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To cite this version:
Florent Brissaud, Heber Varela, Bruno Declerck, Nicolas Bouvier. Production availability analysis for oil and gas facilities: Concepts and procedure. 11th International Probabilistic Safety Assessment and Management Conference and the Annual European Safety and Reliability Conference, Jun 2012, Helsinki, France. pp.4760-4769. hal-01107310

HAL Id: hal-01107310
https://hal.archives-ouvertes.fr/hal-01107310
Submitted on 20 Jan 2015
Production availability analysis for oil and gas facilities:
Concepts and procedure

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Abstract: Since oil and gas facilities can be in multiple states (i.e. operate at different production levels) ranging continuously from nil to full production, the availability, which measures the expected proportion of time in a single (up) state, is too restrictive for performance evaluations. The concept of production availability has then been defined in ISO 20815 as the ratio of production to a reference level (e.g. the design or contracted rate), over a specified period of time. It is notably used to verify production objectives and requirements, identify critical items, compare alternatives and help for economic optimisation.

The present paper discusses the concepts of production availability (definitions, objectives), and presents a procedure to perform production availability analyses. The procedure, which is provided with several references, consists in objectives and preparation, study basis, model development, and production availability analyses. Contributions of the paper include a review of factors that have an impact on production availability (events included in the scope, system configuration, equipment and maintenance characteristics, scheduled shutdowns, operational constraints, etc.); the modelling principle (regarding state-based approach or behavioural modelling) and the use of flow capacity block diagrams (FCBD); the analysis principle (time-dependent and average computations, Monte Carlo simulations) and importance measures.

Keywords: Production availability, ISO 20815, oil and gas, flow capacity block diagram

1. CONCEPTS

The expedience of oil and gas exploitation depends on the availability of processing facilities. The availability is standardized as the ability of an item to be in a state to perform a required function under given conditions at a given instant of time, or in average over a given time interval, assuming that the required external resources are provided [1]. It is based on time, and on a single state (the up state) of an item. Since oil and gas facilities can be in multiple states (i.e. operate at different production levels) ranging continuously from nil to full production, an up state can be assumed when the actual production is equal to or greater than a reference level (e.g. a contracted or a design rate). However, because this availability does not differentiate states where production is slightly or highly below the reference level, even if the impact on resulting production can be important, it is too restrictive for performance evaluations of production systems.

Other availability (or “regularity” [2]) measures have then been proposed, notably those discussed by T. Aven, 1987 [3]. One measure accepted by the ISO 20815 international standard [4] (which derived from the Norsok Z-016 Norwegian standard [5]) is the production availability, defined as the ratio of production to planned production, or any other reference level, over a specified period of time. (The latter is volume-based instead of state-based; besides, the resulting measure is not a probability.)

The production availability analysis takes part in the production assurance of oil and gas projects, that is, the activities implemented to achieve and maintain a performance that is at its optimum in terms of the overall economy and at the same time consistent with applicable framework conditions [4]. It is especially suitable for projects with medium to high technical risk, and during the first life-cycle phases (feasibility, conceptual design, and engineering). The production availability analyses are then used to:

- predict production-performance, and verify compliance with objectives and requirements (specified in the production-assurance programme (PAP) [4]);
- identify operational conditions, subsystems and equipment items that are critical, and find measures for performance improvement;
- compare alternatives, and enable selection/optimisation of equipment items, configurations, maintenance actions, and operations, with economic considerations (under project, technical, operational, health, safety, environmental, and regulatory constraints).
“Production-performance analyses should be consistent and assumptions and reliability data traceable” (ISO 20815 [4], informative part). To fulfil this common sense guidance, a procedure for production availability analysis has to be followed. The four-step procedure presented in the following sections is based on the authors’ experience in reliability, availability, and maintainability (RAM) analyses for oil and gas facilities, and meets the recommendations of the general framework given by the ISO 20815 [4].

2. OBJECTIVES AND PREPARATION

According to the current phase of the project, more or less decisions regarding the system design have already been made, and the purpose of production availability analysis is therefore not the same. During the feasibility phase, the objective can be to optimise the asset-development plans by analysing several alternatives; during the conceptual design, the optimisation problem is usually reduced to two or three alternative field-layout configurations; and during the engineering phase, only a few alternative design solutions are still possible, and the production availability analysis is mainly used to verify compliance with requirements, for sparing recommendations and spare-parts optimisation.

