\,\tau})\sum_{k=-\infty}^{\infty}\left(\sum_{r=-\infty}^{\infty}[A_{k-r}]\{r_r\}\right)\mathrm{e}^{\mathrm{j}(\omega_c + k\,\Omega)t}. \tag{7}$$

The dynamic modal forces and displacements can be related using the dynamic properties of the mechanical structure. Therefore, considering the relative frequency response function (FRF) of the system [Φ] between tool (t) and workpiece (w), the next expression can be written for each harmonic component:

$$\{r_k\} = [\Phi(\omega_c + k\,\Omega)]\{F_k\}. \tag{8}$$

Following Budak and Altintas' development, it is possible to obtain a closed loop formulation [Budak and Altintas, 1998]. The main equation relates different harmonics of the displacement vector r_k taking into account the Cartesian directional factor matrices (7) for each harmonic and FRF matrices (8) evaluated at different harmonics. Therefore:

$$\{r_k\} = \left(\frac{K_t\, a_p\, Z}{2\,\pi\, \sin\kappa}\right)(1-\mathrm{e}^{-\mathrm{j}\omega_c\,\tau})\sum_{r=-\infty}^{\infty}[\Phi(\omega_c + k\,\Omega)][A_{k-r}]\{r_r\}\,. \tag{9}$$

Finally, the stability problem results in an infinite dimensional matricial expression, that is,

$$\begin{Bmatrix} \vdots \\ \{r_{-h}\} \\ \vdots \\ \{r_0\} \\ \vdots \\ \{r_h\} \\ \vdots \end{Bmatrix} = \left(\frac{K_t\, a_p\, Z}{2\pi\,\sin\kappa}\right)(1-\mathrm{e}^{-\mathrm{j}\omega_c\,\tau}) \begin{bmatrix} \ddots & \vdots & & \vdots & & \vdots & \iddots \\ \cdots & [\Phi_{-h}] & \cdots & [0] & \cdots & [0] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & & \\ \cdots & [0] & \cdots & [\Phi_0] & \cdots & [0] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & \vdots & \\ \cdots & [0] & \cdots & [0] & \cdots & [\Phi_h] & \cdots \\ \iddots & \vdots & & \vdots & & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \ddots & \vdots & & \vdots & & \vdots & \iddots \\ \cdots & [A_0] & \cdots & [A_{-h}] & \cdots & [A_{-2h}] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & & \\ \cdots & [A_h] & \cdots & [A_0] & \cdots & [A_{-h}] & \cdots \\ & \vdots & \ddots & \vdots & \ddots & \vdots & \\ \cdots & [A_{2h}] & \cdots & [A_h] & \cdots & [A_0] & \cdots \\ \iddots & \vdots & & \vdots & & \vdots & \ddots \end{bmatrix} \begin{Bmatrix} \vdots \\ \{r_{-h}\} \\ \vdots \\ \{r_0\} \\ \vdots \\ \{r_h\} \\ \vdots \end{Bmatrix}, \tag{10}$$

where $\{r_k\}$ are the amplitudes of vibration of all the considered modes for the kth modulated frequency, and $[\Phi_k]=[\Phi(\omega_c+k\Omega)]$ and $[B_k]$ the Frequency Response Function (FRF) matrix and the modal directional factor matrix evaluated at the k_{th} modulated chatter frequency.

In a theoretical basis, the size of the matrices is infinite, but in practice the FRF takes very small values for frequencies far from the considered natural frequencies. Therefore, the system can be truncated without noticeable loss of accuracy.

The solution of this equation results in an eigenvalue problem where the obtained eigenvalues are related to the spindle speeds Ω/ Z and depth of cuts a_p. There is not a semianalytical general multi-frequency solution for this spindle speed dependent equation, but different numerical methods have been proposed [Budak and Altintas, 1998], [Merdol and Altintas, 2004].

