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Alessandro Chiancone

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Alessandro Chiancone,\textsuperscript{1,2,3,4}\ast

\textsuperscript{1}Rome,
Italy

\textsuperscript{2}Laboratoire Jean Kuntzmann \& INRIA Rhone-Alpes, team Mistis,
Inovallee, 655, av. de l’Europe, Montbonnot, 38334 Saint-Ismier cedex, France

\textsuperscript{3}GIPSA-Lab, Grenoble INP,
Saint Martin d’Heres, France

\textsuperscript{4}Institute of Statistics Graz University of Technology
Kopernikusgasse 24/III A-8010 Graz, Austria

\ast Alessandro Chiancone; E-mail: al.chiancone@gmail.com.

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Abstract

This paper wants to investigate the deepest meaning of the word class that is often used in machine learning and classification as a well-defined concept. This adventure will lead the reader to the fundamentals of Mathematics like set theory from Zermelo-Fraenkel. This will be our start, like is all Mathematics, to understand how well defined is the class concept. A broader theory will be outlined with the courageous attempt to give an homogenous framework to deal with machine learning problems.

Introduction

Yes, I need to cite all of sudden the paper that inspired the title [Blaschke and Strobl, 2001], this was the first time that I started thinking about classification. Thanks to this paper I tried to
clear my mind on some problems affecting image processing. But, what’s wrong with classes? I would say nothing is wrong by default, we just need to be sure to agree on the same definition. Machine learning has been recently applied to face complex problems and I strongly believe that image processing (remote sensing, medical imaging,...) will be in the near future, much more than now, a powerful tool to improve our quality of life. To make future the present, I think, we need to deeply analyze what classification is, in general. I would say that a lot of effort of human kind is represented by classification results. For example, DNA could be seen (also) as a very good feature to overcome the incredible work of the Swedish Linnaeus. But why do we need to classify things? Humans developed a powerful tool, the language (spoken and written), that is a way to give speed to communication and something much more important. The main difference between animal language and human language, I think, is the possibility that gives to refer to facts that are not happening in the very exact moment of communication. Past and Future. This is a beautiful idea (dangerous to control sometime, not being anymore capable of enjoying moments) that gave us the possibility to build what we call Knowledge. This paper wants to underline, explore and try to catch which are the axioms of our language, that are coming with strengths and limitations.

1 A good example to start: Colors

Thanks to internet some years ago I had the luck to watch several videos talking about colors. What is a color? It is clearly a class, since spectra of light is continuos (and oriented), colors can be seen as a regression problem with the function $f : \mathbb{R} \to A \subseteq \mathbb{N}$. Being, e.g., $A = 1, 2, 3, \ldots, 2^{30}$ like the HDMI specification 1.073 billion colors. A good question is: do we have 1.073 billion words for colors in our languages? No. As Raymond Queneau reported in the preface of Cent Mille Milliards des Poemes citing Alan Turing: Seul une machine peut aprecier un sonnet ecrit par une autre machine (more on this can be found in [Jefferson, 1949]). This
means that we are really not capable, or just do not want because of the small benefit, to invent such an enormous number of words for just colors. Our memory is limited after all. In this example we can see how language is the attempt to describe a continuous variable (which in our example is the spectra of light) with a discrete number of classes. A good question is: is it possible, in theory, to give a word to each point in the spectra? This is a very good question and we should be thankful to Cantor if we can give the answer. Which is No. The problem is that even if we suppose our language to have an infinity of words (crazy assumption, but I have a degree in Mathematics, forgive me) there is no way to make correspond points in the line with the infinite words created. The cardinality of the two infinities is different and the one of language is $\aleph_0$, Aleph-null, the smallest infinite cardinal number (thank you Claudio Bernardi for the beautiful course of fundamentals of Mathematics). I now dare to define social sciences, in the common meaning, as the brave attempt to describe an infinity of higher cardinality with the one of language that, even at its full-capability, will be $\aleph_0$. To sum up we need to be careful, the world we want to describe is much wider than we may think, and our tool, which is language, is limited. Sometime is hard to fully agree on colors. If we imagine colors as intervals on the spectra the closer we are to the limits the more uncertainty is the interval to which we will assign the color: is a shade of Blue or Green?

2 Empathy for the Machine

When I read this paper [Blaschke and Strobl, 2001] I started working on object-based classification. We proposed a hierarchy of segmentation to insert spatial information in classification [Chini et al., 2014b]. What I was trying to do at that time was an honest attempt to improve the state of the art which was already full of interesting papers [Dalla Mura et al., 2010; Aytekin and Ulusoy, 2011; Tarabalka et al., 2012; Chini et al., 2014a,b]. In remote sensing, almost every time, the ground truth is plotted by hand on the image and then a percentage of pixels in the
different classes are used to train the classifier, the rest is used for validation. The question is: is a per-pixel classifier watching the same image we are watching? Well, of course yes, but for a per-pixel classifier the following two images (Fig.1, Fig. 2), once the training set is given, are exactly the same. When we look at the result we are usually disappointed, the so called salt and pepper effect is affecting our solution. But, if we start from the Fig. 2, the same problem becomes more challenging. If we empathize with the machine, and remember what Touring said, we should understand that the only problem is language. The solution of the per-pixel classifier is not expressed in our language. What are we missing then? Knowledge

3 Calvino and the Theory of Knowledge

At the beginning of my Master’s thesis, back in Rome, I cited a beautiful passage of an incredible book which is Le città Invisibili, probably the best book I have ever read so far on classification and much more. The book is organized in cities, and the city which opened my mind was Zoe. What is really Knowledge? Knowledge, in my opinion, is what we agreed on, as a society. It has not general value but only relative, there are many different kinds of Knowledge and, among all societies, our personal ideas developed on what we agreed on is Culture. Different cultures are arising from what I will call Knowledge diversity. Now I need the help of Mathematics, the most self-aware of the languages, to define Knowledge.

