Fiber Orientation Distribution from Non-Negative Sparse Recovery

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- **Fiber-orientations** modelled as rank-1 tensors: \( C = \lambda c \otimes 2n \)

  Dirac delta as: (a) SH-8, (b) SH-12, (c) rank1-16, (d) rank1-24

- **FOD tensor** modelled as the sum of \( r \) (unknown) rank-1 tensors:

\[
F = \sum_{i=1}^{r} \lambda_i c_i \otimes 2n
\]

- Discretize the FOD convolution integral \( S(q) = \int_{S^2} R(q,u)F(u)du \):

\[
\min_{\{w\}} \|Bw - s\|^2 \quad \text{st.} \quad w \geq 0, \quad (s_i = S_i, \ w_i = \lambda_i)
\]

- **Non-Negative Least Squares**: find \( w \) with sparsity & non-negativity.
  (start with \( r_0 = 321 \) and recover \( r \) from sparsity: [Ghosh 2013, 2014])
Fiber Orientation Distribution from Non-Negative Sparse Recovery

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1 Introduction

The Fiber Orientation Distribution (FOD) [3] is a high angular resolution diffusion imaging (HARDI) model for robustly estimating crossing white-matter fiber bundles from q-ball acquisitions. However, its angular resolution depends on the spherical harmonic (SH) / tensor basis order, which implies a large number of acquisitions: 45, 66, 91 for typically used orders such as 8, 10, 12. Further, it is still necessary to compute the fiber orientations from the FOD. In the literature two ways have been adopted for this purpose: maxima detection and tensor decomposition.

To overcome this two step approach (FOD estimation + fiber detection), we have proposed a novel FOD model and estimation method based on non-negative sparse recovery [1, 2]. The method has the following advantages: (i) it naturally estimates non-negative FODs, (ii) it computes both the FOD (tensor) and the fiber-orientations together – making tensor decomposition (which is NP-hard) or maxima detection unnecessary, (iii) it doesn’t require the number of fiber-compartments to be predefined and (iv) it can estimate very high order FOD tensors from a minimal number of acquisitions (20 or 30). We adopt this method for single shell data of this challenge.

2 Methods

In [1, 2], we model a single fiber orientation (or an oriented Dirac delta) function as a rank-1 tensor of order 2n: $C = \lambda c^{\otimes 2n}$ and the FOD tensor as the sum of (unknown) r rank-1 tensors: $F = \sum_{i=1}^{r} \lambda_i c_i^{\otimes 2n}$. Thus, a delta function along c is described by $C_n(u) = \mu_{1j2\ldots} c_{1j2\ldots} u_{1j2\ldots} \cdot \ldots \cdot u_{j2n} = C \cdot (2n) u$, where $u \in S^2$ and $\mu$ is the multiplicity. The FOD is described by $F(u) = F \cdot (2n) u$. This model allows us to choose arbitrarily high tensor orders without increasing the number of unknowns. In this model, the unknown $r$ (the tensor rank), represents the number of fiber compartments and $\lambda_i$ represent the weights of the fiber compartments and cannot be negative. The response function is considered to be a bipolar Watson function $R(q, u, b) = e^{-b|q \cdot u|^2}$ and is estimated in voxels with high fractional anisotropy (FA>0.8) to ensure “single-fiber” configurations.

The FOD estimation is performed by first discretizing the convolution integral $S(q_j) = \int_{S^2} R(q_j, u) \cdot F(u) \, du$ and computing the least squares fitting problem (Eq. 1): $\min_{||w||} ||Bw - s||^2$ st. $w \geq 0$, where $s_j = S(q_j)$, $w_i = \lambda_i$, $B_{ij} = \sum_{l=1}^{L} R(q_j, u_i) (c_i \cdot (2n) u_i) \Delta u_i$, $L$ represents the discretization of the convolution and $\Delta u_i$ becomes $\Delta u_i$. As $w_i = \lambda_i$, cannot cannot be negative and $r$, the length of $w$, is unknown apriori, we set $r = 321$, $\{c_i\} = \{c_1\}$: a 321-uniform discretization of the unit sphere (which implies an FOD with a 321 crossing-fiber configuration) and solve Eq. 1 with both sparsity and positivity constraints. This is accomplished using the non-negative least squares (NNLS) [2]. Finally, we heuristically clean small fiber weights and unnaturally narrow crossings that can occur due to signal noise and re-fit the resulting configuration to estimate the final FOD (tensor & fiber-orientations both).

3 Data & Results

Here, we consider the q-ball acquisition with 20 gradient directions and $b = 2000s/mm^2$. We estimate 24th order FODs and provide snapshots of both the FOD and the fiber-orientations since both are estimated together.

References

