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1 **Calculation of river sediment fluxes from uncertain and infrequent measurements**

2

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13

14 Keywords: River flow, sediment transport, uncertainty analysis, rating curves.

15 **Abstract**

16

17 This paper addresses feasibility issues in the calculation of fluxes of suspended particulate matter  
18 (SPM) from degraded-quality data for flow discharge ( $Q$ ) and sediment concentration ( $C$ ) under the  
19 additional constraints of infrequent and irregular sediment concentration samplings. A crucial setting  
20 of the scope involves establishing the number of data required to counterbalance limitations in the  
21 measurement accuracy and frequency of data collection. This study also compares the merits and  
22 drawbacks of the classical rating curve ( $C=aQ^b$ ) with those of an improved rating curve approach  
23 (IRCA:  $C=aQ^b+a_1\delta S$ ) in which the correction term is an indicator of the variations in sediment storage,  
24 thus relating it to flow dynamics. This alternative formulation remedies the known systematic  
25 underestimations in the classical rating curve and correctly resists the degradation in data quality and  
26 availability, as shown in a series of problematic though realistic cases. For example, monthly  
27 concentration samplings (in average) with a random relative error in the [-30%, +30%] range  
28 combined with daily discharge records with a systematic relative error in the [-5%, +5%] range still  
29 yield SPM fluxes within factors of 0.60-1.65 of the real value, provided that 15 years of data are  
30 available. A shorter 5-day time interval (on average) between samplings lowers the relative error in  
31 the SPM fluxes to below 10%, a result directly related to the increased number of Q-C pairs available  
32 for fitting. [For regional-scale applications, this study may be used to define the data quality level](#)  
33 [\(uncertainty, frequency and/or number\) compatible with reliable computation of river sediment](#)  
34 [fluxes. Provided that at least 200 concentration samplings are available, the use of a sediment rating](#)  
35 [curve model augmented to account for storage effects fulfils this purpose with satisfactory accuracy](#)  
36 [under real-life conditions.](#)

37

38 **1. Introduction**

39

40 Sediment fluxes in fluvial systems reflect the denudation processes that occur on the earth's surface  
41 and exert a controlling effect on the fates of nutrients, organic pollutants and heavy metals.  
42 Therefore, assessment of sediment fluxes aids in characterising the impact of particulate transfer on  
43 water quality throughout the river system. However, the estimation of realistic sediment budgets  
44 requires long-term discharge-concentration data (Walling and Webb 1985, Ludwig and Probst 1998,  
45 Webb et al. 2003, Delmas et al. 2009), which often suffer from poor availability and reliability  
46 (Meybeck et al. 2003, Walling and Fang 2003). In particular, uncertainties in both the sampling and  
47 calculation methods affect the measurement of sediment concentration (SPM) fluxes (Rode and Surh  
48 2007). Moreover, significant drifts in the measured quantities may arise due to the location of the  
49 sampling in the river section because the suspended sediment concentration varies within cross-  
50 sections of the rivers, thus necessitating a series of depth- and width-integrated measurements  
51 (Horowitz, 1997). However, in most cases, modellers and decision-makers use the daily discharge  
52 records combined with infrequent sediment concentration samplings, which are generally collected  
53 at a single location and exhibit high temporal variability.

54

55 Previous studies have indicated how increased sampling frequencies and total periods of data  
56 collection can reduce uncertainties in the predicted SPM or [trace element](#) fluxes ([Horowitz et al.](#)  
57 [2001](#), Coynel et al. 2004, Moatar and Meybeck 2005, Rode and Surh, 2007), but a complementary  
58 and perhaps broader strategy would involve treatment of the analysis in terms of the interactions  
59 between the number of data, the time interval between samplings (affected by a realistic random  
60 component) and the total period of the data collection. This analysis would be particularly pertinent  
61 in the numerous areas where available the sediment concentration data are by-products of programs  
62 for water quality monitoring that involve infrequent (approximately monthly) sampling (Delmas et al.  
63 2012). A relevant and widely used methodology is the reconstruction of continuous (daily)

64 fluctuations in the sediment concentration from the empirical relationships ( $C(Q)$  rating curves) that  
65 link the sediment concentration ( $C$ ) to the water discharge ( $Q$ ) values. The classical  $C(Q)=aQ^b$  rating  
66 curve is often used for this purpose. Nevertheless, for sediment transport studies, the occasional  
67 strong non-linearity in the  $C(Q)$  relationship and the presence of only a few extreme events point to  
68 well known and problematic cases when fitting power laws (Laherrere 1996, Goldstein et al. 2004).  
69 These problematic cases suggest the use of truncated intervals of  $Q$  values (Moatar et al. 2012) or  
70 data subdivision and hypothesising of the distinct  $C(Q)$  relationships with the seasons, basin sizes or  
71 properties (Walling and Webb 1981, Smart et al. 1999, Quilbé et al. 2006; Delmas et al. 2009).  
72 Another possibility is the addition of correction terms (Laherrere and Sornette 1998) with a user-  
73 defined physical meaning. Ferguson (1986, 1987) chose the latter option in his inaugural papers that  
74 tackled the advantages and limitations in estimating sediment fluxes from power-law rating curves.  
75 Similar to other research domains, an open question is whether to attribute a deterministic physical  
76 meaning to the pre-factor and the exponent (Peters-Kümmerly 1973, Morgan 1995, Asselmann 2000)  
77 or to consider them as conceptually imperfect though demonstrably statistically relevant. Addressing  
78 these questions with a clear physical sense opens the door for a wide series of improved rating  
79 curves and variants, as advocated by Phillips et al. (1999), Asselmann (2000), Horowitz (2003) and  
80 Delmas et al. (2011), among others. For example, Picouet et al. (2001) hypothesised two sediment  
81 sources (bank and hillslope erosion), and Mano et al. (2009) introduced explicit climatic and  
82 topographic factors.

83  
84 The objective of the current paper is to address the precision and sensitivity issues that exist in  
85 prediction of SPM fluxes for typical cases with data of degraded-quality gathered from infrequent  
86 samplings. We consider the  $C$  and  $Q$  measurements to be affected by errors of different types and  
87 magnitudes. As commonly reported, we hypothesised a higher probability of error for concentration  
88 than for discharge measurements. Concentration data were considered prone to random errors in a  
89 [-30%, +30%] interval around the measured values, whereas five systematic biases were tested for

90 discharge data in the [-20%, 20%] interval around the daily records. Moreover, infrequent  
91 concentration measurements were simulated together with slightly perturbed sampling periods. The  
92 framework of this study compares the merits and drawbacks of the classical rating curve with those  
93 of an improved rating curve approach. The latter includes a correction term as an indicator of  
94 sediment storage, thus relating the variation to flow dynamics. This work relies on a USGS dataset  
95 encompassing multiple rivers, basin typologies and years.

