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A computationally effective dynamic hysteresis model taking into account skin effect in magnetic laminations

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2 **Abstract**

3 We propose a simplified dynamic hysteresis model for the prediction of magnetization behavior of
4 electrical steel up to high frequencies, taking into account the skin effect. This model has the advantage
5 of predicting the hysteresis loop and loss behavior versus frequency with the same accuracy provided
6 by the Dynamic Preisach Model with a largely reduced computational burden. It is here compared to
7 experimental results obtained in Fe-Si laminations under sinusoidal flux up to 2 kHz.

8

9 I. INTRODUCTION

10 The problem of high frequency behavior of magnetic laminations is of primary importance in
 11 modern electrical engineering problems [1], but difficult problems arise in the prediction and
 12 assessment of magnetization process and losses, because the skin effect compounds with the nonlinear
 13 hysteretic response of the material. In order to cope with the inhomogeneous profile of the flux density
 14 over the lamination thickness, the magnetic loss is generally calculated by numerically solving the
 15 diffusion equation over the sample cross-section, and using a dynamic hysteresis model for the material
 16 constitutive law [2]. The Dynamic Preisach Model (DPM) is the model of choice, because it is accurate
 17 and solidly established from the physical viewpoint [3][4]. Its application is, however, particularly time
 18 consuming, because calculations must be done for each finite element of the spatial mesh until
 19 convergence is reached. A faster approach, preserving the special virtues of DPM, would therefore be
 20 appropriate. We apply in this paper the DPM to the broadband behavior of nonoriented Fe-Si
 21 laminations through a simplified method, drastically reducing computing time and complexity of the
 22 full method, requiring the computation of the dynamics of each elementary hysteron distributed in the
 23 Preisach plane [5]. We start our discussion from the differential relation found by Bertotti [6] (formula
 24 (9), page 4609, of reference [6]) for the excess magnetic field due to the dynamic behavior of the
 25 magnetization process

$$H_{\text{exc}}(t) \equiv H(t) - H_{\text{stat}}(t) = \text{sign}(\dot{H}_{\text{stat}}) \frac{4}{3} \sqrt{\frac{|\dot{H}|}{k_d}}, \quad (1)$$

26 where $H(t) = H_a(t) - H_{\text{cl}}(t)$ is the difference between the applied field $H_a(t)$ and the counterfield $H_{\text{cl}}(t)$
 27 generated by the macroscopic eddy currents (classical field). $H_{\text{stat}}(t)$ is the field that would provide
 28 under static conditions the same irreversible polarization J_{irr} and $k_d [\text{A}^{-1}\text{s}^{-1}\text{m}]$ is the DPM constant. Eq.
 29 (1) is derived under the simplifying assumptions of triangular $H_a(t)$ and uniform Preisach distribution
 30 function. The extent to which such a restriction can be circumvented and the full DPM approach for

generic exciting conditions and shape of the Preisach density function can be approximated will be discussed in the following. We find first that a numerical analysis based on (1) (Model 1) does not lead to highly accurate dynamic loop shapes, especially at high frequencies. A slight modification of (1) is therefore proposed (Model 2) and implemented in a non-linear magneto-dynamical model for the computation of the field distribution inside Fe-Si laminations, taking into account skin effect. The numerical results are compared with the experiments performed under sinusoidal flux up to 2 kHz.

II. THE SIMPLIFIED MODEL

A. The simplified DPM-Model 2

The Preisach distribution function, including the reversible contribution, was determined in a 0.35 mm thick Fe-(3.2wt%)Si-(0.5wt%)Al lamination, and the dynamic constant $k_d=350 \text{ A}^{-1}\text{s}^{-1}\text{m}$ was identified. The full and the simplified (Model 1) DPM are compared in terms of the excess field. In this model benchmark, a sinusoidal dynamic field $H(t) = H_a(t) - H_{cl}(t)$ (peak value $H_p = 100\text{A}\cdot\text{m}^{-1}$, frequency $f = 200 \text{ Hz}$) has been applied. For the full DPM case, the excess field $H_{\text{exc}}(t) = H(t) - H_{\text{stat}}(t)$ is derived applying the DPM to $H(t)$ in order to get the irreversible polarization $J_{\text{irr}}(t)$, and then using the inverse static Preisach model to compute the static field $H_{\text{stat}}(t)$ from $J_{\text{irr}}(t)$. For the model 1, a numerical solution of (1) directly provides $H_{\text{stat}}(t)$ from the sinusoidal $H(t)$. A comparison between the results of two approaches is illustrated in Fig. 1 (the J_{irr} waveform obtained from the DPM has also been represented). Discrepancies are found between the two predicted $H_{\text{exc}}(t)$ waveforms, particularly around the reversal points of the irreversible magnetization J_{irr} . With the simplified Model 1 the zero of H_{exc} is, according to (1), coincident with the maximum of $H(t)$, whereas from the same picture, it appears that it occurs when J_{irr} is maximum. Consequently, (1) is formulated as

$$H_{\text{exc}}(t) \equiv H(t) - H_{\text{stat}}(t) = \text{sign}(\dot{H}_{\text{stat}}) \frac{4}{3} \sqrt{\frac{|\dot{H}_{\text{stat}}|}{k_d}}, \quad (2)$$

on account of the fact that the sign of $\dot{H}_{\text{stat}}$ is the same as that of $\dot{J}_{\text{irr}}$. The simplified DPM based on

(2) (DPM-Model 2) appears now to provide (Fig. 1) an $H_{\text{exc}}(t)$ waveform in good agreement with the one provided by the full DPM.

