Set-Membership Method for Discrete Optimal Control

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1. Problem

Considered System

We consider a control system, defined by the differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \quad (1)$$

$\mathbf{x}(t) \in \mathbb{R}^n$ the state vector
$\mathbf{u}(t) \in \mathbf{U}$ the control vector

The system is studied over $[t_0, t_f]$

$$t_k = t_0 + k \times \delta_t,\ t_k \le t_f,\ k \in \{1, \cdots, m\} \quad (2)$$

It is assumed that $\mathbf{u}(t_k)$ is bounded over $[t_k, t_{k+1}]$ so it is possible to determinate a box $[\mathbf{u}_k]$ such that $\mathbf{u}(t_k) \in [\mathbf{u}_k]$ over $[t_k, t_{k+1}]$

The flow map of the system is defined as

$$\varphi(t_0, t_k; \mathbf{x}_0, \mathbf{u}(t)) = \mathbf{x}(t) \quad (3)$$

The reachable set of the system at time t_k is

$$\varphi(t_0, t_k; \mathbf{X}_0, \mathbf{U}) = \{\varphi(t_0, t_k; \mathbf{x}_0, \mathbf{u}(t)) | \varphi(t_0, t_0; \mathbf{x}_0, \mathbf{u}(t)) = \mathbf{x}_0 \text{ and } \varphi : [t_0, t_k] \times \mathbf{X}_0 \times \mathbf{U} \rightarrow \mathbb{R}^n \text{ is a solution of (1) for some } \mathbf{u}(t) \in \mathcal{U}\} \quad (4)$$

where $\mathcal{U} = \{\mathbf{u} : [t_0, t_{k-1}] \rightarrow \mathbf{U} | \mathbf{u} \text{ is continuous over } [t_k, t_{k+1}]\}$ denotes the set of admissible controls and $\mathbf{X}_0$ a set of possible initial values $\mathbf{x}_0$

Objective

Evaluate $\mathbf{C}_{t_0,t_f}$ the subset of initial states of $\mathbf{K}$ (state constraint) from wich there exists at least one solution of (1) reaching the target $\mathbf{T}$ in finite time t_f starting at a time t_0:

$$\mathbf{C}_{t_0,t_f} = \{\mathbf{x}_0 \in \mathbf{K} | \exists \mathbf{u}(t) \in \mathcal{U}, \varphi(t_0, t_f; \mathbf{x}_0, \mathbf{u}(t)) \in \mathbf{T}\} \quad (5)$$