96

## 97 **2. Material & Methods**

98

### 99 *2.1 Database*

100

101 The present study relies on daily discharge-concentration data collected over several years at 22  
102 USGS stations taken from the larger dataset available at <http://waterdata.usgs.gov/>. The selected  
103 stations cover a wide range of river basin typologies and sizes and are associated with specific  
104 discharge and SPM statistics (Table 1). For the Des Moines River station, we chose to split the records  
105 into two distinct periods because we observed notably large discharge variations over the first years  
106 of data collection (1961-1977) followed by rather limited variation over the most recent years (1977-  
107 2004). Several stations display notably low water levels and even close to intermittent flows under  
108 severe winter conditions (e.g., station 21, Salinas River near Spreckels), summer droughts (e.g.,  
109 station 18, Kaskaskia River at Cook Mills) or both (e.g., station 7, Hocking River at Athens). **However,**  
110 **stations in the USGS database that exhibit complete drying or glaciations have been discarded from**  
111 **the analysis because they require different treatment. This choice tends to omit basins with small**  
112 **drainage areas, especially those that experience sharp climatic conditions. Nevertheless, various**  
113 **temperate to continental climates are accounted for at stations in California (CA), Illinois (IL), Iowa**  
114 **(IA), Missouri (MI), North Carolina (NC), Ohio (OH) and Virginia (VA). Another merit of sufficiently**  
115 **large basins for the present study is response times that are greater than the reference daily**

116 sampling period for the discharge and concentration measurements. In addition to the temporal  
117 arguments, large basins are also more integrative than smaller basins because they offer additional  
118 spatial variability with respect to the sediment sources and delivery processes combined with larger  
119 climatic variability.

120

121 [TABLE 1 ABOUT HERE]

122

## 123 2.2 Rating curves

124

125 In addition to the classical  $C=aQ^b$  rating curve (RC), different expressions and strategies have been  
126 tested and described in detail by Delmas et al. (2011). Among these, emphasis is placed in this work  
127 on the Improved Rating Curve Approach (IRCA), which hypothesises  $C=aQ^b+a_1\delta S$ , where  $a_1$  is a  
128 parameter,  $S$  is the sediment storage index and  $\delta S$  is the daily variation. This method operates in two  
129 steps. First, the method fits the  $C=aQ^b$  model, thus freezing the  $a$  and  $b$  coefficients, and  
130 subsequently fits the  $a_1$  parameter, which is the only remaining degree of freedom in the  $a_1\delta S$   
131 correction term. The IRCA was implemented to remedy the known limitations of the RC, especially its  
132 inability to include flow dynamics or antecedent flow conditions in the prediction of  $C$  values.

133

134 For this purpose, the sediment storage index is described as a function of discharge dynamics:

$$S(t) = \exp\left(-A \frac{q(t)}{Q_0}\right) \quad [1]$$

135 where  $A$  is a dimensionless coefficient that controls the amplitude of  $S$  variations, and  $q(t)$  is the  
136 instantaneous  $Q(t)$  value on the rising limbs of the hydrograph or a sliding average of  $Q(t)$  values on  
137 the falling limbs. This double status of  $q(t)$  ensures asymmetry between quick sediment loading and  
138 delayed sediment storage, which is the working hypothesis that underlies the IRCA. The sliding  
139 average extends itself backwards in time towards the beginning of the current falling limb but is

140 limited to 20 days at most, which is the chosen approach used to account for antecedent flow  
141 conditions.

142

143 The IRCA also requires statistical analysis to establish the upper limit of the base flow, given as  $Q_0$ .  
144 From the adopted assumptions,  $S$  varies between 0 for extreme discharges (all possible sediments  
145 present in the flow) and 1 when flow ceases (all material stored and available). The choice of  $A=0.1$   
146 yields  $S=0.95, 0.75, 0.50, 0.25$  and  $0.05$  for  $q/Q_0$  ratios of approximately 0.5, 3, 7, 14 and 30,  
147 respectively. This choice also implies that  $S \approx 0.9$  for  $q=Q_0$ , and thus the “sediment availability” would  
148 be approximately 90%.

149

150 Figure 1 shows the variations in the sediment storage index in an application to the Hocking River at  
151 Athens (station 7, Table 1). The figure displays the main characteristics of this approach: asymmetry  
152 between the delayed sediment storage and quick sediment mobilisation, damping of the high-  
153 frequency oscillations and maximum sediment storage associated with long-lasting low discharges.  
154 The intent of the IRCA is to use the daily variations of the sediment storage index as relevant  
155 information for the flow dynamics, as handled by the  $a_1\delta S$  correction term in the  $C=aQ^b+a_1\delta S$  model.

156

157 [FIGURE 1 ABOUT HERE]

158

159 In a strict sense, both the  $C=aQ^b+a_1\delta S$  and  $C=aQ^b+a_1\delta S$  expressions should be considered in this work  
160 as “models” and not “laws”. Laws should be written with an additional explicit error term, either  
161 multiplicative or additive, whose distribution should be assessed from the characteristics and is  
162 missing in the data ( $Q, C$ ) measurements. This option would take into account that the  $C=aQ^b$  and  
163  $C=aQ^b+a_1\delta S$  expressions are exact formulations of the problem, attributing the deviations between  
164 the calculated and observed quantities to variations in the associated error terms. Instead, the  
165 current option views the  $C=aQ^b$  and  $C=aQ^b+a_1\delta S$  expressions as imperfect models with inherent

166 errors. Therefore, the scope of this work focuses on the dispersion of the model predictions with  
167 alterations of the (Q, C) dataset.

168

### 169 *2.3 Fitting procedures*

170

171 All fittings were automated using a multi-stage procedure centred on the PEST parameter estimation  
172 software (Doherty, 2004) but also resorted to further programming. Figure 2 displays the simplified  
173 flow chart, which is briefly described. The user selects a  $C(Q)$  model with  $p$  parameters to fit, entering  
174 the flow chart with the initial  $p_0$  vector of parameter values and also providing a series of  $Q_i$   
175 discharge samplings. This  $(Q_i, p)$  set is processed via a program that calculates the “fitted”  $C_i^{(f)}$  value  
176 associated with each  $Q_i$  value for the selected power law. Next, PEST carries out an overall  
177 optimisation based on the minimisation of an objective function, which is calculated as the sum of  
178 the squared residuals between the observed  $C_i$  values and the fitted  $C_i^{(f)}$  values. Once the  
179 optimisation has been completed, PEST estimates the goodness-of-fit by calculating the coefficient of  
180 determination, known as  $R$ . The  $R$  value may therefore be used for comparisons between fittings.  
181 One of the interesting features in PEST is its ability to handle power laws with or without log  
182 transformations, and the choice is left to the user’s discretion. Slightly better results have been  
183 obtained in this work by fitting the untransformed expressions.

184

185 [FIGURE 2 ABOUT HERE]

186

### 187 *2.4 Methods for calculating fluxes and errors*

188

189 The real amount of exported sediment in the  $[0, T]$  total time period is the unknown SPM flux:

$$F = \int_0^T Q(t)C(t)dt \quad [2]$$

190 where  $Q$  is the discharge,  $C$  is the concentration of suspended particulate matter (SPM) and  $t$  is time.

191

192 The straightforward approximation for direct calculation of  $F$  is:

$$F_N = \sum_{i=1}^N Q_i C_i \delta t_i \quad [3]$$

193 where  $Q_i$  and  $C_i$  are time-averaged values over the  $\delta t_i$  intervals covering the  $[0, T]$  time period.