The production-performance measure also depends on the project phase and relating objectives (and should agree with the PAP). To model more exhaustively the performance of a production system, the (volume-based) production availability (at time \( t \) or in average) is usually preferred than the (state-based) availability (cf. Section 1). A “reference level” of production has therefore to be defined. To this end, the design rate (i.e. maximum input feed rate that can be treated) is often used in early project phases because it is usually time-independent, convenient for any part or subpart of the production chain (independently of other systems), and does not require sales contract or well-production rates to be defined yet. In more advanced phases of a project, the planned production volume assuming no (planned or unplanned) downtime can be preferred, taking the constraints of sales contract and well-production potentials (e.g. through the actual input feed rate) into account once available. However, to avoid time-dependent constraints in the system description and modelling, it is often more convenient to reason in terms of design rate during the study basis and model development (cf. Sections 3 and 4), and to translate in terms of other reference levels only during the production availability analyses (cf. Section 5). Table 1 given examples of production availability definitions according to different reference levels.

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| design rate \( r_d \) \( r_d \) is the reference level: \( A^d(t) = \min(r_d(t), r \times C_s(t)) \) \( r_d(t) \)
| design rate \( C_s(t) \) \( C_s(t) \) is the reference level: \( A^C(t) = \min(r_C(t), r \times C_s(t)) \) \( r_C(t) \)
| average production availability, general definition: \( A_{[t_1, t_2]} = \int_{t_1}^{t_2} A(t) \) \( dt \) |

The production availability is defined for a single product (which has to be clearly defined, with specifications). If several products are relevant (e.g. oil, gas, condensate), one measure can then be used per product, for informative purpose or multi-objective optimisation issues. Finally, when the effects of compensating elements (e.g. downstream buffer storage, other producers) are available, they should also be included in the complete assessment of the system production-performance.

Once the objectives and the production-performance measures are defined, the preparation of the production availability analysis starts with the collection of all the relevant documents (which are provided by the facilities designer and/or customer). According to the current phase of the project, the available documents can include the design basis, process description and control philosophy, block flow diagrams (BFD), process flow diagrams (PFD), utility flow diagrams (UFD), piping and instrumentation diagrams (PID), operation and maintenance strategies, sparing philosophy, RAM data, production profiles, demand profiles, etc. The review of these documents then allows the study basis to be established.

| Table 1. Definitions of production availability according to different reference levels |
|---------------------------------|---------------------------------|---------------------------------|
| production availability at time \( t \), general definition: \( A(t) = \frac{P(t)}{r_{\text{ref}(t)}} \) | production availability at time \( t \), general definition: \( A(t) = \frac{P(t)}{r_{\text{ref}(t)}} \) | production availability at time \( t \), general definition: \( A(t) = \frac{P(t)}{r_{\text{ref}(t)}} \) |
| with \( P(t) \) the actual system production, and \( r_{\text{ref}(t)} \) the reference level of production | with \( P(t) \) the actual system production, and \( r_{\text{ref}(t)} \) the reference level of production | with \( P(t) \) the actual system production, and \( r_{\text{ref}(t)} \) the reference level of production |
| \( A^d(t) = \frac{1}{r_d(t)} \times C_s(t) = C_s(t) \) | \( A^d(t) = \frac{1}{r_d(t)} \times C_s(t) = C_s(t) \) | \( A^d(t) = \frac{1}{r_d(t)} \times C_s(t) = C_s(t) \) |
| with \( C_s(t) \) the actual system capacity | with \( C_s(t) \) the actual system capacity | with \( C_s(t) \) the actual system capacity |
| \( A^C(t) = \frac{1}{r_C(t)} \times C_s(t) = C_s(t) \) | \( A^C(t) = \frac{1}{r_C(t)} \times C_s(t) = C_s(t) \) | \( A^C(t) = \frac{1}{r_C(t)} \times C_s(t) = C_s(t) \) |
| average production availability, general definition: \( A_{[t_1, t_2]} = \int_{t_1}^{t_2} A(t) \) \( dt \) | average production availability, general definition: \( A_{[t_1, t_2]} = \int_{t_1}^{t_2} A(t) \) \( dt \) | average production availability, general definition: \( A_{[t_1, t_2]} = \int_{t_1}^{t_2} A(t) \) \( dt \) |
3. STUDY BASIS

3.1. Purpose and events included in the scope

The purpose of the study basis is to present the objectives of the study (cf. Section 2), and to set up all the data and assumptions that are required for modelling the system and analysing its production availability. The study basis is based on the collected documentation (cf. Section 2), and prepared with the help of production/performance and operability reviews (POR). If relevant, RAM testing can also be performed. The study basis should be exhaustive enough to allow the model development (cf. Section 4) and the production availability analyses (cf. Section 5) to be made (and reproduced) without additional information. The engaged data and assumptions should be referenced (with the version of the documents specified) and, when required, other choices and expert judgments should be explained.