In general, the accuracy of the modeling of the dynamic milling force increases with the number of harmonics. The magnitude of the harmonics depends on the engagement, the direction of the milling force and the mode and the number of flutes. In interrupted cutting, the high order harmonics are important, while in continuous cutting their influence is low.

3. SEMI-ANALYTICAL SOLUTIONS

3.1. Hopf Bifurcation: Zero Order Approximation (ZOA).

If only the zeroth order term is considered a fast semi-analytical solution is possible to trace the linear stability border related to Hopf-bifurcation. This simplification provides a spindle speed independent equation and therefore it can be solved considering different chatter frequencies and obtaining for each one the corresponding depth of cut and spindle speeds [Altintas and Budak, 1995].

$$\{r_0\} = \left(\frac{K_t\, a_p\, Z}{2\pi\, \sin\kappa}\right)(1-e^{-j\omega_c\tau})\left[\Phi_0\right]\left[A_0\right]\{r_0\}. \tag{11}$$

Following Altintas and Budak, the stability analysis drives to an eigenvalue problem where the eigenvalues *($\Lambda = \Lambda_R + \Lambda_I \cdot \boldsymbol{i}$)* can be related with the maximum stable depth of cut and different cutting speeds.

$$\det\{[I] + \Lambda.\,[\Phi_0][A_0]\} = 0, \tag{12}$$

$$a_{\lim} = -\frac{2\pi \cdot \sin\kappa \cdot \Lambda_R}{N \cdot Kt} \cdot \left(1 + \left(\frac{\Lambda_I}{\Lambda_R}\right)\right), \tag{13}$$

$$n = \frac{60}{N \cdot T} = \frac{60 \cdot \omega_c}{N \cdot \left\{\pi - 2 \cdot \arctan\left(\frac{\Lambda_I}{\Lambda_R}\right) + 2\pi \cdot k\right\}}, \quad k = 0,1,2,... \tag{14}$$

Different values for the integer value k form different lobes formed by series of spindle speeds with the same depth of cut.

3.2 Flip Bifurcation: New semi-analytical approach

The period doubling chatter related to flip bifurcation produces an independent family of lobes with some particularities that can help in the definition of the stability boundary. The main characteristic of this double period chatter is that there is a linear relationship between the chatter frequency and the tooth passing frequency and the spindle speed. When a double period or flip bifurcation appears, the chatter frequency follows a straight line in the chatter frequency diagram (see Figure 1).