194

195 The discrepancy between  $F$  and  $F_N$  plausibly increases with increased  $\delta t_i$  values. Nevertheless, certain  
196 rare and fortuitous combinations of large  $\delta t_i$  values may still lead to  $F_N \approx F$ , from compensating  
197 effects between the underestimations and overestimations of SPM fluxes over certain periods.

198 Therefore, the gap between  $F_N$  and  $F$  is not expected to vary in any monotonous or linear manner  
199 with increasing  $\delta t_i$  values. By contrast, the  $\delta t_i$  intervals that are smaller than the characteristic time  
200 period of fluctuations in  $Q$  and  $C$  values tend to ensure the reliability of the  $F_N$  approximation.

201 Because the SPM regimes are related to the basin sizes (Meybeck et al. 2003) and only drainage  
202 areas larger than 1000 km<sup>2</sup> are considered in this work, the  $F_N$  fluxes calculated from the error-free  
203 daily discharge and concentration records are considered as exact solutions ( $F_N \approx F$ ) in the following.

204 The subsequent developments aim to test the deviations from this best-case scenario that appear  
205 when coping with infrequent and/or uncertain data (i.e., random errors in the concentration data  
206 and systematic errors in the discharge data and sparse concentration samplings in this work).

207

208 The missing concentration data in [3] may be replaced by concentration values obtained from the  
209 fitted  $C(Q)$  models. Each  $Q_i$  datum results in a predicted  $C_i^{(f)}$  concentration value, and the exported  
210 SPM flux now can be approximated by:

$$F_{N,f} = \sum_{i=1}^N Q_i C_i^{(f)} \delta t_i \quad [4]$$

211

212 The hypothesis for the missing  $C$  data implies that only  $n < N$  values have been measured. In the  
 213 calculation of  $F_{N,f}$ , one may either use the available concentration data  $C_i$  instead of the fitted values  
 214 or systematically resort to the entire series of  $C_i^{(f)}$  values, as in this study.

215

216 From a theoretical point of view, any given  $C_i^{(f)}$  is an unknown function of the entire dataset via the  
 217 fitting procedure:

$$C_i^{(f)} = C_i^{(f)}(Q_1, Q_2, \dots, Q_N, C_1, \dots, C_n, \Delta t_1, \dots, \Delta t_n) \quad [5]$$

218 with irregular  $\Delta t_k$  intervals between successive  $C$  samplings in the general case.

219

220 For regular  $\Delta t$  intervals between concentration samplings, the  $(\Delta t_1, \dots, \Delta t_k, \dots, \Delta t_n)$  temporal argument  
 221 could be reduced to  $(\Delta t, n)$  or  $(\Delta t, T)$  without any additional loss of information. Combining these  
 222 elements with shortened notations  $Q' = (Q_1, \dots, Q_N)$  and  $C' = (C_1, \dots, C_n)$  would yield:

$$C_i^{(f)} = C_i^{(f)}(Q, C, \Delta t, T) \quad [6]$$

223

224 The assessment of SPM fluxes ( $F_{N,f}$ ) from the source data ( $Q', C'$ ) under experimental conditions  $(\Delta t,$   
 225  $T)$  is a two-stage process that first solves the inverse problem by fitting the optimal  $\{p\}$  parameter set  
 226 in the  $C(Q)$  model (rating curve) and subsequently uses it for direct calculations by means of [4]:

$$(Q, C, \Delta t, T) \rightarrow \{p\} \rightarrow F_{N,f} \quad [7]$$

227 Most of the time, at least in the French river surveillance network, the intervals between  $C$  samplings  
 228 are only approximately regular such that  $\Delta t^{\sim}$  should be used instead of  $\Delta t$ . [The present procedure is](#)  
 229 [assumed to be valid if the average of the  \$\Delta t^{\sim}\$  values over the data collection period remains](#)  
 230 [sufficiently close to  \$\Delta t\$ . In other words, this hypothesis is intended to authorise random fluctuations](#)  
 231 [of  \$\Delta t^{\sim}\$  around  \$\Delta t = T/n\$  and is not assumed to hold for drastic changes in data collection strategy.](#)

232

233 The objective of Step 1 is to test the combined effects of systematic biases in  $Q'$  with random errors  
234 on  $C'$ , thus maintaining the optimal conditions of data collection: daily discharge and concentration  
235 data over several years. Emphasis is placed on the effects of degraded data quality on both the fitted  
236 parameters and the estimated SPM fluxes for the classical ( $C=aQ^b$ ) and improved ( $C=aQ^b+a_1\delta S$ ) rating  
237 curves.

238

239 Five systematic relative errors in the discharge measurements are addressed and are noted as  $Q_r = -$   
240 20%, -10%, 0%, +10% and +20%: if  $Q^*$  is the measured dataset and  $Q'$  is the collection of exact  
241 values, the listed  $Q_r$  cases are also written as  $Q^*/Q'=0.8, 0.9, 1.0, 1.1$  and  $1.2$ , respectively. However,  
242 random relative errors in the concentration measurements ( $C^R$ ) were assumed as uniformly  
243 distributed within the [-30%, +30%] interval around the collected data. This random treatment  
244 involved replicating each station in Table 1 into 100 virtual stations with the intended perturbations  
245 in the  $C$  data.

246

247 The selected objectives are to study the relative variations of the parameters  $\{p_r\}$  and those of the  
248 calculated SPM fluxes. The latter appear in more eloquent representations when displaying the ratios  
249 of calculated and exact fluxes such that the stages of the test procedure may be summarised in:

$$(Q_r, C^R) \rightarrow \{p_r\} \rightarrow F_{N,f} / F \quad [8]$$

250 where the expected results are the mean variation trends in function of  $Q_r$  values with dispersion  
251 effects arising from the randomised  $C^R$  concentration data. The  $\{p_r\}$  set is  $\{a_r, b_r\}$  for the classical  
252 rating curve and  $\{a_r, b_r, a_{1r}\}$  for the IRCA.

253

254 Step 2 focuses on the effects of the sampling frequencies, especially those with increasing time  
255 intervals between  $C$  data, while ensuring that the data quality is unaffected, thus producing error-  
256 free  $Q$  and  $C$  data. Numerous combinations of  $(\Delta t^R, T_c)$  values have been tested, where  $\Delta t^R$   
257 designates an average sampling interval affected by a "small" random perturbation (at most equal to

258  $\Delta t/2$ ), and  $T_C$  is the total period of data collection. The  $\Delta t^R$  treatment is the approach chosen to  
259 account for the irregular  $\Delta t^{\sim}$  intervals previously mentioned. The targets are once again the relative  
260 variations and dispersions of the SPM fluxes, as shown in the ratios of calculated and exact fluxes:

$$\left(\Delta t^R, T_C\right) \rightarrow F_{N,f} / F \quad [9]$$

261

262 Step 3 analyses the combined effects of the degraded  $Q-C$  data quality and infrequent  $C$   
263 measurements on the calculated SPM fluxes within the following procedure:

$$\left(Q_r, C^R, \Delta t^R, T_C\right) \rightarrow F_{N,f} / F \quad [10]$$

264

265 Finally, step 4 provides an application of the IRCA around the estimation of SPM exports from French  
266 rivers to the sea. Contrary to the preceding sections,  $F$  is unknown and is thus estimated from  
267 “confidence intervals” around the  $F/F_{N,f}$  ratios by relying on the calculated  $F_{N,f}$  fluxes.