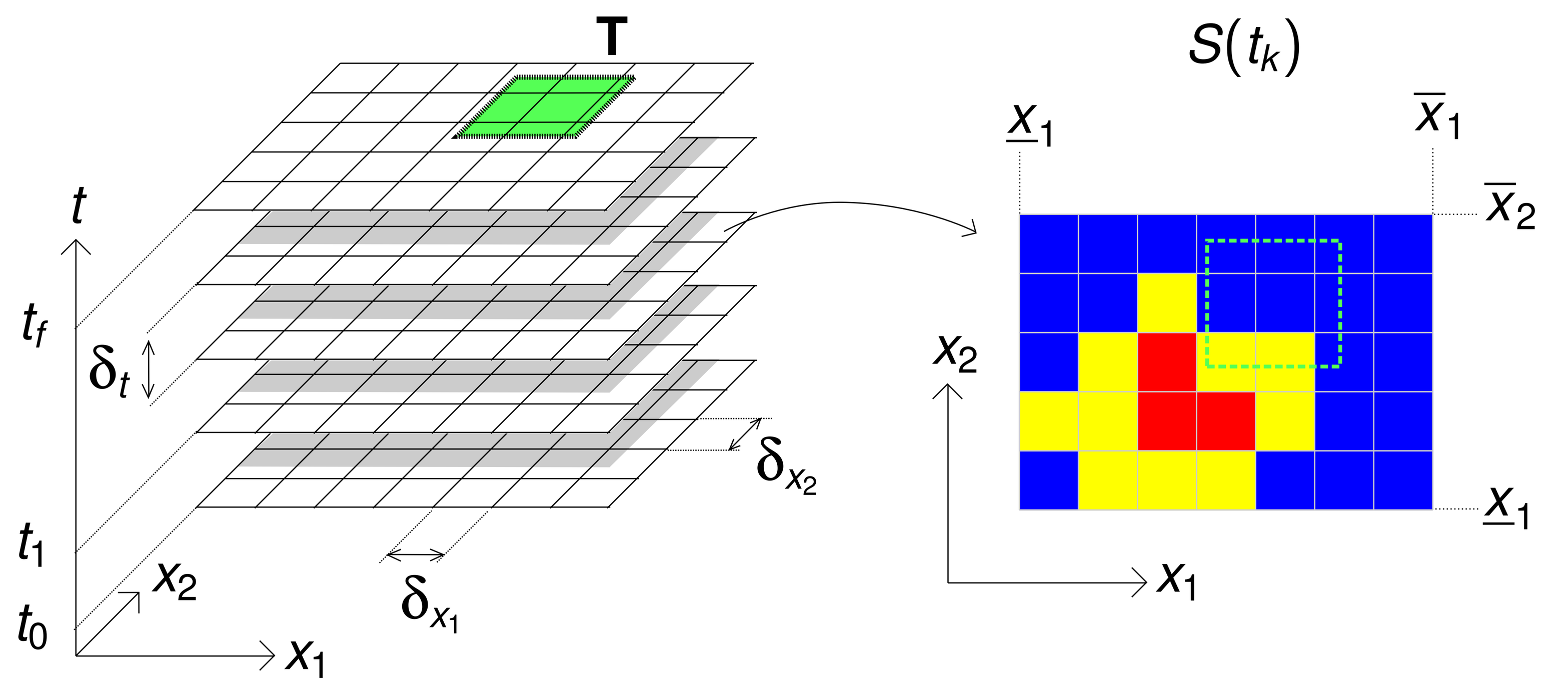
Using interval analysis to compute an inner and an outer caracterisations of $\mathbf{C}_{t_0,t_f}$

$$\mathbf{C}^-_{t_0,t_f} \subseteq \mathbf{C}_{t_0,t_f} \subseteq \mathbf{C}^+_{t_0,t_f} \quad (6)$$