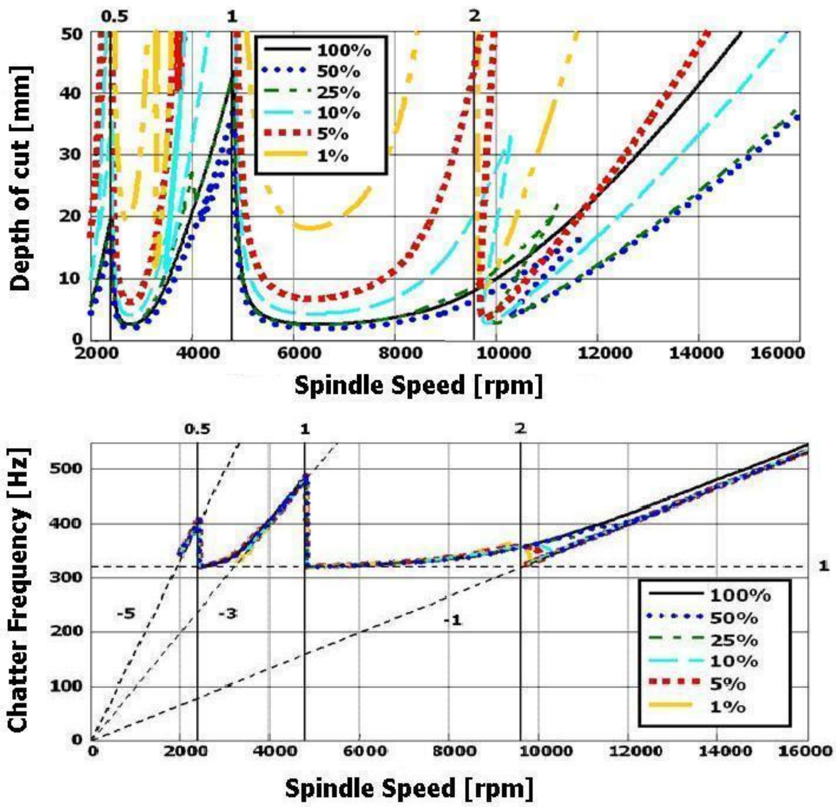


Figure 2. Stability diagram for a single mode case for different engagements (%) for down milling operations [Munoa, 2007].

$$2 \cdot \omega_c = m \cdot \Omega . \tag{15}$$

In terms of frequency domain, the double period chatter or the flip bifurcation happens when the chatter frequency and one of the modulated chatter frequencies are exciting the same mode. Only the odd modulated chatter harmonics can create the double period chatter. The regenerative term explains this effect. This term can be rewritten in function of the involved harmonic and is null for all the even harmonics.

$$f = 1 - e^{-i \cdot \omega_c \tau} = 1 - e^{-i \cdot \omega_c \frac{2 \cdot \pi}{\Omega}} = 1 - e^{-i \cdot m \cdot \pi}, \qquad \begin{matrix} f = 0 & if & m = 0,2,4... \\ f = 2 & if & m = 1,3,5,... \end{matrix}, \tag{16}$$

Applying these effects in the main equation:

$$\begin{Bmatrix} \{r_{-h}\} \\ \vdots \\ \{r_0\} \\ \vdots \\ \{r_h\} \end{Bmatrix} = \left(\frac{K_t\, a_p\, Z}{\pi\, \sin\kappa}\right) \begin{bmatrix} [\Phi_{-h}] & \cdots & [0] & \cdots & [0] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ [0] & \cdots & [\Phi_0] & \cdots & [0] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ [0] & \cdots & [0] & \cdots & [\Phi_h] \end{bmatrix} \begin{bmatrix} [A_0] & \cdots & [A_{-h}] & \cdots & [A_{-2h}] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ [A_h] & \cdots & [A_0] & \cdots & [A_{-h}] \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ [A_{2h}] & \cdots & [A_h] & \cdots & [A_0] \end{bmatrix} \begin{Bmatrix} \{r_{-h}\} \\ \vdots \\ \{r_0\} \\ \vdots \\ \{r_h\} \end{Bmatrix}, \tag{17}$$

where:

$$[\Phi_m] = [\Phi(i(\omega_c + h\cdot\Omega))] = \left[\Phi\left(i\omega_c\left(1+\frac{2h}{m}\right)\right)\right] = \left[\Phi\left(i\Omega\left(\frac{m}{2}+h\right)\right)\right]. \tag{18}$$

The eigenvalue problem is now independent from tooth passing frequency and spindle speed. Therefore, it is possible to obtain the limit of the double period chatter scanning the frequency range like in the method proposed by Altintas and Budak or more straighforward scanning the spindle speed range. For each frequency or spindle speed the eigenvalue problem is solved. This eigenvalues should fulfill these conditions to define a border of the stability lobe.

$$\begin{aligned} &\Lambda = \Lambda_R + \Lambda_I \cdot i, \\ &\Lambda_R > 0, \\ &\Lambda_I = 0. \end{aligned} \tag{19}$$

The stability limits for double period are defined using the eigenvectors with only real values. In fact, the different double period are obtained scanning different lines (m) in frequency domain diagram (m=1,3,5,…).

$$a_{\lim} = \frac{\pi\cdot\sin\kappa\cdot}{Z\cdot Kt}\cdot\Lambda_R, \tag{20}$$

$$N_{ss} = \frac{60\cdot\omega_c}{\pi\cdot Z\cdot m}, \quad \text{where m=1,3,5,…..(2*}N_{lob}\text{-1).} \tag{21}$$

The truncation of the main equation is an important issue to obtain efficiently the exact solution for the double period chatter lobes. A proper frequency domain and number of harmonics is selected in this truncation process. First of all, a frequency range for scanning

(ω_{start}, ω_{end}) is selected considering the main flexibility of the dynamical systems and eliminating frequencies where the system is really stiff compared with the most flexible frequency. Considering the spindle speed range and/or stability lobe order (N_{lob}), the amount of double period chatter scannings is determined (see equation 21).