268

269

270 **3. Results and discussion**

271

272 *3.1 Step 1: Effects of data uncertainty on the calculated SPM fluxes*

273

274 *Effects of data uncertainty on the fitted parameters*

275

276 Theoretical predictions are available from the scale-invariant properties of the  $C=aQ^b$  law: the law  
277 should still hold with a modified pre-factor ( $a^*$  instead of  $a$ ) and unchanged exponent ( $b$ ) when  
278 modifying the argument, i.e., measuring  $Q^*$  instead of the real  $Q$  value:

$$C = aQ^b = a \left( \frac{Q}{Q^*} Q^* \right)^b = a^* Q^{*b} \quad [11]$$

279 with  $a^*=a(Q/Q^*)^b$ .

280

281 For example, the underestimation of  $Q$  by 20% ( $Q_r = -20\%$ ) corresponds to  $Q^*/Q = 0.8$ , thus  $Q/Q^* =$   
282  $1.25$  and  $a^*/a = (1.25)^b$ . A numerical application with a typical  $b$  value of 0.85 yields  $a^*/a=1.21$ . In  
283 such a case, a relative variation  $a_r = +21\%$  is theoretically expected. Similar predictions are available  
284 for the other tested  $Q_r$  values and are represented by the “Theory” dotted lines in Figure 3. Because  
285 the  $b$  exponent is supposedly unaffected, its relative variation  $b_r$  should be zero. No such prediction is  
286 available for  $a_1$  and  $a_{1r}$ .

287

288 [FIGURE 3 ABOUT HERE]

289

290 Figure 3 gathers the  $a_r$ ,  $b_r$  and  $a_{1r}$  variations; the latter is specific to the  $C=aQ^b+a_1\delta S$  model, and the  
291 former two are common with the  $C=aQ^b$  model. All curves display the global averages of  $a_r$ ,  $b_r$  or  $a_{1r}$   
292 for the 22 stations listed in Table 1, and each one is replicated into 100 virtual stations to satisfy the

293 random procedure on  $C$  data. Taking the example of  $a_r$ , the upper (+) and lower (-) limits of the  
 294 dispersion envelope are plotted as:

$$a_r^\pm(Q_r) = \frac{a^*(Q_r) \pm 1.73 \sigma_{a^*}(Q_r)}{a} = a_r(Q_r) \pm 1.73 \frac{\sigma_{a^*}(Q_r)}{a} \quad [12]$$

295 | where  $\sigma_{a^*}$  is the standard deviation in the  $a^*$  values and all quantities are functions of  $Q_r$  except  $a$ ,  
 296 which is the reference value obtained for  $Q_r = 0$ . The  $\sigma_{a^*}/a$  term is a normalised standard deviation.

297

298 The 1.73 factor is usually associated with a 100%-confidence interval for uniform random  
 299 distributions: once the average value (AV) and standard deviation ( $\sigma$ ) are known, the real value falls  
 300 in the  $[AV - 1.73 \sigma, AV + 1.73 \sigma]$  interval. In this work, the uniform random distribution  $C^R$  results in  
 301 nearly uniform random distributions of  $a^*(Q_r, C^R)$  and  $a_r(Q_r, C^R)$  values for each  $Q_r$  value, thus ensuring  
 302 the validity of the calculated dispersion envelope for  $a_r(Q_r, C^R)$ . The same procedure applies to  
 303 parameters  $b$  and  $a_1$ .

304

305 Figures 3a, d and g examine the  $a_r(Q_r)$  trends and compare them with the theoretical predictions for  
 306 increasing values of the PEST coefficient of determination ( $R$ ). The atypical non-monotonous  
 307 variation displayed in Figure 3a is essentially due to poorly fitted data ( $R \approx 0.2$ ) from stations 14, 16, 18  
 308 and 19 in Table 1. At such low  $R$  values, equifinality unavoidably exists such that completely different  
 309  $(a, b)$  couples lead to similar performances in the optimisation. For the cited stations, unexpectedly  
 310 high  $a$  values compensate for the effects of notably low  $b \ll 1$  values, yielding  $R$  values otherwise  
 311 obtained from  $(a, b)$  couples with  $b \approx 1$ .

312

313 In Figure 3a, d and g, the dispersion decreases with increasing  $R$  values and remains limited in the  
 314 sense that variability is explained by the  $a_r(Q_r)$  trends rather than by the "orthogonal" dispersive  
 315 random effects. As expected, the  $a_r(Q_r)$  curves best match the theoretical curves for increasing  $R$

316 values, which indicate a clearer power-law organisation of the dataset. Unfortunately, only 7 stations  
317 are encompassed by the  $R > 0.65$  criteria vs. 17 stations out of 22 for  $R > 0.3$ .

318

319 Rather weak dispersions and low  $b_r$  values (i.e., nearly unchanged  $b$  exponents) in Figure 3b, e, and h  
320 appear to be in good agreement with expectations arising from the scale-invariance properties of  
321 power laws. However, a striking feature is the increasing variability of  $a_r$  and  $b_r$  with decreasing  
322 determination values. The underlying and unanswered question at this point is whether this  
323 variability will endanger the reliability of SPM flux calculations for fittings with low determination  
324 values. This question is the subject of the advocated “composite analysis” performed in the following  
325 paragraphs, from data uncertainty to uncertainty in the fitted parameters as well as in the calculated  
326 fluxes.

327

328 The  $a_{1r}$  variations (Figure 3c, f and i) also show limited dispersion, except for strong discharge  
329 underestimations. Positive relative errors in the discharge ( $Q_r > 0$ ) cause positive relative variations in  
330 the  $a_1$  coefficient ( $a_{1r} > 0$ ). A key point is that the  $\delta S$  term is identical regardless of the  $Q_r$  value  
331 because the  $q/Q_0$  ratio is used in [1], where both  $q$  and  $Q_0$  are affected by the same relative error in  
332 the discharge values. This choice of  $\delta S$ , referred to as “identical by construction”, allows direct  
333 interpretation of the role played by  $a_1$ : the  $a_1 \delta S$  correction behaves similar to  $a_1$ , producing its  
334 strongest relative variations ( $a_{1r} = -30\%$ ) for the strongest discharge underestimations ( $Q_r = -20\%$ ).

