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Traffic-Aware Training and Scheduling for the 2-user MISO Broadcast Channel

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***Abstract*—In this paper we study the stability region of the 2-user MISO broadcast channel where the transmitter employs Zero Forcing precoding when both users are scheduled, taking into account the time overheads needed for uplink channel training. We show that, with proper signalling design, combining a decentralized policy with the baseline centralized one for user selection can increase the stability region of the system.**

I. Introduction

The use of multiple antennas and scheduling in the transmitters have been recognized as powerful means to increase the rate regions and performance of wireless systems. However, to fully achieve the potential of these techniques, channel state information is crucial. This can be done only by feedback or training from the receivers, thus consuming resources (time/bandwidth) . In current standards, like LTE, subset of users can feed back their channel states at each time and they are selected/scheduled by the base station i.e. in a centralized manner [1]. This can be done based on the statistics of the channels of the users. Unfortunately, using such centralized schemes, some scheduled users may have poor current channel states and some users with good current channel states may not be scheduled (i.e. may not feed back), which reduces the system performance. On the other hand, each user knows its own current channel state, and therefore decentralized feedback policies where the users decide based on their current channel states may improve the system performance. This must be done properly as the decentralized policies require additional signalling information that may decrease drastically the improvement. In this paper, we indeed show that, by designing the signalling properly, combining ideas (and enhancing them) from decentralized scheduling policies can improve the stability region of a downlink system with a multiple antenna transmitter using Zero Forcing precoding. Our system works in Time Division Duplexing (TDD) mode and takes into account the timing needed for users to train the channel.

It is worth noting that recent works [2], [3] have shown that, in a network with simple physical layer (e.g. on-off channel, finite discrete channel states,...), decentralized algorithms like the recently proposed Fast CSMA [4] can achieve good performance. In addition, results in [5], [6], show that up-to-date channel state information, which is known at the receivers, is more crucial than accurate queue length information, at least as far as stability is concerned. The scenario considered in this paper is more complicated as compared to the recent work on decentralized scheduling. In fact, in scheduling problems [7] (e.g. OFDMA or TDMA), a user can directly estimate its bit rate using the current channel state. In multi-user MIMO systems, the bit rate of each user depends on the channel states of all users and the user cannot simply estimate its bit rate using its current channel state. This highly complicates the analysis.

Limited feedback in multi-user MIMO systems has been the subject of a intense research in the last years, see for example [8] and [9] and references therein. Most works however focus on sum-rate maximizations, ignoring the aspect of having incoming traffic destined for the receivers. The most relevant work is [10], where the authors study the impact of quantized channel state feedback on the stability regions experienced by the users of a MISO system using Zero Forcing beamforming. However, they consider a centralized scheme where the transmitter selects the users to be scheduled based only on the queue lengths. On the contrary, in our paper we examine the stability region of three approaches in a system where channel estimation is done in TDD mode (i.e. via uplink training from the receivers) and where every user that trains the channel will get scheduled. The motivation is that, since in TDD the training overhead does not depend on the number of antennas and channel reciprocity can be exploited, it is most promising approach when multiple antennas are used [1].

All three policies take into account the training and signaling overhead. The first approach is centralized, in the sense that the transmitter decides which user will be scheduled (i.e. will train) at every slot. The second approach, which we term as "decentralized", is to let the users decide which of them should actually feed back via some contention/coordination scheme. The main idea behind this approach is that every user can know its channel state, therefore a user with a very bad channel state will choose not to feed back (contrary to what can happen in the centralized approach). More specifically, in this case, the transmitter specifies the number of users to be scheduled and lets the users decide in a decentralized manner who will be the ones that will actually get scheduled in the slot. Combined with some (infrequent) signalling regarding the users queue lengths from the base station, we will see in

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[1]Note also that, even if feedback is done in FDD mode the transmitter must wait to collect the feedback from the receivers before precoding [8]

the following that properly combining the decentralized and centralized approaches leads to a bigger achievable stability region than using the centralized approach alone.

Note: Due to space constraints, the proofs of intermediate results are not presented here but can be found in [11].