For each chatter frequency and/or spindle speed, the oriented frequency response matrix is truncated *([Φ][A])* calculating automatically the number of positive and negative significant harmonics, and of course, considering always a positive scanning in the chatter frequency range.

$$h^- = \mathrm{int}\left\{ {(\omega_{end} + \omega_c)}\big/{\Omega} \right\}, \tag{22}$$

$$h^+ = \mathrm{int}\left\{ {(\omega_{end} - \omega_c)}\big/{\Omega} \right\}. \tag{23}$$

3.3 Combined semi-analytical frequency method

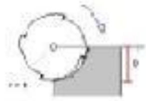
Both frequency domain methods can be combined to create a new semi-analytical frequency domain algorithm. This new method improves the accuracy of the ZOA but the main advantages are maintained: experimental FRF can be introduced directly and the calculation speed is not strongly punished. The combined method adds exact double period chatter lobes (flip bifurcation) to the approximated solution (Hopf bifurcation) proposed by ZOA). The algorithm is described in Figure 3.

4. COMPARATION BETWEEN METHODS.

The accuracy of this new method has been compared with traditional ZOA approximation, multifrecuency solution and semidiscretization method. Several works have been compared semidiscretization and multifrequency methods concluding that the two methods are providing the same result [Munoa et al, 2009]. The conparation has been carried out selecting three representative examples from the literature. The ZOA simulations has been carried out using the algorithm described by Altintas and Budak [Altintas and Budak, 1995], the multifrequency solution has been obtained following Budak and Altintas [Budak and Altintas, 1998], and finally the semi-discretization model has been implemented considering the method proposed by Insperger and Stepan [Insperger and Stepan, 2000].

Input Parameters

Dynamic Parameters | Cutting Parameters | Cutting Conditions

$[\Phi_0]$  $\{k\}$  ϕ_e ϕ_s 

Determination of calculation bands

Determination of double period scanning lines → m=1,3,5,.....(2*N_{lob}-1)

Determination of frequency range → (ω_{start}, ω_{end})

Calculation of the frequency related to the maximum flexibility.

The frequency range is selected collecting point with flexibility bigger than a certain value related to the maximum flexibility.

Calculation of the maximum directional matrix → $[A]$

Determination of the truncation harmonics.

Calculation of the directional factor matrix

Chatter Frequency scanning:

Frequency scanning → $\omega_c = \omega_{start}$ .... ω_{end}

Truncation for zero order calculation. → $[A_0]$

Calculation of the oriented frequency response matrix → $[\Phi_0][A_0]$

Calculation of the eigenvectors for ZOA. → $\det\{[I] + \Lambda \cdot [\Phi_0][A_0]\} = 0$

Verification

Calculation of the depth of cut → $a_{\lim} = -\dfrac{2\pi \cdot \sin\kappa \cdot \Lambda_R}{N \cdot Kt} \cdot \left(1 + \left(\dfrac{\Lambda_I}{\Lambda_R}\right)\right)$

Calculation of the different spindle speeds for different lobes.

Lobe scanning for double period chatter LOB=lobstart,......lobend

Determination of the spindle speed and tooth passing frequency → $N_{ss} = \dfrac{60 \cdot \omega_c}{\pi \cdot Z \cdot m}$

Determination of the truncation harmonics → $h^+ = \operatorname{int}\left\{(\omega_{end} - \omega_c)/\Omega\right\}$ $\quad h^- = \operatorname{int}\left\{(\omega_{end} + \omega_c)/\Omega\right\}$

Truncation for double period chatter.