335

### 336 *Effects of data uncertainty on the calculated SPM fluxes*

337

338 Contrary to the fitted coefficients, the calculated SPM fluxes exhibit almost no dependence on the  
339 determination ( $R$ ) values. Therefore, Figure 4 plots the SPM fluxes and dispersions obtained for all  $R$   
340 values. As shown previously, the upper (+) and lower (-) limits of the dispersion envelope are written:

$$\left(\frac{F_{N,f}}{F}\right)^{\pm}(Q_r) = \frac{F_{N,f}(Q_r) \pm 1.7 \sigma_{F_{N,f}}(Q_r)}{F} = \left(\frac{F_{N,f}}{F}\right)(Q_r) \pm 1.7 \frac{\sigma_{F_{N,f}}(Q_r)}{F} \quad [13]$$

341

342 [FIGURE 4 ABOUT HERE]

343

344 Figure 4a and b show weak dispersions and similar variation trends because the calculated SPM  
 345 fluxes increase with the estimated discharge values. The relative variations in the calculated fluxes  
 346 are near-linear functions of the systematic relative error present in the discharge measurements for  
 347 both the classical  $C=aQ^b$  (Figure 4a) and the improved  $C=aQ^b+a_1\delta S$  (Figure 4b) rating curves. The  
 348 difference between these parallel curves is the “offset” added in Figure 4b by the  $a_1\delta S$  correction,  
 349 which perfectly remedies the underestimation due to the classical method. The gain obtained from  
 350 the  $a_1\delta S$  correction is approximately 5% along the “calculated on exact” vertical axis, with even  
 351 weaker dispersion in the results. For example, if the discharge is known with a 10% uncertainty (5%  
 352 on each side of the  $Q_r=0$  line), then  $F_{N,f}/F$  lies within the 0.88-1.04 and 0.93-1.07 ranges of the  
 353 classical and improved rating curves, respectively.

354

355 Looking at the nearly stable  $b$  values ( $b_r \approx 0$  in Figure 3b, e and h), it is worth noting that increases in  
 356 the fitted  $a$  values ( $a_r > 0$  in Figure 3d and g) were not sufficient to counterbalance the influence of  
 357 the discharge underestimations ( $Q_r < 0$ ). The calculated fluxes still underestimate the real values  
 358 when the discharge is underestimated for both methods in Figure 4. However, the overall effect of  
 359 the storage term is an increase in the calculated ( $F_{N,f}$ ) SPM fluxes, which is validated by  
 360 improvements of approximately 5% in the determination values.

361

362 *3.2 Step 2: Effects of data infrequency on the calculated SPM fluxes*

363

364 This subsection leaves data quality concerns aside to focus on data availability, especially on the  
365 problem of infrequent  $C$  data. This subsection also tackles the sampling frequency issue by  
366 addressing the interplay between the number ( $n$ ) of available data, the total collection period ( $T$ ) and  
367 the sampling period ( $\Delta t$ ), as commented next to [6]. As previously mentioned, the sampling periods  
368 have always been tested with slight random perturbations ( $\Delta t^R$ ) of the predefined values to account  
369 for more realistic field conditions, resulting in approximations ( $\Delta t^{\sim}$ ) of the predefined  $\Delta t$  intervals.  
370 Figure 5 displays the ratio of the calculated and exact SPM fluxes, the medians of the absolute errors  
371 on this ratio and the statistics for the absolute errors on this ratio in the  $\Delta t^{\sim}30$  day case for various  
372 ( $n, T, \Delta t$ ) triplets, thus allowing comparisons between the fitted  $C=aQ^b$  and  $C=aQ^b+a_1\delta S$  models.

373

374 [FIGURE 5 ABOUT HERE]

375

376 Figure 5a targets the evolution of the “calculated on exact” ratios of SPM fluxes for decreasing values  
377 of the sampling interval ( $\Delta t$ ) and also shows the positive impact of increasingly long periods of data  
378 collection ( $T$ ) for the given sampling intervals. A clear increase in dispersion occurs for  $\Delta t > 5$  days  
379 combined with  $T < 4$  years. Further on the left of the diagram, the rather typical monthly sampling  
380 period ( $\Delta t^{\sim}30$  days) requires at least an 8-year collection period ( $T=8$  years) for the “calculated on  
381 exact” ratios to lie within the gross [0.5, 5.0] interval. Figure 5b also begins with highly dispersed  
382 values for combinations of high sampling periods, even with long collection periods, and exhibits  
383 additional oscillations before achieving convergence because it is obviously more seriously affected  
384 by weak  $T$  values. The main difference is the highest dispersion for  $\Delta t^{\sim}5$  days and  $T=4$  years and for  
385  $\Delta t^{\sim}20$  days and  $T=8$  years. A common point appears to be the trend of overestimating the exact SPM  
386 fluxes, even for the most relevant estimations that rely on low  $\Delta t$  and high  $T$  values.

387

388 A complementary view is given by Figure 5c and d and shows the median of the absolute error of the  
389 calculated against exact ratio as a function of the number of available concentration data ( $n$ ) by

390 plotting a dedicated curve for each of the total tested collection periods ( $T=1, 4, 8, 12$  and  $15$  years).  
391 Figure 5c shows that  $n=150$  takes the error under  $20\%$ , whereas  $n=300$  is required for errors lower  
392 than  $10\%$ . These results are nearly independent of  $T$  and thus hold for sampling intervals  $\Delta t=T/n$  with  
393  $n$  above the mentioned thresholds. For example, it takes  $\Delta t\sim 20$  days for  $T=8$  years to dispose of  
394  $n=150$  concentration data. Equivalently,  $T=13.7$  years is the minimum time period required to fulfil  
395 the  $20\%$  error criterion when performing monthly samplings ( $\Delta t\sim 30$  days). By contrast, the trends in  
396 Figure 5d are less uniform because the error values exhibit a clear dependence on the  $T$  values,  
397 which calls in question analyses that only involve the number  $n$  of concentration data and make a  
398 case for combined  $(n, T)$  criteria. For example,  $n>200$  and  $T\geq 8$  years will confine the relative error to  
399 under the  $20\%$  mark. For monthly samplings, the  $n>200$  threshold alone corresponds to  $T>18$  years,  
400 which *de facto* verifies  $T\geq 8$  years. This result tends to indicate that the condition for the number of  
401 data remains the most restrictive and could still be maintained alone.

402

403 Figures 5e and f focus on the case of monthly concentration samplings. The monotonic and  
404 exponential-like decrease of the error in Figure 5e emphasises the regular gain in the stability of the  
405  $C=aQ^b$  method for total data collection periods increasing from  $T=1$  to  $15$  years. Figure 5f shows the  
406 same trend with a few irregularities. The dynamic definition chosen for the storage term ( $S$ ) and its  
407 variations ( $\delta S$ ) plausibly requires sufficiently dense concentration data for significant gains in  
408 precision, with potentially better performances than the  $C=aQ^b$  method but apparently with slightly  
409 more restrictive conditions of application.

410

### 411 3.3 Step 3: Combined effects of data uncertainty and infrequency on the calculated SPM fluxes

412

413 Subsection 3.1 established the ability of the  $C=aQ^b+a_1\delta S$  model to better predict the SPM fluxes, as  
414 compared with the  $C=aQ^b$  model, using degraded-quality data. Subsection 3.2 showed that the  
415  $C=aQ^b+a_1\delta S$  model was slightly more sensitive to data infrequency. The present subsection tests both

416 models against combined data uncertainty and infrequency to simulate eventual real-life conditions:  
417 systematic relative errors in the discharge measurements (poorly-gauged stations), random errors in  
418 the concentration measurements (uncertain techniques) and possibly sparse concentration  
419 measurements (limited credits).