II. System Model

A. Physical Layer

In this work we consider a single transmitter serving 2 single-antenna receivers with $N \geq 2$ antennas and total transmit power available P. Time is slotted. Channels are i.i.d. in time and users, following Rayleigh block fading, that is the channel of user k can be written as an N-dimensional complex vector $\mathbf{h}_k(t) \sim \mathcal{CN}(0, \bar{g}\mathbf{I}_N)$ where $\mathcal{CN}$ denotes the complex normal distribution. The noise power at each receiver is assumed σ^2. In this setting, in a slot, the transmitter can serve either one or both receivers. We will focus on the case of Zero Forcing precoding if both receivers are served. Thus, if only receiver k is scheduled then the signal for this user will be precoded with the vector $\mathbf{w}_k(t) = \sqrt{P}\frac{\mathbf{h}_k(t)}{||\mathbf{h}_k(t)||}$ and with $\mathbf{w}_k(t) = \sqrt{\frac{P}{2}}\frac{\left(\mathbf{I}_N - ||\mathbf{h}_j(t)||^{-2}\mathbf{h}_j(t)\mathbf{h}_j^H(t)\right)\mathbf{h}_k(t)}{||\left(\mathbf{I}_N - ||\mathbf{h}_j(t)||^{-2}\mathbf{h}_j(t)\mathbf{h}_j^H(t)\right)\mathbf{h}_k(t)||}$ if both receivers are served. Power is split equally among users for tractability purposes; in general joint scheduling and power allocation for multiuser MIMO is a very challenging problem even with perfect CSI acquisition at no cost. A scheduled receiver can be served by a rate of R bits per channel use if the corresponding SNR exceeds the threshold for correct decoding $\hat{S}$. The transmitter needs the realizations of the channel states for the scheduled receiver(s) in order to calculate the precoder. Here we assume that this is done via uplink training with sequence length of β channel uses per user. In addition, we assume that there is no error in the channel estimation; this can be argued if the power of the training sequence is high enough. For this reason, a downlink pilot β_p is sent in the beginning of each slot to allow receivers estimate their channel magnitude.

B. Queuing model and impact of training

Each of the receivers has an incoming traffic process $a_k(t)$, which is an integer-valued process, measured in bits, i.i.d. in time and independent across users with $\mathbb{E}\{a_k(t)\} = \lambda_k$ and $a_k(t) < A_{max}$ for some finite constant A_{max}, which is assumed known to the transmitter and receivers. Data for receiver k is stored in a respective buffer until transmission and let $q_k(t)$ denote its size in bits at the beginning of slot t.

Denote now $z_k(t)$ as the schedule in timeslot t, that is $z_k(t) = 1$ if user k is scheduled for this timeslot (i.e. if user k has actually reported its channel to the base station). Let $\tau(t)$ the number of channel uses used for training and signalling in the slot t. If the rate supported to user k at timeslot t is $r_k(\mathbf{W}(t), \mathbf{H}(t))$ bits per channel use, we have

$$q_k(t+1) = [q_k(t) - (T_s - \tau(t))r_k(\mathbf{W}(t), \mathbf{H}(t))z_k(t)]^+ + a_k(t), t \geq 0. \quad (1)$$

What we are interested in is the aspect of stability of the system. Formally we have:

Definition 1. *A queueing system with K queues is called strongly stable if:* $\lim_{t\to+\infty} \sup \frac{1}{t}\sum_{t'=0}^{t-1}\mathbb{E}\{q_k(t')\} < +\infty$.

If the arrivals and service rate processes are such that the Markov chain is irreducible and aperiodic with a single communicating class, strong stability is equivalent to positive recurrence of the chain [12]. In this work we are interested in this form of stability, therefore "stable" will imply "strongly stable" in the rest of the paper.

The arrival processes involved in the above definition have fixed mean arrival rates, which leads to the concept of a stability region.

Definition 2. *The stability region Λ of the system is the set of mean arrival rate vectors $\lambda = [\lambda_1, ..., \lambda_K]^T$ for which the system is strongly stable.*

For the rest of the paper, when describing stability regions we will mean that the system is stable in the *interior* of the calculated region (thus behaviour on the boundary will not be examined - usually for the boundary points the system is stable in at least a weaker sense, i.e. mean rate stable [12]). Informally, a system is stable when the average service rate of each user is bigger than the corresponding mean arrival rate. Equation (1), thus, implies that training affects essentially the service rate, and thus the stability region, in two ways: First, more time devoted to training leads to lower service rate for the users actually scheduled in the timeslot. On the other hand, if more users participate in the training, more users can get scheduled in a timeslot, thus overall a user can get higher mean service rate. The focus of the paper is, thus, this tradeoff and how to efficiently design user selection strategies to achieve large stability regions. In our model scheduling user(s) is equivalent to deciding which users will participate in the uplink training in every timeslot.