Once the static field is known, $J_{\text{irr}}(H_{\text{stat}})$ is computed by means of the Static Preisach Model, while the reversible component $J_{\text{rev}}(H)$ is calculated ignoring any dynamic effect linked to the reversible contribution [1]. This procedure, which permits to obtain the constitutive law of the material $J(H)$, is summed up in Fig. 2. An example of hysteresis loop prediction (in this case with nested minor loop) is illustrated in Fig. 3, confirming the good agreement between the results provided by the full DPM and the simplified DPM-Model 2.

A. Numerical implementation of DPM-Model 2

This model requires solving the non-linear differential equation (2) for H_{stat} knowing the field H . This can be done putting (2) under an equivalent canonical form and adding the periodicity conditions (period $T=1/f$):

$$\begin{cases} \dot{H}_{\text{stat}} = \text{sign}(H(t) - H_{\text{stat}}(t)) \cdot \frac{9k_d}{16} \cdot (H(t) - H_{\text{stat}}(t))^2 \\ H_{\text{stat}}(0) = H_{\text{stat}}(T) \end{cases} \quad (3)$$

and numerically solving it using a Runge-Kutta method. The application of this newly defined dynamical model to the computation of flux distribution inside the steel lamination can dramatically reduce the computational burden, as demonstrated in the next section.

III. MAGNETO DYNAMICAL MODELING AND EXPERIMENTAL

A. Magneto-dynamical model of the lamination

The solution of the diffusion equation over the lamination thickness with a dynamic hysteretic constitutive law requires a special numerical treatment of the non-linearity using the Fixed Point method [3][7]. The method proposed in [3] has been here implemented. The diffusion problem on the lamination thickness is one-dimensional, with the spatial coordinate x ranging over the lamination

74 cross-section $[-e/2; e/2]$ (where $e = 0.35$ mm is the lamination thickness). To solve the diffusion
 75 equation, the non-linearity of the material constitutive law $J(H)$ is contained in a spatio-temporal
 76 function $R(x, t)$, called the residual [3] and the relationship between H and J is written as

$$H(x, t) = v_{\text{FP}} \cdot B(x, t) + R(x, t), \quad (4)$$

77 where v_{FP} is a properly chosen constant, ensuring convergence [1]. Using (4), the diffusion equation
 78 can be written, with imposed mean flux density $B_{\text{MEAN}}(t)$, in terms of vector potential A on a half
 79 lamination $[0; e/2]$:

$$\begin{cases} v_{\text{FP}} \cdot \frac{\partial^2 A}{\partial x^2} - \sigma \frac{\partial A}{\partial t} = \frac{\partial R}{\partial x} \\ A(0, t) = 0 \text{ and } A(e/2, t) = -B_{\text{MEAN}}(t) \cdot e/2 \\ A(x, t) \text{ is } T\text{-periodic} \end{cases} \quad (5)$$

80 The time derivative and the time periodic condition are dealt with using temporal Fourier series [3].
 81 More precisely, the residual function $R(x, t)$ is decomposed into complex Fourier series for what
 82 concerns the time dependence. Equation (5) is then solved for each time harmonic, the time derivative
 83 becoming an algebraic multiplication by the corresponding harmonic pulsation. An inverse Fourier
 84 transformation permits to retrieve the function $A(x, t)$. For each time harmonic, the spatial second order
 85 derivative and the boundary conditions are dealt with using a numerical finite difference scheme (the
 86 half lamination is subdivided into 50 intervals). At the beginning of the iterative procedure, the residual
 87 R is initialized to zero and at each iteration step (index number i), the following process is performed:

- 88 1. Knowing the residual at the previous stage $R^{(i-1)}(x, t)$, the differential equation (5) is solved to
 89 compute the vector potential $A^{(i)}(x, t)$. The magnetic field is obtained as

$$90 \quad H^{(i)}(x, t) = -v_{\text{FP}} \frac{\partial A^{(i)}}{\partial x} + R^{(i-1)}(x, t).$$

- 91 2. A dynamic hysteresis model, providing the dynamic constitutive law $J(H)$, is used to evaluate $J^{(i)}$

from $H^{(i)}$ and the new induction value $B^{(i)}(x,t)=J^{(i)}(x,t)+\mu_0\cdot H^{(i)}(x,t)$ is calculated. In [3], the dynamic hysteresis model is the DPM, and is replaced in this paper by the simplified model proposed in Fig. 2.