420

421 Figure 6 uses the 100%-confidence intervals, similar to these already defined in [12] and [13], to draw  
422 dispersion envelopes associated with the random treatment of concentration data for daily ( $\Delta t \sim 1$   
423 day), decadal ( $\Delta t \sim 10$  days) and monthly ( $\Delta t \sim 30$  days) concentration samplings. The  $\Delta t \sim 1$  day case  
424 corresponds to the results shown in Figure 4. In Figure 6, sparser concentration samplings lead to  
425 wider dispersion envelopes that are somewhat shifted towards higher flux predictions. The drift and  
426 dispersion are a bit more pronounced for the storage method with too many data lacking, although  
427 the curves are quite similar between sketches (a) and (b). Figure 6 also proves that infrequent and  
428 limited-quality data may still be used for flux predictions (at least with caution and while staying  
429 within the tested ranges for errors) because they do not lead to completely divergent, uncontrolled  
430 or unpredictable results.

431

432 [FIGURE 6 ABOUT HERE]

433

434 As a mean behaviour among all stations, if one commits a systematic relative error within the [-5%,  
435 +5%] interval on  $Q$  together with a random relative error within the [-30%, +30%] interval on  $C$  while  
436 only disposing of  $C$  data each 30 days on average, the calculated SPM flux still lies within a factor of  
437 0.60-1.60 of the real value with the classical rating curve. The improved rating curve gives the  
438 estimation in the 0.65-1.65 range. A narrower 0.80-1.40 range would be obtained if considering 60%-  
439 confidence intervals instead of 100%-confidence intervals. The following subsection therefore  
440 addresses possible applications in bounding sediment budgets.

441

442 *3.4 Step 4: Application to sediment exports from French rivers*

443

444 In real-life situations, the predicted flux is known, and the objective is to define a plausible interval  
445 for the real flux. This process may be carried out by examining the “exact against calculated” ratios  
446 along the y-axis instead of the “calculated against exact” ratios shown in the previous figures. Figure  
447 7a presents dispersion envelopes corresponding to the 100%-confidence intervals, indicating where  
448 the exact SPM flux may lie for sampling periods of  $\Delta t \sim 10$  days and  $\Delta t \sim 30$  days. Taking a 10%  
449 uncertainty for the discharge measurements (between -5 and +5%), retaining the 60% random  
450 uncertainty in the concentration data (between -30% and +30%) and assuming a worst-case sampling  
451 interval of  $\Delta t \sim 30$  days for the concentration data, the real flux (F) lies between 0.6 and 1.53 times the  
452 calculated value ( $F_{N,t}$ ).

453

454 In the French sediment budget proposed by Delmas et al. (2012) for the major rivers to the sea, the  
455 calculations were performed from monthly sediment concentration samplings ( $\Delta t \sim 30$  days) over  
456 more than 25 years, except for the Rhone river, for which daily measurements were available in  
457 certain periods, yielding an average of  $\Delta t \sim 10$  days. Figure 7b shows the induced uncertainty in the  
458 calculated sediment fluxes. The estimated total sediment load for the selected rivers is 13.9 Mt/yr.  
459 From the uncertainties calculated in this work, the real sediment export from these rivers is  
460 contained between 10.05 Mt/yr and 17.7 Mt/yr.

461

462 [FIGURE 7 ABOUT HERE]

463

464

#### 465 4. Conclusion

466

467 This paper tackled feasibility and precision issues in calculation of fluxes of suspended particulate  
468 matter (SPM) by assuming systematic errors in the water discharge ( $Q$ ) and random errors in  
469 sediment concentration ( $C$ ) data and under the additional constraint of infrequent sediment  
470 concentration samplings. The chosen framework compared the merits and drawbacks of the classical  
471 rating curve ( $C=aQ^b$ ) with those of an improved rating curve approach (IRCA,  $C=aQ^b+a_1\delta S$ ) in which  
472 the correction term is an indicator of the variations in sediment storage and is thus related to flow  
473 dynamics. Successive steps of the analysis were: (1) to establish the effects of data uncertainty on  
474 the fitted coefficients ( $a$ ,  $b$ ,  $a_1$ ) and on the calculated SPM fluxes, (2) to examine how infrequent  
475 sediment concentration data affects these estimates and (3) to combine both effects into the  
476 definition of realistic cases, thus allowing an application to sediment exports from French rivers.

477

478 Step 1 involved systematic relative errors in the flow discharge (-20%, -10%, 0%, +10%, +20%) and  
479 random relative errors in the sediment concentration in the [-30%, +30%] interval. Increasing  $a$   
480 values and stable  $b$  values were observed when gradually moving from -20% to +20% errors in the  
481 discharge, together with decreasing dispersion in the fitted ( $a$ ,  $b$ ) coefficients with the increasing  
482 goodness-of-fit. Such results globally meet the expectations arising from the scale-invariant  
483 properties of power laws. As a complement, the dispersion in the magnitude of the  $a_1\delta S$  correction  
484 term appeared far stronger for the most severe discharge underestimations than for all other tested  
485 errors. The general trend was that of an upward correction due to the  $a_1\delta S$  term: the IRCA remedies  
486 the small systematic underestimations observed with the classical rating curve method. [The IRCA](#)  
487 [always performs better than the classical rating curve \(and does not require additional information\)](#)  
488 [when frequent concentration measurements are available, regardless of the period of data collection](#)  
489 [and the tested errors in the discharge and concentration.](#)

490

491 Step 2 hypothesised error-free discharge and concentration measurements to focus on the effects of  
492 sampling frequencies on the calculated sediment fluxes. The results show that the number ( $n$ ) of  
493 available C measurements is more discriminating than the sampling frequency itself, at least within  
494 the tested data collection strategies, and the provided data measurements captured sufficient  
495 variability in the river behaviour according to the Wilcoxon test of data representativity. The criteria  
496 on  $n$  were revealed as the most restrictive with respect to the performances of the rating curves:  
497  $n > 200$  guaranteed that the calculated SPM fluxes would lie within the [-20%, +20%] interval around  
498 the exact values.

499

500 Step 3 consisted of combining the previous aspects and checking whether degradation of the results  
501 remained progressive and controlled, or if data uncertainty and infrequency would yield diverging  
502 results and render the methods inapplicable. This question was especially pertinent for the IRCA, that  
503 was slightly better in dealing with uncertain data but was slightly more sensitive to data infrequency.  
504 Bounding the calculated SPM fluxes within the confidence intervals around the real values allowed  
505 the following observations: the calculations are slightly higher with the improved rating curve, and  
506 the dispersion also grows slightly wider with increasing errors in discharge, but the results always  
507 stay within the acceptable margins of errors.

508

509 In the chosen worst-case scenario, poor sampling intervals of 30 days (on average) over  
510 approximately fifteen years combined with relative errors in discharge between -5% and +5% and  
511 random errors within the [-30%, +30%] interval for the sediment concentration values resulted in  
512 calculated fluxes that are still bounded within a factor of 0.60 to 1.65 of the real values. However, all  
513 other realistic cases yield far better estimates. For example, there is no technical improvement, but a  
514 shorter 5-day sampling interval on average reduces the relative error to below 10%. Finally, the  
515 application to French rivers illustrated the reliability of the method for a wide variety of irregular  
516 sampling intervals and flow discharge and sediment concentration ranges, provided that sufficient

517 concentration data were available. In addition to technical improvements, a key issue is to better  
518 address the irregular samplings and thus to extract additional information from the discharge  
519 dynamics, especially the antecedent flow conditions, through ongoing developments around the  
520 sediment storage term in the IRCA.