III. Proposed Policies

In this Section we detail the scheduling policies to be considered. Figures 1,2,3 illustrate their operation. These policies are generally based on a modification of the celebrated max weight algorithm [13], by trying to minimize the drift of a given quadratic Lyapunov function. Since the channel realizations are unknown to the transmitter, decisions have to be taken based on the channel distributions. If F receivers are served, the following result will be useful to calculate the average rates:

Proposition 1. *It holds [11]:*

$$\bar{p}(1) = \mathbb{P}\{SNR_k > \hat{S}|F=1\} = 1 - \frac{\gamma\left(\frac{\hat{S}}{\bar{g}}; N\right)}{\Gamma(N)}$$

$$\bar{p}(2) = \mathbb{P}\{SNR_k > \hat{S}|F=2\} = 1 - \frac{\gamma\left(\frac{2\sigma^2}{\bar{g}P}\hat{S}; N-1\right)}{\Gamma(N-1)}.$$

On the above, $\Gamma(N)$ is the Gamma function, $\gamma(x; N)$ the lower incomplete Gamma function with parameter N.

A. Centralized policy

In this policy, in every slot t the transmitter selects either one or both the receivers to be scheduled. In the latter case, there is an overhead of $2\beta_c$ channel uses to broadcast the IDs of the two users and in the former, of $\beta_c + 1$ to broadcast the ID of the scheduled user and a signal that the control period is over.

The expected service that a receiver gets if both users are scheduled or if this receiver only is scheduled at timeslot t is given by

$$\bar{\mu}_c(2) = (T_s - (\beta_p + 2\beta_c + 2\beta))\bar{p}(2)R, \\ \bar{\mu}_c(1) = (T_s - (1 + \beta_p + \beta_c + \beta))\bar{p}(1)R \quad (2)$$

,respectively. The set to be scheduled at stot t is then chosen at the beginning of this slot by the rule that follows:

- $\mathcal{F}_c(t) = \{1,2\}$, if $(q_1(t) + q_2(t))\bar{\mu}_c(2) \geq \max\{q_1(t), q_2(t)\}\bar{\mu}_c(1)$
- $\mathcal{F}_c(t) = \{\arg\max\{q_1(t), q_2(t)\}\}$, otherwise .

This policy is actually the one achieving the largest stability region of all centralized policies: this can be proved by Lyapunov drift analysis and is not presented here.

B. Decentralized Policy

The main observation here is that the user knows its current channel but not its queue length. Our idea is to let the transmitter use slot $mT, m = 0, 1, ...$ to broadcast the queue lengths to the receivers. Here $T > 1$ is an (arbitrarily large) finite integer. For all slot between times $mT + 1$ and $(m + 1)T - 1$, the users have an outdated information of their queues (i.e. only $\mathbf{q}(mT)$). Since the control rate is limited in practice, the broadcasted $\mathbf{q}(mT)$ should be quantized. We propose the following quantization scheme: Define $Q = \max\{T_s R(T-1), TA_{max}\}$, which is the biggest difference that can possibly happen to a queue between slots $(m-1)T$ and mT. Then, at slot mT, the length of each queue belongs to one of the intervals $[lQ, (l+1)Q), l = 0, 1, ...$ For each queue, the broadcasted message contains first information about if it has stayed in the same interval as in the previous broadcast or moved to any of the adjacent ones; the additional part comprises of the place of the quantized queue length inside the interval assuming uniform quantization. The quantized queue length of user k in the beginning of slot mT will be denoted as $\hat{q}_k(m)$. The transmitter then decides the *number* $F(m)$ of users to get scheduled in the next $T-1$ slots. Depending on $F(m)$, we have the following possibilities:

1)Contention procedure: If $F(m) = 1$, in each of these slot the receivers are given a contention period of τ_c channel uses to decide which one is to be scheduled based on the (quantized and outdated) queue length information they have and the realization of their channels. This can be done using a contention scheme, assuming contention in continuous time e.g. like [4], where each user waits until time $\frac{\tau_c}{\hat{q}_k(m) r_k(t)}$: if both have the same timer, e.g. the user with the smallest ID is scheduled. Another alternative, that can be used thanks to our model, is to divide the contention period into minislots (TDMA manner) where each receiver sends a signal in its corresponding minislot if its SNR is above the threshold $\hat{S}$. If both receivers send a signal, in their corresponding minislots, then the receiver with the largest broadcasted queue length gets scheduled for training (this analysis/comparison can be done independently by each receiver since the queue lengths of all receivers are broadcasted). Otherwise, if only one user sends a signal in a minislot, then this user will be scheduled for training. Then, the user to be scheduled sends its ID to the base station, taking β_c channel uses, and trains. Using the above "decentralized" procedure, the user that will eventually get served in the slot will be the one with the maximum product of quantized queue length at mT times achievable rate. Due to our model here, denoting $SNR_k^{(1)}(t) = \frac{P||\mathbf{h}_k(t)||^2}{\sigma^2}$, the user $k^*(t)$ to be scheduled will be

- If $\forall k = 1, 2$ holds $SNR_k^{(1)}(t) > \hat{S}$, then $k^*(t) = \arg\max[\hat{q}_1(mT), \hat{q}_2(mT)]$ and user 1 in case of a tie.
- The user for which $SNR_k^{(1)}(t) > \hat{S}$ otherwise

The scheduled receiver will always be given rate of R bits per channel use, except in the case where no one has sufficiently high SNR, in which no receiver can be scheduled anyway. Defining the permutation $k(1), k(2)$, where $\hat{q}_{k(1)}(mT) \geq \hat{q}_{k(2)}(mT)$, the average service rates of these users under $F = 1$ for the next $T - 1$ slots are

$$\bar{\mu}_{k(1)}^{d,(1)}(t) = (T_s - (\beta_p + \tau_c + \beta))\bar{p}(1)R := \bar{\mu}_d(1) \\ \bar{\mu}_{k(2)}^{d,(1)}(t) = (T_s - (\beta_p + \tau_c + \beta))\bar{p}(1)(1 - \bar{p}(1))R. \quad (3)$$

2) F(m)=2: Both users train just after the coordination period. The average rate per slot for each user in this case will be

$$\bar{\mu}_d(2) = (T_s - (\beta_p + \tau_c + 2\beta))\bar{p}(2)R \quad (4)$$

Based on the above, the transmitter decides at $t = mT$ the number of users to get scheduled for the next $T - 1$ slots by:

- $F(m) = 2$, if: $(\hat{q}_{k(1)}(m) + \hat{q}_{k(2)}(m))\bar{\mu}_{k(1)}^{d,(1)}(t) \geq (\hat{q}_{k(1)}(m) + \hat{q}_{k(2)}(t)(1 - \bar{p}(1))\bar{\mu}_{k(1)}^{d,(1)}(t)$
- $F = 1$ otherwise: In this case, the contention procedure is followed. .

C. Mixed Policy

The mixed policy is a combination of both the ideas behind the centralized and decentralized policies. As in the decentralized policy, slot mT is used to broadcast signalling regarding the quantized queue lengths and the action that specifies how scheduling will be done in the next $T - 1$ slots.

The transmitter can choose in the signalling slot one of the following actions: $\mathcal{F} = \{1\}$, $\mathcal{F} = \{2\}$, $\mathcal{F} = \{1, 2\}$ and $F = 1$. In the first three actions the receiver(s) specified train directly in the uplink for the $T - 1$ slots after the signalling slot, without any control or contention/uplink of the IDs phase. In the case of $F = 1$ one receiver is scheduled according to the contention procedure explained in III-B. In detail, for the rates at a slot t corresponding to each of the base station actions and assuming $\hat{q}_{k(1)}(m) \geq \hat{q}_{k(2)}(m)$ we have for $t \in \{mT + 1, ..., mT + T - 1\}$:

$$\mathbb{E}\{\mu_1(t)\} = (T_s - (\beta_p + \beta))\bar{p}(1)R, \mu_2(t) = 0, \text{if} \mathcal{F} = \{1\}$$
$$\mathbb{E}\{\mu_1(t)\} = 0, \mu_2(t) = (T_s - (\beta_p + \beta))\bar{p}(1)R, \text{if} \mathcal{F} = \{2\}$$
$$\mathbb{E}\{\mu_1(t)\} = \mathbb{E}\{\mu_2(t)\} = (T_s - (\beta_p + 2\beta))\bar{p}(1)R, \mathcal{F} = \{1, 2\}$$
$$\mathbb{E}\{\mu_{k(1)}(t)\} = \bar{\mu}_d(1), \mathbb{E}\{\mu_{k(2)}(t)\} = (1 - \bar{p}(1))\bar{\mu}_d(1), F = 1$$