Before going in depth into the system description, the scope of the study has to be specified in terms of events that are included or excluded in the modelling and production availability analyses. According to the performance objectives and requirements, these events may cover: process equipment failures (e.g. critical, degraded, incipient failures); safety system failures and procedures (e.g. safety shutdowns, spurious trips); pipe failures; systematic failures (related in a deterministic way to errors in design, manufacturing, operational procedures, etc.); software failures (for programmable equipment items); corrective (unplanned) maintenance; preventive (planned) maintenance; turnarounds (e.g. general overhauls) and modifications (e.g. design changes); catastrophic events (e.g. fires and explosions, earthquakes and seismic sea waves, sabotage).

Once the scope of study is specified, the study basis is composed of two main parts: the system description and the RAM data.

3.2. System description

The system description first comprises the system definition (i.e. the units included in the scope, for example taking part of the wells, gathering network, process facilities, utilities, storage, and export) with boundaries relative to its surroundings (e.g. the first and the last valves included in the scope, for each relevant line); the system design rate (i.e. maximum input feed rate that can be treated); and the system design life (i.e. the production lifetime, which is the time period relevant for the analyses) organised, if relevant, in operating phases (e.g. start-up, normal operation, production plateau period, operation with partial loads, operation with new facilities, run-down) with respective rates (i.e. actual input feed rate and/or contracted rate to be treated for each product or kind of product). A flow (capacity block) diagram (cf. Section 4.2) (or a flow network) is suitable to depict the routing of the product(s) through all the system units (considered as “branches”), with specifying their respective design rates (expressed in absolute value) or equivalent design capacities (expressed in percentage of the system design rate).

Then, a detail description of each unit included in the scope is provided. When relevant, a unit can be divided in series/parallel branches, with respective design rates or capacities. These descriptions shall also explain failure mode and effect analyses (FMEA) designed for production systems, that cover all the equipment items with regards to the events included in the scope (cf. Section 3.1). Such FMEA can include the following items:

- tag (identification of the equipment item);
- unit (name of the unit that includes the equipment item);
- branch (name of the unit branch that includes the equipment item);
- equipment item (component or group of components, of which outages may impact production);
- equipment design rate (expressed in absolute value), if different from the branch design rate;
- type of equipment item (e.g. process equipment, safety system);
- equipment configuration (design rate, redundancy/sparing of the equipment item);
- failure mode(s) (list of possible failures of the equipment item, including possible failure combinations among its redundant/sparing parts);
- local effect of each failure mode on the production (relative capacity of the equipment item during the failure mode, expressed in percentage of the equipment design rate);
- global effect of each failure mode on the production (if relevant);
mitigation measures (specific equipment or operations used to compensate for the effect of the failure mode e.g. bypass, flaring, venting, buffering, off-spec production);

maintenance requirements (characteristics that can affect the maintenance duration, e.g. the need for specific maintenance crews, tools, and assets);

planned/preventive maintenance (“known unavailability” of the equipment item);

effects of maintenance operations on the production (if different from the effects between the failure mode occurrence and the maintenance start);

other information (including potential common cause failures or on-demand sparing failures);

comments (notably to justify unusual data).

By nature, one FMEA limitation is the lack of representation of failure mode interactions. In some cases, it is therefore required to enclose notes that describe such interactions (e.g. additional production losses when a specific combination of failure modes occurs).

Finally, the system description has to specify the general operational conditions. These features include notably possible ramp-up (when the production starts at zero e.g. following a process depressurization, this is the time required for the production reaching its design rate), turndowns (rate below which the system cannot produce, basically, the system functions in closed loop until this turndown limit is reached), general shutdowns (turnarounds, modifications), and specific regulation constraints (e.g. maximum quantities for flaring and venting, minimum frequencies for preventive maintenance and turnarounds).

3.3. Reliability, availability, and maintainability (RAM) data

The next part of the study basis presents the RAM data. These data can include:

- failure rates of the equipment items (one failure rate per relevant failure mode);
- rates of common cause failures (failures of different equipment items resulting from the same direct cause, occurring within a relatively short time);
- on-demand probabilities of sparing failures (probability that a spare part is not activated when required) and corresponding (mean) times to repair;
- times to repair the equipment items (one time to repair per relevant failure mode), or set of parameters that describes the corresponding distributions (e.g. extreme and/or average values);
- number of maintenance crews, regarding all the types of equipment items/failure modes;
- time periods where maintenance can be carried out;
- logistic delays (accumulated time during which maintenance cannot be carried out due to the necessity to acquire maintenance resources), or set of parameters that describes the corresponding distributions;
- number of spare parts required for maintenance (and, if relevant, stock and supply constraints)
- frequencies and durations of “known unavailability” for each equipment item (preventive maintenance) and for the whole system (turnarounds, modifications);
- expected frequencies and durations of safety shutdowns (to prevent hazard, or due to spurious trips);
- if relevant, design, use, environmental, and human factors.