521

522 In its present formulation, the IRCA perfectly corrects the known underestimation of the classical  
523 rating curve, if frequent concentration samplings are available, combined with the uncertainties of  
524 discharge and concentration values. Both methods perform well when concentration samplings  
525 become sparse if the sampling period and number of concentrations remain sufficiently large. Finally,  
526 only the IRCA is subject to improvements for better exploitation of the information contained in the  
527 temporal dynamics of discharge.

528

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532

533

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640

641

642 **Table caption**

643

644 Table 1 – Names and locations of the USGS stations used in this study together with the drained  
645 areas of the monitored rivers and statistics of their discharge (Q) and concentration (C) values. The  
646  $Q_0$  base-flow limit is an indicator introduced in the IRCA-Improved Rating Curve Approach, and  $\sigma(Q)$   
647 and  $\sigma(C)$  report the standard deviations of the Q and C data, respectively.

648

649

650 **Table 1**  
 651

USGS station	Drained area	Min Q	Base-flow $Q_0$	Median Q	Mean Q	Max Q	$\sigma(Q)$	Min C	Median C	Mean C	Max C	$\sigma(C)$	
# <i>River name and station location</i>	$km^2$	$m^3 s^{-1}$						$g L^{-1}$					
1 Rappahannock River at Remington, VA	1603	0.1	17.4	11.2	19.0	1296.9	32.9	1	11	39	2070	105	
2 Roanoke River at Randolph, VA	7682	5.1	72.6	49.6	79.3	1996.3	102.1	1	34	76	2060	142	
3 Dan River at Paces, VA	6700	6.9	64.4	53.2	77.7	1795.3	91.6	5	60	122	2260	193	
4 Yadkin River at Yadkin College, NC	5905	9.3	68.7	62.3	83.7	1868.9	87.0	1	70	150	2970	224	
5 Muskingum River at Dresden, OH	15522	13.0	89.1	93.2	165.7	1067.5	177.5	1	30	60	1600	84	
6 Muskingum River at McConnelsville, OH	19223	16.1	111.7	171.0	256.6	1350.0	229.7	2	48	77	1710	100	
7 Hocking River at Athens, OH	2442	0.7	24.3	9.3	25.0	883.5	49.3	1	14	56	1320	116	
8 Scioto River at Highby, OH	13289	7.1	121.6	61.2	133.3	3596.2	196.8	1	41	99	2520	177	
9 Little Miami River at Milford, OH	3116	1.5	39.8	17.4	38.6	863.7	59.4	1	40	103	4850	216	
10 Great Miami River at Sydney, OH	1401	0.8	10.0	6.7	15.9	250.3	23.6	1	50	72	1710	98	
11 Stillwater River at Pleasant Hill, OH	1303	0.3	14.5	4.0	12.2	373.8	25.6	1	23	52	1970	108	
12 Maume River at Waterville, OH	16395	0.5	95.0	53.0	148.6	3199.8	247.7	1	39	82	2240	129	
13 Upper Iowa River near Dorchester, IA	1994	2.2	9.1	7.4	12.7	268.2	17.5	1	43	192	10000	677	
14 Iowa River at Iowa City, IA	8472	1.4	21.2	37.9	64.0	410.6	65.2	1	55	103	7540	215	
15 Des Moines River near Saylorville, IA	15128	0.4	44.9	27.8	69.8	1333.7	109.6	1	120	240	5400	356	
16 Des Moines River near Saylorville, IA	15128	2.1	53.4	48.4	101.1	1254.4	129.4	0.7	32	46	1210	51	
17 Illinois River at valley City, IL	69264	37.7	3.6	549.3	743.0	3398.0	572.3	13	120	182	3720	202	
18 Kaskakia River at Cooks Mills, IL	1225	0.0†	5.8	4.9	12.7	274.4	22.8	1	46	60	1710	67	
19 Kaskakia River near Venedy Station, IL	11378	2.0	35.8	53.8	109.3	1379.0	145.5	5	81	124	2590	157	
20 Mississippi River at St. Louis, MO	1805222	1166.7	3808.6	4898.8	6154.1	29732.7	3841.3	21	217	340	6720	375	
21 Salinas River near Spreckels, CA	10764	0.0‡	16.4	0.2	9.3	1121.3	42.6	1	36	306	24000	1273	
22 Sacramento River at Sacramento, CA	60883	112.4	358.2	461.6	656.7	2797.7	495.7	8	47	75	1960	88	

† Min Q=0.001  $m^3 s^{-1}$

‡ Min Q=0.01  $m^3 s^{-1}$

652 **Figure captions**

653

654 Figure 1 – Sediment storage index  $S(t)$  calculated from discharge values  $Q(t)$  and  $Q_0$ , in an application  
655 to the Hocking River at Athens, Ohio (station 7, Table 1).

656

657 Figure 2 – Simplified flow chart of the fitting procedure.

658

659 Figure 3 – Relative variation and dispersion of the fitted coefficients in  $C=aQ^b$  (a, b, d, e, g, h) and  
660  $C=aQ^b+a_1\delta S$  models (c, f, i) for increasing values of the coefficient of determination. Dispersion  
661 envelopes correspond to the 100% confidence intervals. Lines in the middle of the envelopes show  
662 averages over all stations listed in Table 1.

663

664 Figure 4 – Ratios and dispersion of calculated on exact SPM fluxes obtained from the fitted  $C=aQ^b$  (a)  
665 and  $C=aQ^b+a_1\delta S$  models (b) without any criterion for the coefficient of determination. *These results*  
666 *were obtained from daily samplings over the entire sampling periods at each USGS station, i.e.,*  
667 *station-specific time periods.* Dispersion envelopes correspond to the 100% confidence intervals.  
668 Lines in the middle of the envelopes show the averages over all stations listed in Table 1.

669

670 Figure 5 – Ratios of calculated on exact SPM fluxes (a, b), medians of the absolute errors of these  
671 ratios (c, d) and statistics for the absolute errors of these ratios for sampling periods  $\Delta t \sim 30$  days (e,  
672 f). The results are obtained from the fitted  $C=aQ^b$  and  $C=aQ^b+a_1\delta S$  models for various combinations  
673 involving the number (n) of available concentration data, the total time period for data collection (T)  
674 and the sampling period ( $\Delta t$ ) for concentration data.

675

676 Figure 6 – Dispersion envelopes for the ratios of calculated and exact SPM fluxes obtained from  
677 uncertain and infrequent concentration data by fitting  $C=aQ^b$  (a) and  $C=aQ^b+a_1\delta S$  models (b). These

678 results were obtained over the entire [sampling period at each USGS station, i.e., station-specific time](#)  
679 [periods](#). These dispersion envelopes correspond to the 100% confidence intervals averaged over the  
680 entire dataset listed in Table 1. Each station has been affected with systematic errors in discharge,  
681 random errors in concentration, and increased sampling periods.