We define further

$$\bar{\mu}_m(\{k\}) = (T_s - (\beta_p + \beta))\bar{p}(1)R, \\ \bar{\mu}_m(\{1, 2\}) = (T_s - (\beta_p + 2\beta))\bar{p}(2)R. \quad (5)$$

The mixed policy selects, at every slot mT, the following action to maximize $\sum_{k=1}^{2} \hat{q}_k(m)\mathbb{E}\{\mu_k(t)\}$:

- $\mathcal{F} = \{k(1)\}$, if: $\hat{q}_k(1)(mT)\bar{\mu}_m(\{k\}) > \max\left[(\hat{q}_1(mT) + \hat{q}_2(mT))\bar{\mu}_m(\{1, 2\}), (\hat{q}_1(mT) + (1 - \bar{p}(1))\hat{q}_2(mT))\bar{\mu}_d(1)\right]$
- $\mathcal{F} = \{1, 2\}$, if: $(\hat{q}_1(mT) + \hat{q}_2(mT))\bar{\mu}_m(\{1, 2\}) \leq \max\left[\ \hat{q}_k(1)(mT)\bar{\mu}_m(\{k\}), (\hat{q}_1(mT) + (1 - \bar{p}(1))\hat{q}_2(mT))\bar{\mu}_d(1)\right]$

- $F=1$ if: $(\hat{q}_1(mT)+(1-\bar{p}(1))\hat{q}_2(mT))\bar{\mu}_d(1) > \max\big[\ \hat{q}_k(1)(mT)\bar{\mu}_m(\{k\}), (\hat{q}_1(mT)+\hat{q}_2(mT))\bar{\mu}_m(\{1,2\})\big]$.

IV. Stability Results

This Section contains the main result of the paper, namely the stability regions of the policies considered. Theorem 2 gives the analytical characterization, where $\mathcal{CH}$ denotes the convex hull. The regions are shown graphically in Fig. 4.

Theorem 2. *The stability regions for the centralized, decentralized and mixed policies are*

a) $\Lambda_c = \mathcal{CH}\big\{(0,\bar{\mu}_c(1)),(\bar{\mu}_c(2),\bar{\mu}_c(2)),(\bar{\mu}_c(1),0)\big\}$,

b) $\Lambda_d = \left(1-\frac{1}{T}\right)\mathcal{CH}\big\{(0,\bar{\mu}_d(1)),(\bar{\mu}_d(1)(1-\bar{p}(1)),$
$\bar{\mu}_d(1)),(\bar{\mu}_d(2),\bar{\mu}_d(2)),(\bar{\mu}_d(1),\bar{\mu}_d(1)(1-\bar{p}(1))),(\bar{\mu}_d(1),0)\big\}$,

c) $\Lambda_m = \left(1-\frac{1}{T}\right)\mathcal{CH}\big\{(0,\bar{\mu}_m(\{k\})),(\bar{\mu}_d(1),$
$(1-\bar{p}(1))\bar{\mu}_d(1)),(\bar{\mu}_m(\{1,2\}),\bar{\mu}_m(\{1,2\})),$
$((1-\bar{p}(1))\bar{\mu}_d(1),\bar{\mu}_d(1)),(\bar{\mu}_m(\{k\}),0)\big\}$,

respectively.

The rest of this Section is devoted to the proof of the stability region for the decentralized policy. The proofs for the other regions can be done similarly and are omitted due to lack of space. Define $\tilde{\mathbf{q}}(m)=\mathbf{q}(mt)$. This defines a system that, for every step m takes the values of the original system "sampled" every T timeslots $\mathbf{q}(mT)$. It can be proven that the stability regions of both are the same [11], therefore we will restrict our attention to $\mathbf{q}(m)$ for the rest of the analysis.

The proof consists in four parts. For the first two parts we compute the stability region for policies that select all the time $F=2$ and $F=1$. In the third, we prove that the decentralized policy achieves the convex combination of the two and finally we prove the converse.