2. Caracterisation computation

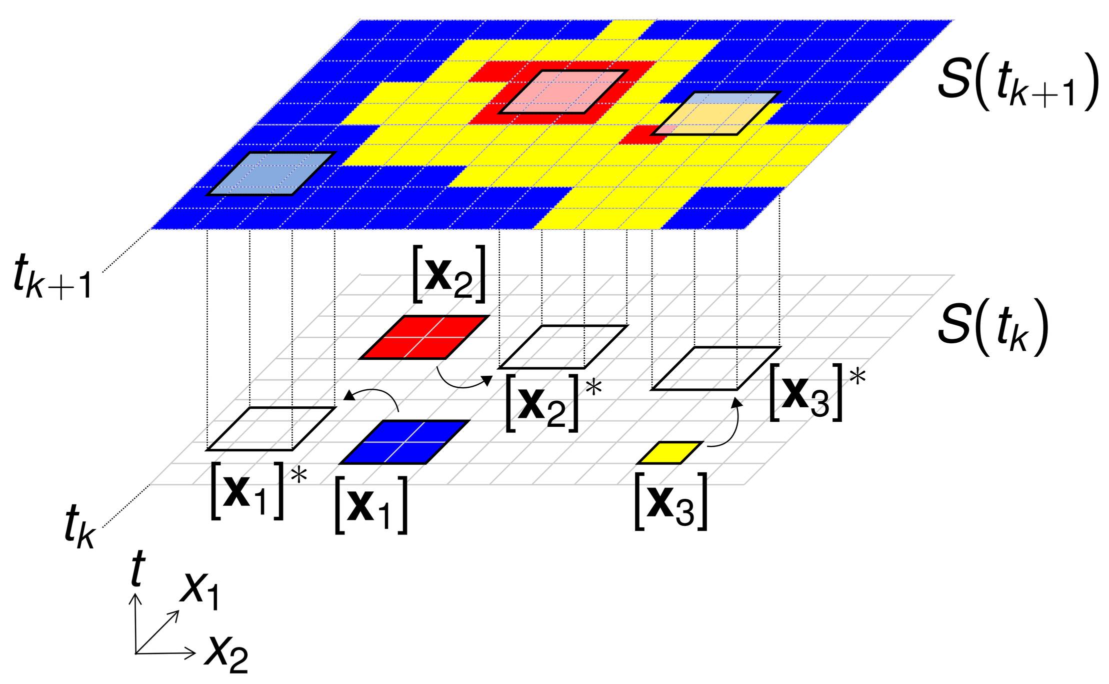
Proposed approach

For each time t_k the algorithm computes a gridding of $\mathbf{K}$ (a slice), noted $S(t_k)$. The resolution of the gridding is $\delta_{\mathbf{K}} = (\delta_{x_1}, \cdots, \delta_{x_i}, \cdots, \delta_{x_n})$ where δ_{x_i} corresponds to the i^{th} dimension of $\mathbf{K}$



Slice computation

We propose an iterative algorithm that classifies the cells of each slice in three categories:
- *unreachable* (blue), no state inside the cell allows the system to reach the target at time t_f
- *reachable* (red), all the states inside the cell allow the system to reach the target at time t_f
- *indeterminate* (yellow), neither *reachable* nor *unreachable*



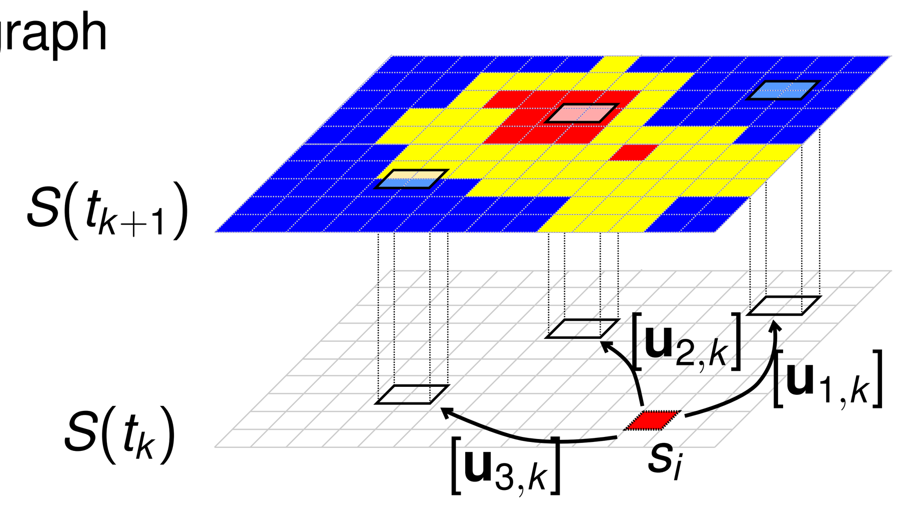
The slices are built from $S(t_f)$ to $S(t_0)$