Calculation of the eigenvectors for lobe number LOB

Verification

Calculation of the depth of cut → $a_{\lim} = \dfrac{\pi \cdot \sin\kappa \cdot}{Z \cdot Kt} \cdot \Lambda_R$

Graphical representation:

All the lobes are described in same figure.

Figure 3. Combined semi-analytical frequency method algorithm

4.1. Case 1: Heavy duty milling with face milling cutter.

The case of a heavy duty operation with a face milling cutter has been considered first [Munoa, 2007]. The dynamic properties have been obtained making a curvefitting on DS630 machining center for different positions. In this machine, the dynamic behaviour is changing inside the workspace, and hence, the workspace has been discretized in 16 different positions. In each position 8 cutting process simulations have been performed for different cutting directions with high engagement (80%). C45 steel has been chosen in these simulations.

Table 1. Parameters for simulation of case 1 [Munoa, 2007].

<table>
<tr><td colspan="6">Tool</td></tr>
<tr><td colspan="2">Diameter (D)</td><td colspan="2">Number of flutes (Z)</td><td>Lead angle (κ)</td><td>Helix angle</td></tr>
<tr><td colspan="2">125mm</td><td colspan="2">8</td><td>45º</td><td>º0</td></tr>
<tr><td colspan="6">Cutting conditions & Coefficients</td></tr>
<tr><td colspan="2">Engagement</td><td>Feed Direction</td><td>Kt [N/mm2]</td><td>Kr</td><td>Ka</td></tr>
<tr><td colspan="2">100mm (Down Milling)</td><td>(1,0,0)</td><td>1889</td><td>0.4105</td><td>0.1928</td></tr>
<tr><td colspan="6">Dynamic Parameters</td></tr>
<tr><td>Mode</td><td>ω_0 [Hz]</td><td>ζ [%]</td><td>k [N/μm]</td><td>m [kg]</td><td>Orientation</td></tr>
<tr><td>1</td><td>36.1</td><td>5.1</td><td>56.6</td><td>1105</td><td>(0.128,-0.674,0.728)</td></tr>
<tr><td>2</td><td>50.6</td><td>3</td><td>42</td><td>415</td><td>(0,1,0)</td></tr>
<tr><td>3</td><td>84.6</td><td>2.5</td><td>52.6</td><td>186</td><td>(0.969,-0.246,-0.013)</td></tr>
<tr><td>4</td><td>89.8</td><td>4.3</td><td>48.9</td><td>154</td><td>(0.267,0.961,-0.075)</td></tr>
<tr><td>5</td><td>135.1</td><td>2.6</td><td>30.6</td><td>42.5</td><td>(1,0,0)</td></tr>
</table>

In all this simulations the ZOA approximation is able to predict accurately the stability compared with semi-discretization and multi frequency. A representative example is presented in Figure 4.

Considering the machinability issues for the selected tool, spindle speeds 300 and 800 rpm are suitable for steel machining. Hence, the ZOA provides a fast and precise solution at the same time. There is an anecdotic discrepancy around 1800rpm due to the presence of a closed flip instability region. The combined frequency method offers the exact solution.