Step 1: We first find the stability region if $F=2$ for every signalling slot mT. In this case, the mean rate a user gets for each data slot is $\bar{\mu}_d(2)$. Thus, for the system $\tilde{\mathbf{q}}(m)$, the mean arrival rate for user k is $T\lambda_k$ and the mean service rate is $(T-1)\bar{\mu}_d(2)$, thus the stability region here is
$\lambda_k < \frac{T-1}{T}\bar{\mu}_d(2), \forall k=1,2$.

Step 2: We then find the stability region if $F=1$ in every signalling slot. We define a hypothetical policy where the transmitter knows from the start of a data slot the achievable rates for both users and, based on this knowledge, chooses one of the two users to train and get scheduled, probably at random (while keeping the same time for data transmission in the slot as the corresponding in the decentralized policy). More concretely, if only one user can support the rate R then this user should be scheduled, otherwise if both support the rate R then user 1 gets scheduled with some probability π_1 and user 2 with a probability π_2. In this case, taking into account the model for the system $\tilde{\mathbf{q}}(m)$ the mean arrival rates λ_1,λ_2 that can be supported by the system are the one for which there exist probabilities π_1,π_2 such that (the quantities in the right hand side are the mean rates given to each user):

$$\begin{aligned} T\lambda_k &< (T-1)\left((1-\bar{p}(1))\bar{\mu}_d(1)+\pi_k\bar{p}(1)\bar{\mu}_d(1)\right) \\ &:= (T-1)\hat{\mu}_{d,k}. \end{aligned} \tag{6}$$

Since $0\le\pi_1+\pi_2\le 1$, this is the algebraic representation of the convex hull of the points $(0,\frac{T-1}{T}\bar{\mu}_d(1))$, $(\frac{T-1}{T}(1-\bar{p}(1))\bar{\mu}_d(1),\frac{T-1}{T}\bar{\mu}_d(1))$, $(\frac{T-1}{T}\bar{\mu}_d(1),\frac{T-1}{T}(1-\bar{p}(1))\bar{\mu}_d(1))$, $(\frac{T-1}{T}\bar{\mu}_d(1),0)$. Let us now prove that their region is indeed achievable by the decentralized policy. For any vector λ inside this region, denoting $\tilde{\mu}_k^d(m)$ be the random variable representing the service receiver k gets at a slot of $\{mT+1,...,mT+T-1\}$, we can show, after some calculations [11], that the drift of the quadratic Lyapunov function $V(\mathbf{x})=x_1^2+x_2^2$ is, for a constant $\tilde{B}$: $\Delta V(\tilde{\mathbf{q}}(m)) \le \tilde{B} +$
$T\sum_{k=1}^2\tilde{q}_k(m)\lambda_k-(T-1)\sum_{k=1}^2\tilde{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^d(m)|\tilde{\mathbf{q}}(m)\right\}$.
Recall that $\hat{\mathbf{q}}(m)$ is vector containing the quantized versions of the queue lengths at the beginning of the signalling slot, therefore $|\hat{q}_k(m)-\tilde{q}_k(m)|\le Q$. Defining
$\tilde{C} = \tilde{B}+TQ\sum_{k=1}^2\lambda_k+(T-1)KQR$ we then get: $\Delta V(\tilde{\mathbf{q}}(m)) \le$
$\tilde{C}+\sum_{k=1}^2\hat{q}_k(m)\left(T\lambda_k-(T-1)\mathbb{E}\left\{\tilde{\mu}_k^d(m)|\tilde{\mathbf{q}}(m)\right\}\right)$
$\le \tilde{C}+\sum_{k=1}^2\hat{q}_k(m)\left(T\lambda_k-(T-1)\hat{\mu}_{d,k}\right) \le$
$\tilde{C}-\epsilon\sum_{k=1}^2\hat{q}_{d,k}(m)$ for some $\epsilon>0$. The second inequality follows because in the decentralized policy the user with the maximum $\hat{q}_k(m)\mathbb{E}\left\{\tilde{\mu}_k^d(m)|\tilde{\mathbf{q}}(m)\right\}$ is eventually selected. The last inequality follows from (6). The drift is negative for $\sum_{k=1}^2\hat{q}_k(m) > \tilde{C}/\epsilon \implies \sum_{k=1}^2\hat{q}_k(m) > 2Q+\tilde{C}/\epsilon$, thus, from the Foster-Lyapunov criterion, the system under the decentralized policy achieves indeed the stability region given by (6).