It should be noted that several RAM data can depend on process variables. For example, the failure rate of an equipment item can be reduced when its operation (regarding the flow of product through it) is reduced or intermittent. In particular, when an equipment item is in stand-by due to a nil flow (e.g. if the equipment item is in spare/“cold redundancy” or during a turnaround), its failure rates are sometimes assumed negligible.

To present the RAM data in the study basis, the equipment items can be classified by categories (e.g. valves, vessels, compressors, pumps, heat exchangers, etc.), then by failure modes (e.g. incipient, degraded, critical), and the equipment failure rates and parameters for times to repair are assigned according to them. To this end, the RAM feedback data issued from the same kind of applications (same equipment items, same operational conditions, same environment) should be preferred, provided that they are sufficient enough to be assumed significant (based on confidence intervals of statistic estimates). By lack of such feedback knowledge, RAM data handbooks can be used. For example, the OREDA [6] is commonly used for process equipment items (mainly for offshore applications), and the PDS data handbook [7] is concerned with safety (instrumented) systems. Using these handbooks, a special attention has to be paid on the boundary definition of the equipment items (what parts are assumed in the given data, including instrumentation and pipes), and
also on the definition of the failure modes (which can be application-dependent). Moreover, it is often necessary to apply corrective coefficients, for example to the given failure rates in order to take influencing factors (regarding design, use, environment, etc.) into account [8]. The data selection (issued or not from databases) then often requires an important part of expert judgement (regarding the equipment type and technology, maintenance and operational conditions, data significance, relevant failure modes, equipment boundaries, various influencing factors, etc.) and should be agreed upon by the involved parties.

4. MODEL DEVELOPMENT

4.1. Basis of model development

The model development is performed using the information, data and assumptions given in the study basis (cf. Section 3). The modelling of production availability first consists in expressing the time-dependent system capacity $C_s(t)$ (i.e. proportion of the design rate that can be actually treated by the system), expressed in percentage of the system design rate $r_s$ (specified in the study basis, cf. Section 3.2). Then, the actual time-dependent system production $P(t)$ can be simply expressed, assuming the design rate $r_s$ and, for example, the input feed rate $r_d(t)$ and the contracted rate $r_c(t)$, depending on the defined reference level (cf. Table 1). Of course, the system capacity $C_s(t)$, and thus its actual production $P(t)$, depend on the time-dependent system (random) state $X_s(t)$, forming stochastic processes. Due to these stochastic characteristics, a “right” (single) value of $C_s(t)$ (as well as $P(t)$ and $X_s(t)$) cannot exist (assuming no deterministic case such as planned nil production), and model development should therefore focuses on the expectancy, denoted $E[C_s(t)]$.

Commonly, $X_s(t) = (X_1(t), X_2(t), ..., X_n(t))$, where $X_i(t)$ is the state of equipment item $i$, and $n$ is the number of defined equipment items in the whole system. (It is also often required to define “fictitious items” for modelling some factors including maintenance characteristics, scheduled shutdowns, mitigation measures, and other operational conditions.) Basically, an equipment item can be in an “operating state” with full capacity, in various “failed states” (between the failure mode occurrence and the maintenance start) and “repair states” with different corresponding capacities (from nil to full) according to the failure modes. The use of “passive” equipment items may also require some specific states to be defined (e.g. “ready to operate state”). The transitions between equipment states can then be determined by deterministic and/or random variables (e.g. time to failure, time to repair, logistic delay), and statements (i.e. predicates), which depend on other variables (e.g. maintenance crew availability, requirements for sparing activation).

For example, Figure 1 depicts a production according to time, for one equipment item. First, a failure causes a partial production loss (e.g. the equipment capacity is reduced due to a degraded failure mode). Then, the maintenance starts after a logistic delay, and causes a full production loss (the equipment capacity is nil during the maintenance). Once the maintenance is finished, a ramp-up starts. However, as long as the capacity does not allow a production that exceeds the turndown limit, the production is nil. Then, the production may restart and continues to grow with the ramp-up, up to the full rate.