3. The new residual is then computed by $R^{(i)}(x,t)=H^{(i)}(x,t)-v_{\text{FP}}\cdot B^{(i)}(x,t)$. The process is repeated until convergence of the residuals is obtained.

B. Results

The dynamic hysteretic constitutive law $J(H)$ of the material, which was in [3] given by the full DPM, has been here replaced by the model proposed in Fig. 2 where the Model 2 of the local excess field is applied. The Preisach distribution function has been identified following the method proposed in [1], in the previous 0.35 mm thick Fe-(3.2wt%)Si-(0.5wt%)Al lamination (conductivity $\sigma = 1.773\cdot 10^6 \text{ S}\cdot\text{m}^{-1}$, $k_d=350 \text{ A}^{-1}\text{s}^{-1}\text{m}$). The magnetic loss and hysteresis loops have been measured under sinusoidal flux at different frequencies and peak inductions up to 2 kHz. The magnetic flux distribution and loss have been computed by applying the dynamical model, using both the full and the simplified DPM (Model 2). Fig. 4 provides an example of computed and experimental loops ($J_p = 0.5 \text{ T}$, frequency $f = 1 \text{ kHz}$). In Fig. 5 the comparison is made for energy loss versus frequency behavior. It shows the good predicting capability of the simplified dynamic Model 2, in spite of a computing time reduced by a factor around 60 with respect to the full DPM. Fig. 5 also shows the predicted loss behavior if the skin effect is ignored. In this case the induction is assumed uniform across the sample thickness and equal to the mean flux density $B_{\text{MEAN}}(t)$. The classical loss is thus calculated according to

$$W_{\text{class}} = \frac{\sigma e^2}{12} \int_0^T \left(\frac{dB_{\text{MEAN}}}{dt} \right)^2 dt \quad (6)$$

It is found that ignoring the skin effect brings about a large overestimation of the loss above about $f = 400 \text{ Hz}$, for the considered $B_{\text{MEAN}}(t)$ peak value of 0.5 T considered in Fig. 5. An example of calculated induction profile $B(x, t_0)$ across the lamination thickness at a given instant of time t_0 is shown in Fig. 6.

114 It refers to $f = 1$ kHz and a time t_0 corresponding to the average flux density $B_{\text{MEAN}}(t_0) = 0$. Again, the
115 full and simplified DPMs predict very close results.

116 IV. CONCLUSIONS

117 In this work we have discussed a novel dynamic model for energy losses and hysteresis loops in soft
118 magnetic laminations based on simplified treatment of the Dynamic Preisach Model (DPM). It is
119 applied on experiments performed up to the kHz range in Fe-Si sheets, in the presence of substantial
120 skin effect, showing good agreement both with the experimental results and the prediction made using
121 the full machinery of the DPM. A remarkable advantage in computing time, which is reduced by a
122 factor around 60, is obtained substituting the full DPM with the present model.

123

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- 138

139 **Figure captions**

140 Fig. 1- Excess fields provided by the full DPM, Model 1, and Model 2, for a sinusoidal dynamic
141 field $H(t) = H_a(t) - H_{cl}(t)$ (frequency $f = 200$ Hz, field peak value $H_p = 100\text{A}\cdot\text{m}^{-1}$) and corresponding
142 irreversible polarization J_{irr} provided by the DPM in a 0.35 mm thick Fe-(3.2wt%)Si-(0.5wt%)Al
143 lamination (dynamic constant $k_d=350\text{ A}^{-1}\text{s}^{-1}\text{m}$).

144 Fig. 2 - Block diagram of the simplified constitutive law $J(H)$ of the material, using the Model 2 as a
145 link between the field H and the static field H_{stat} (computation of the local excess field).

146 Fig. 3 - Example of the reconstructed $J(H)$ hysteresis loop with nested minor loops ($f = 200$ Hz, $H_p =$
147 $150\text{A}\cdot\text{m}^{-1}$)

148 Fig. 4 – Experimental and theoretical hysteresis loops at $f = 200$ Hz in the 0.35 mm thick Fe-
149 (3.2wt%)Si-(0.5wt%)Al lamination for sinusoidal induction ($B_p=0.5$ T). The measurements have been
150 performed on Epstein strips. Closed results are obtained by the full and the simplified DPM.

151 Fig. 5 – Same as Fig. 4 for the energy loss versus magnetizing frequency (DC – 2 kHz).

152 Fig. 6: Instantaneous induction profile $B(x, t_o)$ across the lamination thickness as obtained by the full
153 and the simplified (Model 2) DPM ($f = 1$ kHz, $-0.175\text{ mm} \leq x \leq 0.175\text{ mm}$). The time t_o considered in
154 this figure is the one for which the mean flux density $B_{MEAN}(t_o) = 0$.

155

Fig. 1

Fig. 2

160
161
162

Fig. 3

Fig. 4

Fig. 5

Fig. 6