Step 3: Here we prove that Λ_d is achievable by the decentralized policy. Consider a randomized policy between $F=1$ and $F=2$ with probabilities $\pi(F=1)$ and $\pi(F=2)$ (independent on anything), respectively and the randomized hypothetical policy for the case of $F=1$ given in the above paragraph. The mean arrival rates supported under this policy should then be such that there exist these probabilities while satisfying the conditions

$$\begin{aligned} T\lambda_k &< (T-1)\big(\pi(F=1)\big((1-\bar{p}(1))\bar{\mu}_d(1) \\ &+\pi_k\bar{p}(1)\bar{\mu}_d(1)\big)+\pi(F=2)\bar{\mu}_d(2)\big) := (T-1)\hat{\mu}'_{d,k}. \end{aligned} \tag{7}$$

Since $0\le\pi_1+\pi_2\le 1$ and in addition it holds that $0\le\pi(F=1)+\pi(F=2)\le 1$, the region defined by the above equations is the convex hull of the two regions for $F=1$ and $F=2$, thus Λ_d. Under the decentralized policy, using the same calculations as above, the drift of the quadratic Lyapunov function becomes $\Delta V(\tilde{\mathbf{q}}(m)) \le \tilde{C}+T\sum_{k=1}^2\hat{q}_k(m)\lambda_k-(T-1)\sum_{k=1}^2\hat{q}_k(m)\mathbb{E}\{\tilde{\mu}_k^d(m)|\tilde{\mathbf{q}}(m)\} \le \tilde{C}+\sum_{k=1}^2\hat{q}_k(m)(T\lambda_k-(T-1)\hat{\mu}'_{d,k}) \le \tilde{C}-\epsilon\sum_{k=1}^2\hat{q}_k(m)$ for some $\epsilon>0$, where the second inequality follows from the fact that by definition of the policy the quantity $\sum_{k=1}^2\hat{q}_k(m)\mathbb{E}\{\mu_k^\pi(m)\}$ is maximized and the third from (7). By the same reasoning as above, the decentralized policy stabilizes the system, i.e. achieves Λ_d.

Step 4: Now we prove the converse, that is any mean arrival rate vector λ for which the system under the decentralized policy is stable lies in the interior of the set Λ_d. To do that assume any λ such that the system under the decentralized policy is stable. It holds that the system evolves an aperiodic Markov chain with countable state space $(\mathbb{Z}_+^2)$ and a single communicating class, thus strong stability implies ergodicity of the chain, therefore existence of an invariant distribution $\pi(\mathbf{q})$ [12]. The mean service rate receiver 1 gets is $\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}\sum_{t=mT+1}^{mT+T-1}\mu_1(t)=$

$(T-1)\big(\bar{\mu}_d(1)\phi_{1,1}+(1-\bar{p}(1))\bar{\mu}_d(1)\phi_{1,2}+\bar{\mu}_d(2)\phi_{1,2}\big)$ where $\phi_{1,1}=\sum_{\mathbf{q}\in\mathbb{Z}_+^2:F(\mathbf{q})=1,\hat{q}_1\geq\hat{q}_2}\pi(\mathbf{q})$, $\phi_{1,2}=\sum_{\mathbf{q}\in\mathbb{Z}_+^2:F(\mathbf{q})=1,\hat{q}_1<\hat{q}_2}\pi(\mathbf{q})$ and $\phi_{1,3}=\sum_{\mathbf{q}\in\mathbb{Z}_+^2:F(\mathbf{q})=2}\pi(\mathbf{q})$ and similar for receiver 2. This means that the vector of mean service rates (per slot) is indeed written as a convex combination of the corner points of Λ_d. By assumption the system is stable therefore $T\lambda_k<\lim_{M\to\infty}\frac{1}{M}\sum_{m=0}^{M-1}\sum_{t=mT+1}^{mT+T-1}\mu_k(t)$ for both users. Combining the above,we get that $\lambda\in\Lambda_d$.