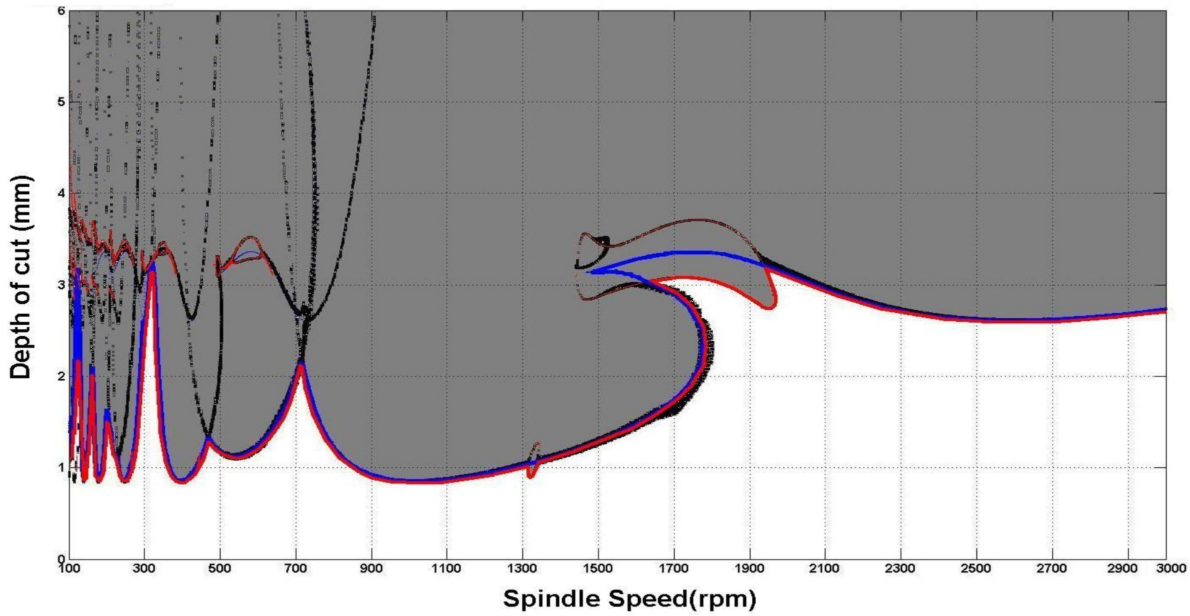


Figure 4. Stability diagram for case 1 comparing ZOA (blue line), combined frequency method (red line) and Multi frequency (shadow zone).

The ZOA is a powerful tool for heavy duty cutting where the engagement and the number of flutes is high, perfect to make series of simulation taking into account different positions, cutting planes, cutting directions and milling senses.

3.2. Case 2: Highly interrupted cutting.

Insperger and Stepan reported an example of highly interrupted cutting using 1 fluted tool. This is example is far from a real cutting case but it has been experimentally verified and used for comparison with the multi frequency model in the literature [Merdol and Altintas,2004].

Milling tests were performed with an experimental flexure designed to mimic a single .d.o.f. system. Aluminium (7075-T6) test samples of width 6.35 mm were mounted on the flexure and centrally milled by a 19.05 mm diameter carbide end mill with a single flute (the second flute was ground off to remove any effects due to asymmetry or runout).

Table 2. Parameters for case 2 [Insperger et al, 2003][Merdol and Altintas, 2003].

<table>
<tr><td colspan="7">Tool</td></tr>
<tr><td colspan="2">Diameter (D)</td><td>Number of flutes (Z)</td><td colspan="3">Lead angle (κ)</td><td>Helix angle</td></tr>
<tr><td colspan="2">19.05mm</td><td>1</td><td colspan="3">90</td><td>0</td></tr>
<tr><td colspan="7">Cutting conditions & Coefficients</td></tr>
<tr><td colspan="2">Engagement</td><td>Feed Direction</td><td>Kt [N/mm2]</td><td colspan="2">Kr</td><td>Ka</td></tr>
<tr><td colspan="2">6.35mm (Centered)</td><td>(1,0,0)</td><td>550</td><td colspan="2">0.364</td><td>0</td></tr>
<tr><td colspan="7">Dynamic Parameters</td></tr>
<tr><td>Mode</td><td>ω_0 [Hz]</td><td>ζ [%]</td><td>k [N/μm]</td><td colspan="2">m [kg]</td><td>Orientation</td></tr>
<tr><td>1</td><td>146.8</td><td>0.38</td><td>2.2</td><td colspan="2">2.586</td><td>(1,0,0)</td></tr>
</table>