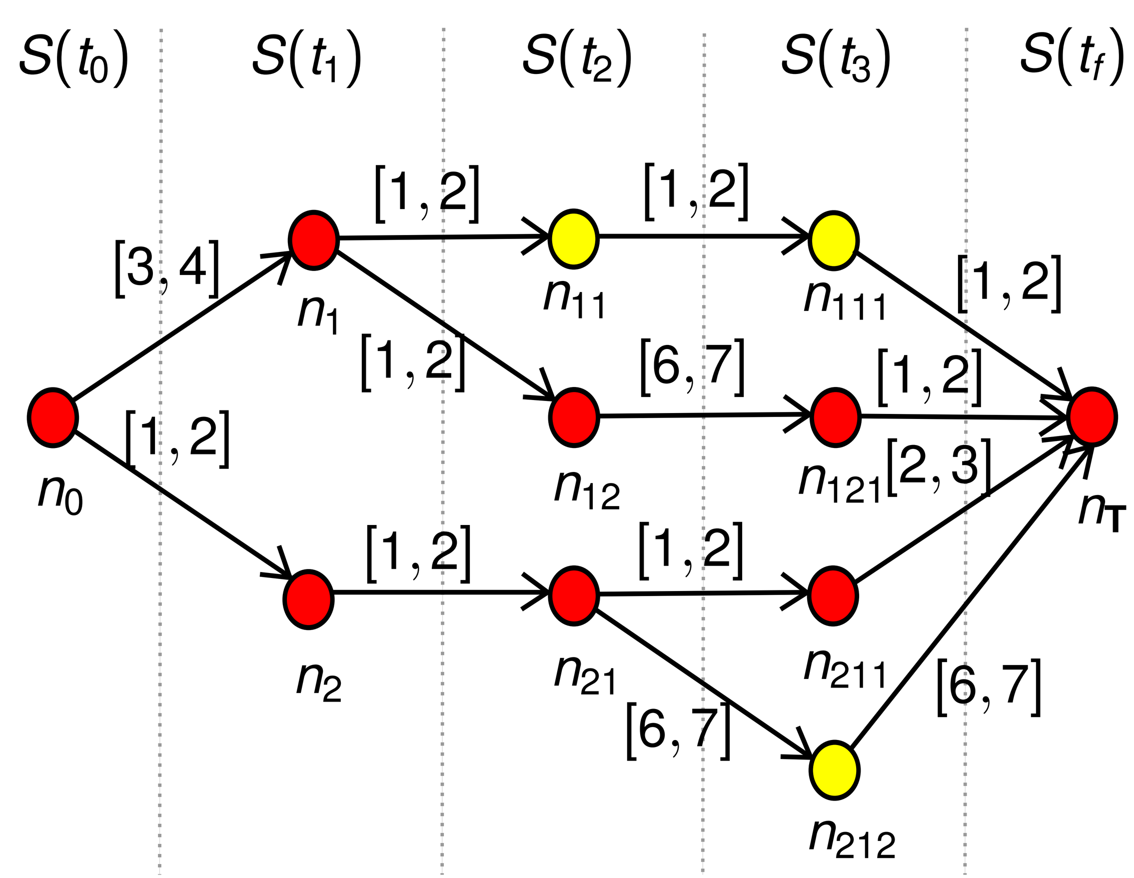
3.Optimal discrete path evaluation

Slice modification and graph building

For each cell $s_i \in S(t_k)$ is defined a set of input vectors $\mathbf{U}(s_i)$ that leads s_i to reachable or indeterminate cells of $S(t_{k+1})$

Gather the cells into nodes and build a graph





$$J(P(n_0, n_1, n_{11}, n_{111}, n_{\mathbf{T}})) = [6, 10]$$
$$J(P(n_0, n_1, n_{12}, n_{121}, n_{\mathbf{T}})) = [11, 15]$$
$$J(P(n_0, n_2, n_{21}, n_{211}, n_{\mathbf{T}})) = [5, 9]$$
$$J(P(n_0, n_2, n_{21}, n_{212}, n_{\mathbf{T}})) = [14, 18]$$

Obtained paths

Using the graph and a shortest path algorithm (e.g. Interval Dijkstra) it is possible to compute:
- an enclosure $(\mathbf{P}^*)$ of the optimal discrete control vector to reach the target from an initial state $[\mathbf{x}_0] \in \mathbf{C}_{t_0,t_f}$
- an evaluation of the cost $(J(\mathbf{P}^*))$ of this control vector

For instance

$$\mathbf{P}^* = \{P(n_0, n_1, n_{11}, n_{111}, n_{\mathbf{T}}), P(n_0, n_2, n_{21}, n_{211}, n_{\mathbf{T}})\}$$
$$J(\mathbf{P}^*) = [6, 10] \cup [5, 9] = [5, 9]$$

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