V. Comparison and Discussion

In the centralized policy the scheduling is based only on the queue lengths on the beginning of each slot. This has the benefit of knowing the "priority" a user has to get scheduled in real time. On the other hand, letting the users decide according to their instantaneous channel states leads to scheduling eventually a user with good channel condition (in the case where the base station selects $F=1$). The idea behind the mixed policy is to combine the strong points of the decentralized and centralized policies. In fact, for a suitable choice of T the mixed policy can lead to a greater stability region than the centralized:

Proposition 3. *A sufficient condition for the mixed policy to achieve a bigger stability region than the centralized policy is*

$$T>\max\left[\frac{T_c-\beta_p-\beta}{1+\beta_c},\frac{T_c-\beta_p-2\beta}{2\beta_c}\right].$$

In this case, $\Lambda_m\supseteq\rho(T)\Lambda_c$ *with* $\rho(T)=\frac{T}{T-1}\min\left[\frac{T_c-\beta_p-\beta}{T_c-(\beta_p+\beta_c+1+\beta)}\frac{T_c-\beta_p-2\beta}{T_c-(\beta_p+2\beta_c+2\beta)}\right]$.

The idea of the proof is to show expansion on the directions of the axes and $\lambda_1=\lambda_2$; expansion in all the other directions then follows from the shapes of the policies. This proof uses only points achieved by the periodic centralized policy. That is, the increase comes from the fact that a smaller overhead for training and signalling in the data slots is needed and the necessary overhead for scheduling is in the slots mT instead. The use of decentralized scheme in the mixed policy helps enlarge the stability region above the lines connecting the point $((1-1/T)\bar{\mu}_m(\{1,2\}),(1-1/T)\bar{\mu}_m(\{1,2\}))$ with the points on the axes (refer to Fig. 4) thus yielding more gains with respect to the centralized region for traffic demands in these directions. As $T\to\infty$ this increase is bounded and the bound depends only on the parameters of the system; this limit is the highest stability region the mixed scheme can achieve. Finally, increasing T leads to bigger stability region, but it may also lead to bigger delays and slower convergence of the system to its stationary behaviour.

VI. Conclusions

In this paper we have demonstrated that a feedback/training policy that combines decentralized schemes for user selection along with the traditionally applied centralized ones can achieve greater stability region in the case of a MISO broadcast system. This suggests that, in future systems, decentralized methods should be considered for feedback and/or user scheduling along with the traditional centralized ones.

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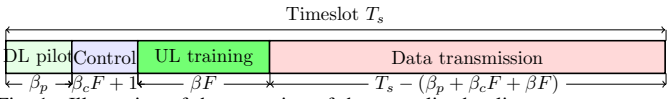


Fig. 1. Illustration of the operation of the centralized policy

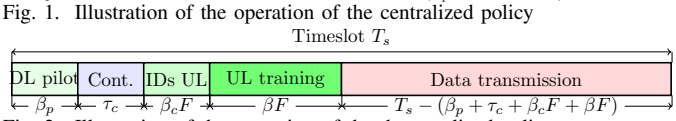


Fig. 2. Illustration of the operation of the decentralized policy

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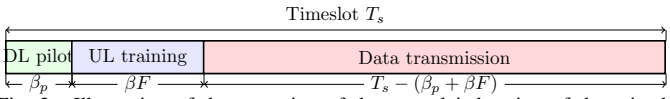


Fig. 3. Illustration of the operation of the centralzied action of the mixed policy

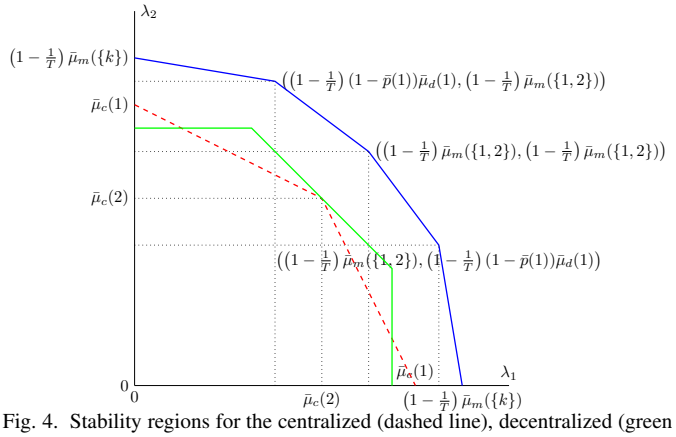


Fig. 4. Stability regions for the centralized (dashed line), decentralized (green continuous line) and mixed (blue continuous line) policies