Definition

Knowledge \( K \) is the set of all functions from \( X \) to \( Y \). Let \( X \) be the Information set and \( Y \) the set of Representations (the fruit of Knowledge).

Remarks

We can look at \( X \) as the set of Information we filter from real World through our senses (sensors
if machines), $Y$ will be the set of notions, ideas, theories... The fruit of Knowledge. History taught us that if we fix $y \in Y$:

$$\bigcap_{f \in K} f^{-1}(y) = \emptyset$$  \hspace{1cm} (1)

i.e. the same notions (ideas, theories, etc..) can arise from different sets of information. Agriculture was developed independently by at least three civilizations. It is in the end reasonable to think that there is no surjective $f \in K$.

Communication is the first reason to develop a language, Bernard Shaw once said: *The single biggest problem of communication is the illusion that it has taken place.* Suppose that we have two entities $A$ and $B$ and let us denote with $X_A$ and $X_B$ as the information that they filter from real World. And let us suppose that $Y_A = Y_B$, in other words the set of Representations are the same (i.e. they speak the same language). If we fix $y \in Y$ and denote $X_A^y = f_A^{-1}(y)$ and $X_B^y = f_B^{-1}(y)$ as the set of information that are sent in the same representation $y$:

$$X_A^y \bigcap X_B^y \neq \emptyset \text{ and } X_A^y \neq X_B^y.$$  \hspace{1cm} (2)

To make it clear suppose that $y = chair$, it is sure that the intersection of $X_A^y$ and $X_B^y$ when the word is pronounced is non zero but it is also reasonable to think that there will be something which is considered chair by only one of the entities and viceversa. What Shaw was trying to say, I believe, is that often we exchange only representations and rarely try to estimate functions $(f_B^{-1}, f_A^{-1})$ capable of revealing $X_B^y$ to the entity $A$ and $X_A^y$ to $B$. I like to define love as the *deepest way to Knowledge*, what I believe we do when we are in love is that we try to overcome the set of representations and connect directly to the set of information that the other is filtering from reality, having indeed access to an infinity of cardinality higher that $\aleph_0$.

I want to thank professor Paolo Piccinni, the most generous of the professors I had at university. He introduced us to the wildlife of Topology. In my opinion the most imaginative part of Mathematics, rarely studied. The pearls of this theory are classification results (e.g. Classi-
fication theorem of closed surfaces), going deep to the fundamental properties of space. How to remove uninteresting properties of the space ending up selecting the best features to distinguish them? Functions. Homeomorphisms in particular. In this theory, that can be seen as the highest point of human classification, features are called topological invariants. Thanks to this definition and the work of many Mathematicians we can see that what is important, most discriminating, are things that even if something changes (also dramatically) do not change. Those are the features we are looking for to solve a specific classification problem.

4 Knowledge vs Information

In machine learning, classifiers can be seen as specific functions, as topology teaches we rather need a set of functions (e.g. Homeomorphism) to properly solve a classification problem. The massive information we have from a series of sensors that nowadays are much better than humans in collecting data is amazing. What we should do, I think, is to be aware that Information is potentially Knowledge, it will be actual Knowledge once we agree, once we classify it. In Grenoble, where I am doing my PhD, I had the chance to attend lectures by E. Candès, his beautiful theory of Compressive Sensing is, in my opinion, so impressive because he is trying to solve a problem equipped with the correct Knowledge (i.e the set of norm functions). Machine learning community should be inspired by this work and realize that once we are properly equipped with the right Knowledge we can solve many problems in an elegant way, like Candès did. Once we are aware of what we are asking to the machine, and once we fully give the correct set of functions, then we can be sure that the result will reflect our language. Features should be defined as the invariant under the set of functions we are considering, those are then related to the choice of Knowledge and not the other way around (trying many different kinds of features will confuse more a poor classifier).
5 Conclusions

Each language is coming with strength and limitations, axioms on which we base our theories can be seen as an orientation of the language. Before solving problems one should be sure that the starting direction will develop a language capable to describe and hopefully solve those problems. I strongly believe that KnowledgeDiversity is the most important thing to fight for in this world. The set $\mathbb{K}$ is not ordered (i.e there is NO element in this set better than others). Machine learning will soon help us to have more time to think. If for a moment we assume the real world being described with $\mathbb{R}$, then let us consider as $\mathbb{Q}$ the set of our language. We know that if we want to select a point $r \in \mathbb{R} \setminus \mathbb{Q}$ in our line then we can go really close i.e. $\forall \epsilon \exists q \in \mathbb{Q}$ s.t. $r = q + \epsilon$. Even if we can come arbitrarily close to point $r$ we will never get there, I believe that the role of science is to give humans time to fill this infinitely small, but always present gap. This research I call it happiness. Happiness is my axiom.

Acknowledgments

I would like, once again, to abuse of Mathematics for my personal benefit. Let $\mathbb{B}$ be the set of all beautiful things in the universe and let $\mathbb{S}$ be the set of Stupidity then:

$$A.\text{Chiancone} = \mathbb{B} \cup \mathbb{S}$$

(3)

I only hope that $\text{card}(\mathbb{S}) = \aleph_0$!

Among this beauty there are people who helped me in my life, the ones I call my family. Thank you each and all of you, I will never forget that.
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IN VARIETATE CONCORDIA

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Fig. 1. From the right to the left my mom Aurora, my dad Marco and Camillo.

Fig. 2. Permutated version of Fig.1
Figure 1: From the left to the right my mom Aurora, my dad Marco and Camillo
Figure 2: Permuted version of Fig.1