![Figure 1. Example of equipment production changes according to time](image-url)
Flow capacity block diagrams (FCBD) are used to describe the product routing through the system units and equipment items. Basic elements are equipment items, and they are grouped in series and parallel branches, up to the whole system level. Each defined element (equipment item, branch, system) should be specified with a design rate (i.e. maximum input feed rate that can be treated by the element, expressed in absolute value) as a parameter, and a relative capacity (i.e. proportion of the element design rate that can be actually treated by the element, expressed in percentage) as a time-dependent random variable. The product flows are depicted by arrows between elements. It is assumed that the whole system has only one input and one output, and that there is no loop in the resulting directed graph.

Let $i$ be the index for equipment items, expressed in Arabic numerals (i.e. $i = 1, 2, ..., n$ with $n$ the number of defined equipment items). The design rate of equipment item $i$ is denoted $r_i$, and its relative capacity at time $t$ is denoted $C_i(t)$. $r_i$ is a parameter that depends on the design of equipment item $i$, and $C_i(t)$ is a random variable that depends on the equipment state $X_i(t)$. Similarly, let $b$ be the index for branches, expressed in Roman numerals (i.e. $b = I, II, ..., ñ$ with ñ the number of defined branches). The design rate of branch $b$ is denoted $r_b$, and its relative capacity at time $t$ is denoted $C_b(t)$. $r_b$ is a parameter that depends on the design rates of the equipment items and/or sub-branches that compose branch $b$, and $C_b(t)$ is a random variable that depends on the relative capacities. Finally, the design rate of the system is denoted $r_s$, and its (relative) capacity at time $t$ is denoted $C_s(t)$.

To specify a branch, the “design capacity” is often preferred (than the design rate). It is defined as the ratio of the branch design rate to the system design rate. The design capacity of branch $b$ is denoted $\omega_b$, and it is therefore equal to $(r_s / r_b) \times 100\%$. Commonly, to specify $p$ branches in parallel with $\omega_p$ design capacity each, the notation “$p \times \omega_p$” is used. If an “$(p-q) \times \omega_p$” configuration already allow a 100% design capacity (i.e. $(p-q) \times \omega_p \geq 100\%$), then $q$ parallel branches can be used as “spare.” In that case, a product flow is routed to them only if it is required for maintaining a full rate of production (e.g. the relative capacity of the $(p-q)$ first branches is reduced below a sufficient level). A spare branch can be depicted on a FCBD by a “broken” line (as a “switch”).

Figure 2 gives an example of FCBD for a system composed by seven equipment items and four branches. Branches I, II, and IV are series, while branch III is parallel (with the two sub-branches I and II). The system can also be considered as a series branch (with the two sub-branches III and IV).

The design rate of a series branch is equal to the minimum design rate among the elements that compose the branch. The relative capacity of a series branch is equal to the minimum of the products of design rate and relative capacity among the elements that compose the branch, divided by the branch design rate. Assuming the FCBD example given in Figure 2, $r_1 = \min[r_1, r_2, r_3]$, $r_6 = \min[r_6, r_3, r_7]$, $r_4 = \min[r_4, r_5]$; $r_1 = \min[r_1, r_5, r_7]$, and $C_2(t) = \min[C_2(t) \times r_1, C_2(t) \times r_2, C_2(t) \times r_3] / r_1$, $r_5 = \min[C_5(t) \times r_6, C_5(t) \times r_7] / r_5$, and $r_3 = \min[C_3(t) \times r_6, C_3(t) \times r_7] / r_3$. It should be noted that it is often more convenient to calibrate the model such as the design rates of the elements that compose a branch are equal to the design rate of the branch (equipment relative capacities have then to be defined accordingly).

The design rate of a parallel branch is equal to the sum of the design rates of the elements that compose the branch. The relative capacity of a parallel branch is equal to the sum of the products of design rate and relative capacity of each element that compose the branch, divided by the branch design rate. Assuming the FCBD example given in Figure 2, $r_{II} = r_I + r_{III}$ and $C_{II}(t) = (C_I(t) \times r_I + C_{III}(t) \times r_{III}) / r_{II}$.

Modelling the capacity $C_s(t)$ of the system at time $t$ can be based on two approaches. The first approach uses the FCBD to establish the set of the possible values of $C_s(t)$ (linked to the system state $X_s(t)$) according to the sets of the possible values of $C_i(t)$ (linked to equipment states $X_i(t)$, with $i = 1, ..., n$). Then, the probability distribution of $C_s(t)$ is obtained by combinations of the probability distributions of $X_i(t)$. This state-based approach is quite simple for basic cases, for example when each equipment item has only two states (binary case) [9] or when the equipment items are independent [3]. For other cases, it can also be based on analytical approaches or on simulations, as described in Section 4.3. The second approach is more “bottom-up” since the equipment states $X_i(t)$ are first simulated, and then the system capacity $C_s(t)$ is computed according to the FCBD. This behavioural modelling is based on simulations, as described in Section 4.4.
4.3. State-based approach