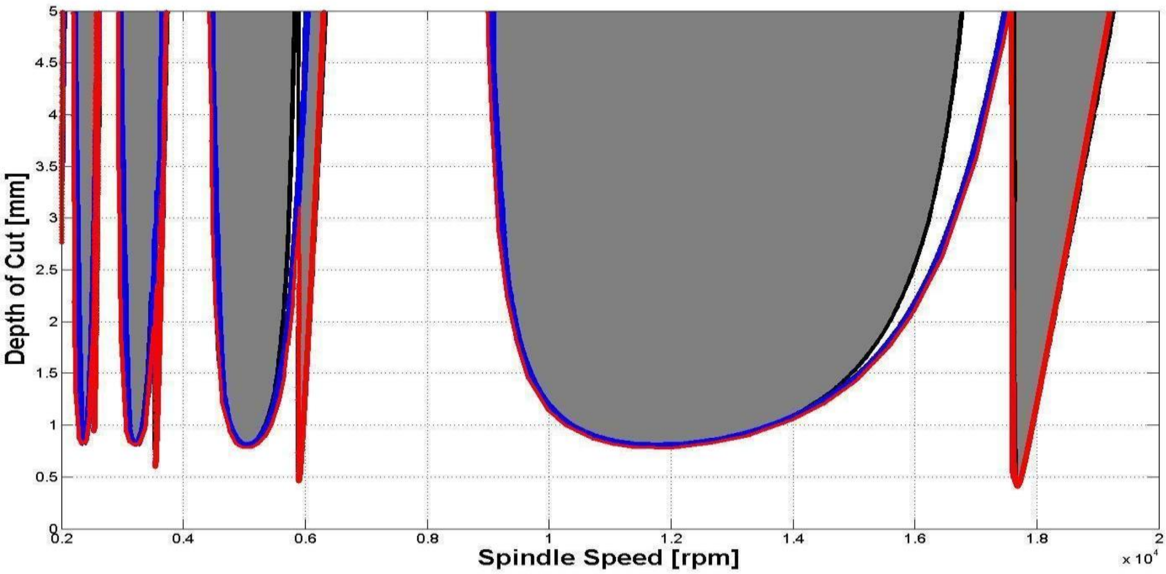


Figure 5. Stability diagram for case 2 comparing ZOA (blue line), combined frequency method (red line) and Semi-discretization (shadow zone).

In this example, the double period chatter (flip bifurcation) defines the minimum stability. In general, the flip lobes are as important as the lobes related to the Hopf bifurcation.

In this case ZOA is not able to describe the double period chatter and therefore important discrepancies are found. The combined method is able to predict exactly the chatter related to the flip bifurcation and therefore this fast prediction is good enough and can have practical applications. The combined method uses the ZOA to predict the traditional chatter (Hopf bifurcation) and it does not capture changes in the shape of the traditional lobes.

3.3. Case 3: Interrupted cutting with mode coupling.

Finally a third example has been chosen considering mode couplings due to existence of more than one significant vibration mode when an interrupted milling process is performed.

Table 3. Parameters for simulation of case 3 [Munoa et al, 2009].

<table>
<tr><td colspan="6">Tool</td></tr>
<tr><td colspan="2">Diameter (D)</td><td colspan="2">Number of flutes (Z)</td><td>Lead angle (κ)</td><td>Helix angle</td></tr>
<tr><td colspan="2">50mm</td><td colspan="2">4</td><td>90</td><td>0</td></tr>
<tr><td colspan="6">Cutting conditions & Coefficients</td></tr>
<tr><td colspan="2">Engagement</td><td>Feed Direction</td><td>K° [N/mm2]</td><td>Kr</td><td>Ka</td></tr>
<tr><td colspan="2">12.5mm (Down Milling)</td><td>(1,0,0)</td><td>2000</td><td>0.3</td><td>0</td></tr>
<tr><td colspan="6">Dynamic Parameters</td></tr>
<tr><td>Mode</td><td>ω_0 [Hz]</td><td>ζ [%]</td><td>k [N/μm]</td><td>m [kg]</td><td>Orientation</td></tr>
<tr><td>1</td><td>45</td><td>4</td><td>30</td><td>375</td><td>(1,0,0)</td></tr>
<tr><td>2</td><td>60</td><td>4</td><td>30</td><td>211</td><td>(0,1,0)</td></tr>
</table>

