Computation and Modelisation of the Sound Radiation of an Upright Piano Using Modal Formalism and Integral Equations
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**Computation and Modeling of The Sound Radiation of An Upright Piano Using Modal Formalism and Integral Equations**

Philippe Dérogis, René Caussé

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**Introduction**

Upright Piano

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**Modal Analysis**

Modal Formalism for general viscous damping

\[
M \ddot{x} + C \dot{x} + K x = p \quad (1)
\]

\[
\begin{bmatrix}
C & M \\
M & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\ddot{x}
\end{bmatrix}
+ 
\begin{bmatrix}
K & 0 \\
0 & -M
\end{bmatrix}
\begin{bmatrix}
x \\
\dot{x}
\end{bmatrix}
= 
\begin{bmatrix}
p \\
0
\end{bmatrix} \quad (2)
\]
\[ A r + B \dot{r} = s \]  \hspace{2cm} (3)

**Eigenvalue Problem**

\[ A y_k = -\lambda_k B y_k \]  \hspace{2cm} (4)

where \( y_k \) can be written as:

\[ y_k = \begin{bmatrix} z_k^* \\ \lambda_k z_k \end{bmatrix} \]  \hspace{2cm} (5)

where \( \{\lambda_k, z_k\} \) are respectively the eigenvalues and the eigenvectors of equation (1).

**Modal parameters extraction**

The response \( a_{rs} \) at the point \( r \) to a sinusoidal excitation at the point \( s \) can be expressed using the eigenvalues and the eigenvectors of equation (1):

\[ a_{rs} = \sum_{k=1}^{N} \left( \frac{A_{r,s(k)}}{j\omega - \lambda_k} + \frac{A_{r,s}(\dot{k})}{j\omega - \lambda_k} \right) \]  \hspace{2cm} (6)

where \( A_{rs(k)} = \frac{a_{rs(k)}}{\rho_k} \)

**Computation of the displacement resulting from any excitation**

The displacement \( x(t) \) resulting from an excitation \( s e^{j\omega t} \) can be computed using the formula:

\[ x(t) = \sum_{k=1}^{N} \left( \frac{1}{j\omega - \lambda_k} \frac{z_k z_k^t}{\rho_k} + \frac{1}{j\omega - \lambda_k} \frac{\dot{z}_k \dot{z}_k^t}{\rho_k} \right) s e^{j\omega t} \]  \hspace{2cm} (7)

where \( \rho_k = \dot{z}_k^t C z_k + 2\lambda_k \dot{z}_k^t M z_k \)

**Measurements and Results**
Sound Radiation

Sound radiation of a baffled plate

The sound pressure $p(r)$ and the acoustic velocity $\vec{v}(r)$ resulting from the vibrations of a baffled plate moving with normal acceleration $\gamma_n$ are given by:

\[ p(r) = \rho_0 \int_{r_0 \in \text{Plate}} \gamma_n(r_0) \frac{e^{-jk\|r-r_0\|}}{2\pi \|r - r_0\|} \, dr_0 \]

(8)

\[ \vec{v}(r) = \rho_0 \int_{r_0 \in \text{Plate}} \gamma_n(r_0) g \frac{-e^{-jk\|r-r_0\|}}{2\pi \|r - r_0\|} \, dr_0 \]

(9)

Sound pressure radiated by the first mode of the soundboard
Active Intensity

Normal Active Intensity

The active intensity is the power radiated per unit surface. It is given by the formula:

\[ \vec{I}(r) = \frac{1}{2} \Re(p(r) \overline{\delta(r)}^*) \]  

(10)

It is interesting to calculate the \( z \) component of the active intensity near the soundboard in order to know which regions produce the acoustic power, in particular:

- If it is positive, the energy travels from the soundboard to the acoustic field
- If it is negative, the energy travels from the acoustic field to the soundboard.

Computation of the normal Active Intensity at 5 centimeters of the soundboard
An example of a loop of active intensity

Active intensity field in a plane orthogonal to the plate for the mode 1: 122 Hz
Radiation efficiency

Power radiated

The total power radiated by a source can be computed using:

\[ \mathcal{W}_a = \oint_S \mathbf{E} \cdot \mathbf{n} \, dS \]  \hspace{1cm} (11)

Power radiated by the soundboard versus frequency for several driving point

- Computation of the displacement of the soundboard using modal formalism
- Computation of the acoustic field
- Computation of the radiated power

Results

Radiated power for several points excited by a 1N force

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**Point #1: Base bridge A1 : 55Hz**

**Point #2: Main bridge A2 : 110Hz**

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**Point #3: Main bridge B2 : 164Hz**

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Radiation efficiency

Radiation efficiency of the soundboard
Estimated radiation efficiency

Radiation efficiency of a soundboard having eigen values (frequency and damping) a factor 1.5 higher.
Conclusion

- The first modes of the soundboard look like the ones of an isotropic supported plate
- The pressure radiated by the modes depends strongly of the position of the observer
- The frequencies of the first modes are well below the coincidence frequency of these modes
- There are loops of active intensity
- The acoustic power depends on the location of the excitation point
- The radiation efficiency is about 15-20%