When the possible states of the system (i.e. the possible values of $X_i(t)$ at any time $t$) form countable and finite set $\{X_i,k\}_{k=1}^{m_s}$, with $m_s$ the number of possible system states, and when the possible capacities of the system (i.e. the possible values of $C_i(t)$ at any time $t$) form countable and finite multiset $\{c_i,k\}_{k=1}^{m_c}$ such as the system capacity is $c_{i,k}$ when its state is $x_{i,k}$ (i.e. $X_i(t) = x_{i,k}$), a state-based approach can be used. ($\{c_i,k\}_{k=1}^{m_c}$ can contain identical members.)

Let assumed that $X_i(t) = (X_1(t), X_2(t),..., X_n(t))$, with $X_i(t)$ the state of equipment item $i$, and $n$ the number of defined equipment items in the system. To fulfill the previous condition, the possible states of each equipment $i$ (i.e. the possible values of $X_i(t)$ at any time $t$) has also to be defined by a countable and finite set $\{x_i,k\}_{k=1}^{m_i}$, with $m_i$ the number of possible states of equipment $i$. In addition, when equipment item $i$ is in state $x_{i,j}$ (i.e. $X_i(t) = x_{i,j}$), its relative capacity is denoted $c_{i,j}$.

The number of possible system states is therefore $m_s = m_1 \times m_2 \times ... \times m_n$. For each of these $m_s$ system states $x_{i,k}$, the corresponding system capacity $c_{i,k}$ has to be determined (assuming that this capacity only depends on the current system state, and not on the ordered sequence of events that lead to this state). To this end, the FCBD (cf. Section 4.2) is used to compute $c_{i,k}$ according to the relevant values of $c_{i,j}$. (After this task, it can be convenient to group together all the system states $x_{i,k}$ that result the same system capacity in order to reduce the total number of outcomes to analyse.) To assign each state $x_{i,k}$ to a (predefined) system capacity, a systematic procedure based on minimal cut sets has been proposed by E. Zio et al. [10].

The expected capacity $E[C_i(t)]$ of the system at time $t$ is expressed as follows:

$$E[C_i(t)] = \sum_{j=1}^{m_s} (Pr[X_i(t) = x_{i,j}] \times c_{i,j})$$

(1)

The challenge of this “multi-state”/“multi-output” problem is then to compute the probability that the system is in state $x_{i,k}$ at time $t$ (i.e. $Pr[X_i(t) = x_{i,k}]$), (or in any group of states that result the same system capacity), which is function of the probability distributions of the time-dependent (random) equipment states $X_i(t)$, with $i = 1, ..., n$. Analytical approaches can be provided for simple cases, for example when the equipment items are independent [11]. For more general cases, classical reliability methods such as reliability block diagrams, fault trees, and Markov models can be able to provide “exact” results [12]. Finally, Monte Carlo simulations can also be performed for more complex cases [10], for example with the support of a stochastic Petri net model [13].

While a state-based approach is efficient for quite simple systems, notably because “exact results” can be obtained, its applications to more complex systems may present difficulties in terms of discrete definition of system states and capacities, modelling of event sequences, and scale problem for large systems.
4.4. Behavioural modelling

A behavioural modelling focuses directly on the equipment states $X_i(t)$ (with $i = 1, ..., n$ and $n$ the number of defined equipment items in the system), and is simulation-based. For equipment item $i$, a transition from state $x_{i,a}$ to state $x_{i,b}$ (i.e. $X_i(t) = x_{i,a}$, then $X_i(t + \Delta t) = x_{i,b}$) can occur randomly (e.g. an equipment item fails according to an exponential distribution defined by a failure rate) or deterministically, based on conditions (e.g. an equipment item of a spare branch is activated according to the equipment states of other branches) and/or time delays (e.g. an equipment item starts to be maintained after a certain delay). The equipment relative capacities $C_i(t)$ (with $i = 1, ..., n$) then follow these state changes. For equipment item $i$, a transition from state $x_{i,a}$ to state $x_{i,b}$ results in a change from relative capacity $c_{i,a}$ to relative capacity $c_{i,b}$ (i.e. $C_i(t) = c_{i,a}$, then $C_i(t + \Delta t) = c_{i,b}$). The FCBD (cf. Section 4.2) is then used to actualise dynamically the system capacity $C_i(t)$ according to the equipment relative capacities $C_i(t)$ (with $i = 1, ..., n$).