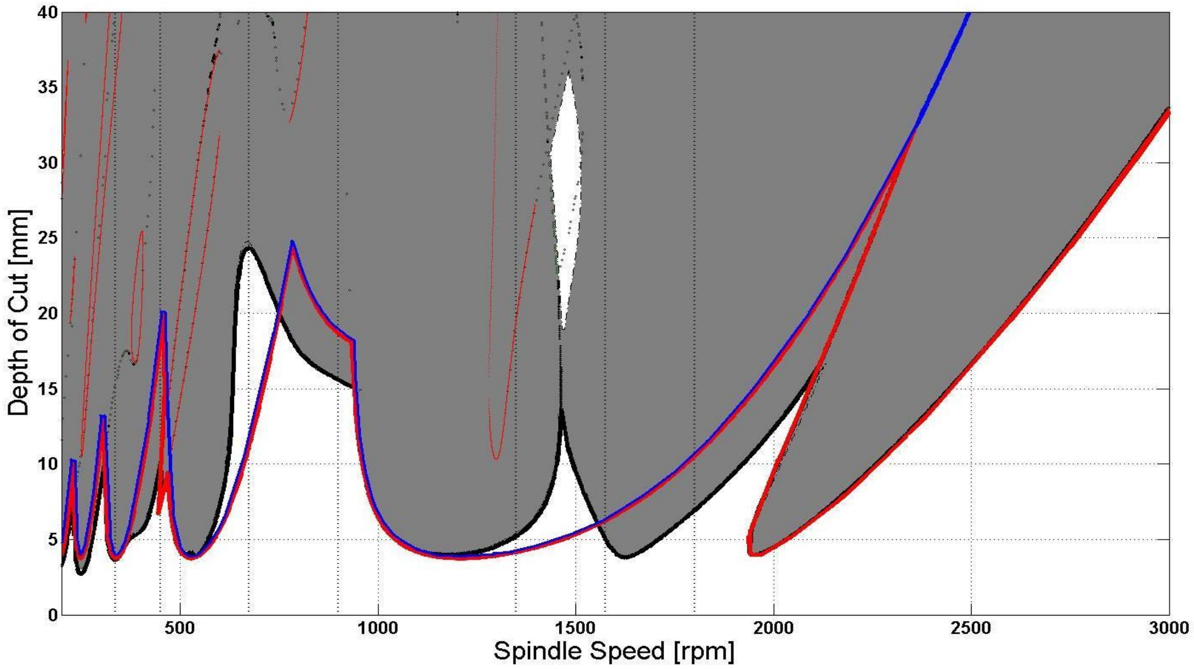


Figure 6. Stability diagram for case 3 comparing ZOA (blue line), combined frequency method (red line) and Semi-discretization (shadow zone).

The combined method is able to improve the accuracy of the ZOA method introducing exact double period lobes, but it is not able to capture the variations due to mode couplings. Therfore, important discrepancies are found in some regions.

5. CONCLUSIONS

The single frequency or zero order approximation method is the main reference in milling stability because it provides an accurate determination of the stability for continuous cutting making possible the introduction of experimental FRF function without any fitting. This method is accurate enough to predict stability in industrial problems like a heavy duty face milling.

In this work, the double period chatter has been calculated directly in frequency domain similarly to the ZOA. This algorithm provides exactly the double period chatter even in the most complex cases. A combined frequency domain method has been proposed based in a single frequency scanning. This combined method has improved the accuracy of the ZOA approximation in milling conditions with interrupted cutting maintaining the main advantages of the ZOA.

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