The expected capacity $E[C_i(t)]$ of the system at time $t$ is estimated by Monte Carlo simulations. This approach is based on statistical results obtained from several simulated histories. Each history simulation consists in moving step by step to the next instant of event occurrence (i.e. state transition). These time instants are determined by a pseudorandom generator, according to the probability distributions of the random variables, and the current values of these variables. At each step, the variables (notably $C_i(t)$ with $i = 1, ..., n$ and $C_i(t)$ are actualised. A history stops once the system design life is reached, and for each history the “path” of $C_i(t)$ is recorded. Finally, $E[C_i(t)]$ is estimated (at each time $t$) as the average value of $C_i(t)$ among all the recorded paths. The confidence interval of this statistic estimate allows judging the sufficiency of the number of histories (cf. Section 5.2).

To support behavioural modelling, tools based on “mode automation,” notably the AltaRica Data-Flow language [14], can be used. Dedicated methods based on extended block diagrams, as well as stochastic Petri nets with predicates, also provide promising tools for these tasks [15], such as the BStoK module of GRIF [16]. Finally, other commercial simulators based on this approach, including MAROS [17], OPTAGON, and MIRIAM Regina, are also available to model and analyse production systems [18].

Since the system states and capacities do not need to be preliminarily defined by countable and finite sets, this approach is very flexible. In particular, additional variables (which can be time-dependent and dependent on each other) can be introduced in the model, and used to define more complex equipment states and relative capacities. However, a main drawback is the Monte Carlo simulations, which can be relatively time-consuming.

5. PRODUCTION AVAILABILITY ANALYSES

5.1. Basis of production availability analyses

Production availability analyses aim at generating indicators that describe production characteristics of the system. They are based on the time-dependent system production $P(t)$ expressed by the model (cf. Section 4.1), and commonly on the reference production level $P_{ref}(t)$ (cf. Table 1). Since $P(t)$ is a random variable, the production availability analyses focus on expectancies and confidence intervals of indicators. The expected production availability $E[A(t)]$ of the system at time $t$ is expressed as follows:

$$E[A(t)] = E \left[ \frac{P(t)}{P_{ref}(t)} \right]$$

(2)

In most cases, an average measure is preferred as indicator. The expected average production availability $E[A_{t_1,t_2}]$ of the system in time interval $[t_1, t_2]$ is expressed as follows:

$$E[A_{t_1,t_2}] = E \left[ \int_{t_1}^{t_2} A(t) \, dt \right] = E \left[ \int_{t_1}^{t_2} \frac{P(t)}{P_{ref}(t)} \, dt \right]$$

(3)

The time interval $[t_1, t_2]$ can be, for example, the system design life or an operating phase defined in the study basis (cf. Section 3.2), or each year of the system design life.

For example, if the reference level of production is defined by the system design rate (i.e. $P_{ref}(t) = r_s$), Equations (2) and (3) can be simplified using the equality $A(t) = C_i(t)$ (cf. Table 1).
Other indicators can, for example, focus on the probability that the system production is over the reference production level (i.e. $Pr(P(t) ≥ P_{ref}(t))$) or greater than 0 (i.e. $Pr(P(t) ≥ 0)$), at time $t$ or in average in a given time interval.

In addition to the system production, intermediate productions at the outlet of system units or equipment items can also be considered (e.g. $C_i(t) × r_i$ for equipment item $i$, or $C_b(t) × r_b$ for branch $b$).

### 5.2. Estimates using Monte Carlo simulations

Monte Carlo simulations are required to perform the production availability analyses when a behavioural modelling is used (cf. Section 4.4), and sometimes also when a state-based approach is used (cf. Section 4.3).

Let $Y$ be the assumed indicator (e.g. defined by one measure given in Table 1). $Y$ is therefore a random variable, which corresponds to an average value in a given interval (e.g. $A_{[t_1,t_2]}$) or a value at a given time $t$ (e.g. $A(t)$). To estimate $Y$, $m$ Monte Carlo simulations (or “histories”) are performed, each providing the observation $y_l$ of indicator $Y$, with $l = 1, ..., m$. The “true” expectancy of $Y$ is denoted $E[Y]$. Even if Monte Carlo simulations cannot provide an “exact” value of $E[Y]$, an estimate $\hat{E}[Y]$ can be obtained by:

$$\hat{E}[Y] = \frac{1}{m} \sum_{l=1}^{m} y_l$$  \hspace{1cm} (4)

If the number $m$ of Monte Carlo simulations is small in face of the variability of the observations $y_l$, then the estimate obtained by Equation (4) can be very uncertain. To judge the sufficiency of the number of simulations, a confidence interval on $\hat{E}[Y]$ has to be assessed (and provided with results). For a value of $m$ relatively high, it is shown by the central limit theorem that:

$$\Pr \left[ |E[Y] - \hat{E}[Y]| ≤ \frac{1.96}{\sqrt{m}} \sqrt{\frac{\sum_{l=1}^{m} (y_l - \hat{E}[Y])^2}{m-1}} \right] = 0.95$$  \hspace{1cm} (5)

It is important to note that the confidence interval given by Equation (5) concerns the estimate of the expectancy of $Y$, and does not provide any information on the probability that a single observation $y_l$ (taking at random) is close or not to $\hat{E}[Y]$ (e.g. will the indicator, which will be observed in practice, be close or not to the estimate expectancy?).

Let assumed that the $m$ observations $y_l$ of indicator $Y$ (with $l = 1, ..., m$), obtained by Monte Carlo simulations, are arranged in such a way that $y_1 ≤ y_2 ≤ ... ≤ y_m$. For a value of $m$ relatively high, a confidence interval on $y_f$, where $y_f$ is an observation of indicator $Y$ that is either taken at random from $\{y_l\}_{l=1}^{m}$ or that will be observed in the future, can be assessed by:

$$\Pr \left[ y_f \left[\frac{\alpha}{2}\right] ≤ \gamma_f ≤ y_f \left[1 - \frac{\alpha}{2}\right] \right] = 1 - \alpha$$  \hspace{1cm} (6)

where $\alpha$ is a probability with $0 ≤ \alpha ≤ 1$ (e.g. $\alpha = 0.05$), and $[b]$ is the integer number that is the closest to $b$.

While the confidence interval obtained by Equation (5) depends on the Monte Carlo simulations and can therefore be reduced by increasing the value of $m$, the confidence interval obtained by Equation (6) depends on the system properties and can only be refined by increasing the value of $m$.

In addition, for obtaining confidence intervals of the results with regards to the assumptions and data taken, sensitivity analyses can be used to assess the changes in the results when input assumptions are modified and input data are set to other possible values, especially for those concerning the most critical parts of the system (cf. Section 5.3).

### 5.3. Importance measure

To assess the importance of an equipment item, system unit, or operational condition for the production availability, several importance measures can be defined, including the criticality for production availability and the contribution to production unavailability. These measures can be assumed for different indicators, at time $t$ or in average (cf. Section 5.1). The examples given below refer to the average production availability $A_{[t_1,t_2]}$ of the system in time interval $[t_1, t_2]$, but can be easily extended to other indicators.
The criticality for production availability of an equipment item or system unit is defined by 100% minus the average production availability that results from a configuration where the equipment item or system unit is “removed,” that is, its relative capacity is set to 0%. For example, the criticality \( \text{Crit}(i) \) of equipment item \( i \) in time interval \( [t_1, t_2] \) is expressed as follows:

\[
\text{Crit}(i)_{[t_1, t_2]} = 100% - E[A_{[t_1, t_2]}](C_i(t) = 0%)
\]  

(7)

By definition (for “coherent” systems), a criticality cannot be lower than the actual average production availability (i.e. \( A_{[t_1, t_2]} \)). Moreover, if \( \text{Crit}(i)_{[t_1, t_2]} = A_{[t_1, t_2]} \), then equipment item \( i \) has not effect on production availability in time interval \( [t_1, t_2] \) (according to the model).

The contribution to production unavailability of an equipment item or system unit is defined by the average production unavailability that results from a configuration where the equipment item or system unit is “perfect,” that is, its relative capacity is set to 100%, minus the actual average production availability. This measure also corresponds to the “potential for production availability improvement” of the equipment item/unit. In other words, how much the production availability can be improved (or production unavailability can be reduced) by reducing the production losses due to these elements (e.g. by increasing design rates, adding redundant/sparing parts, reducing scheduled shutdowns) For example, the contribution to production unavailability \( \text{Cont}(i)_{[t_1, t_2]} \) of equipment item \( i \) in time interval \( [t_1, t_2] \) is expressed as follows:

\[
\text{Cont}(i)_{[t_1, t_2]} = E[A_{[t_1, t_2]}](C_i(t) = 100%) - E[A_{[t_1, t_2]}]
\]  

(8)

The contribution to production unavailability can also be applied to operational conditions such as the number of maintenance crews (comparing the actual case with a configuration with more maintenance crews), the periodicity and duration of scheduled shutdowns, etc. An optimisation policy of the production availability should focus on the equipment items, system units, and operational conditions that have the greater contribution to production unavailability, balancing with the costs of improved solutions.

References