682

683 Figure 7 – Dispersion envelopes (100% confidence intervals) for the ratios of exact on calculated SPM  
684 fluxes obtained from uncertain and infrequent concentration data by fitting the  $C=aQ^b+a_1\delta S$  [model](#)  
685 (a). Application to sediment exports from French rivers, where inner and outer circles bound the real  
686 SPM values (b).

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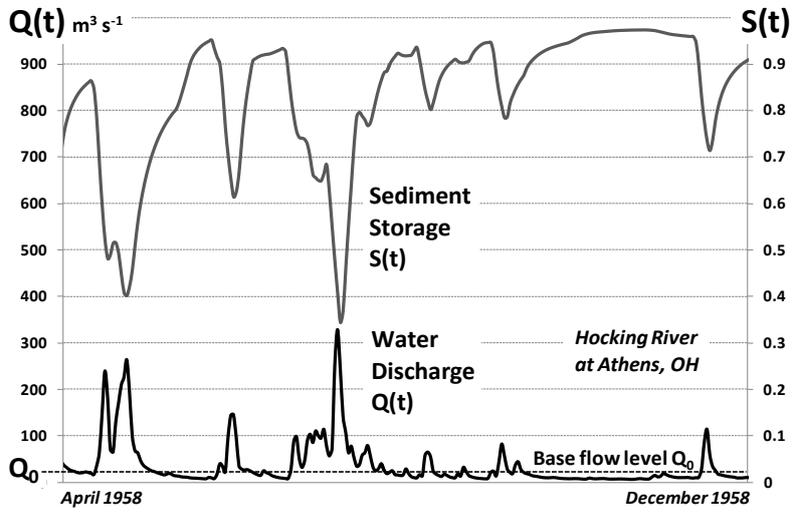
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689 **List of Figures**

690

691 Figure 1:

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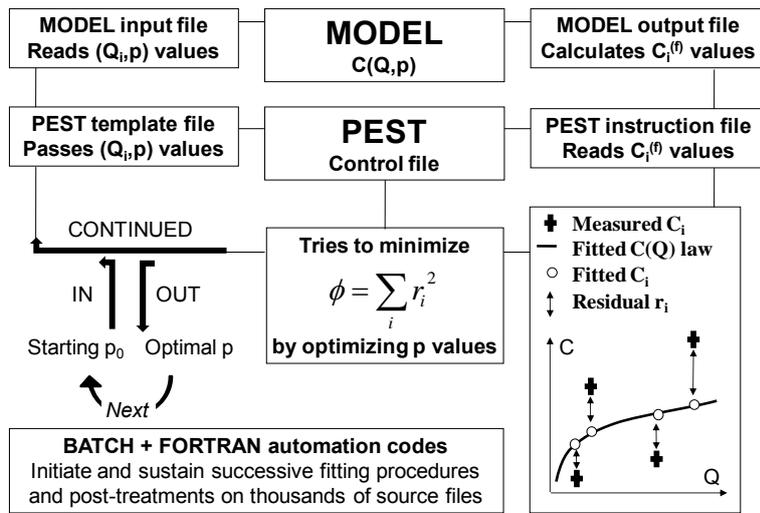


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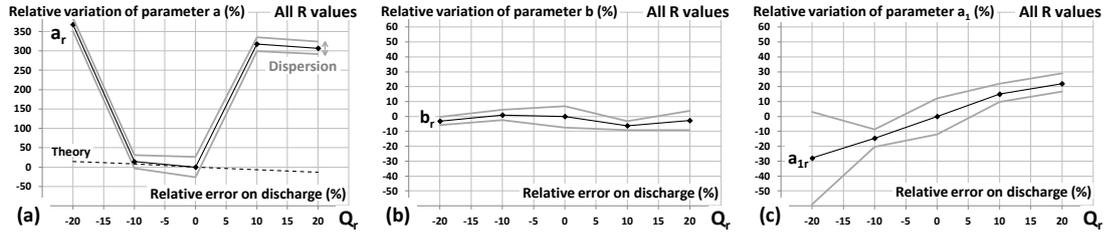
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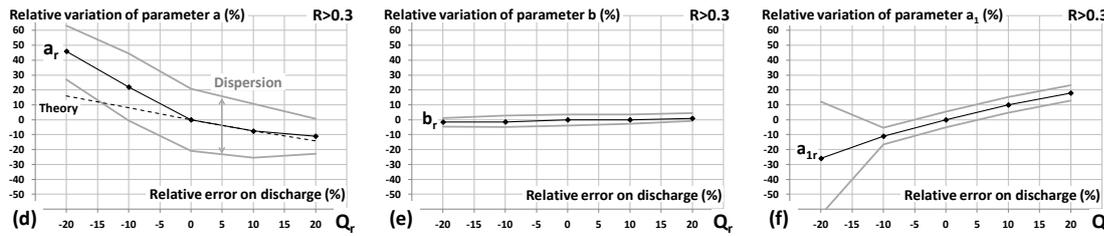
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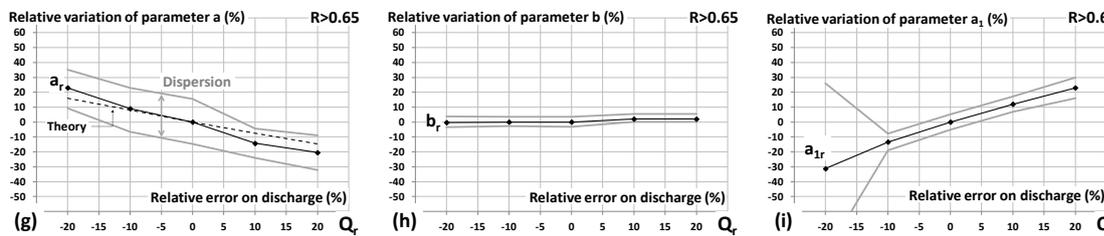
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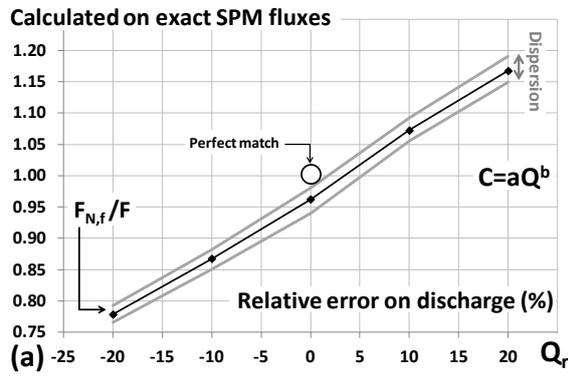
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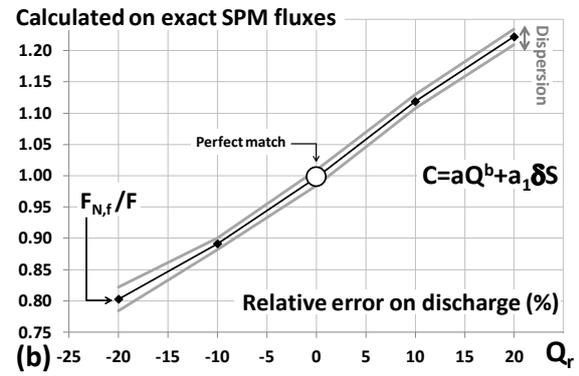
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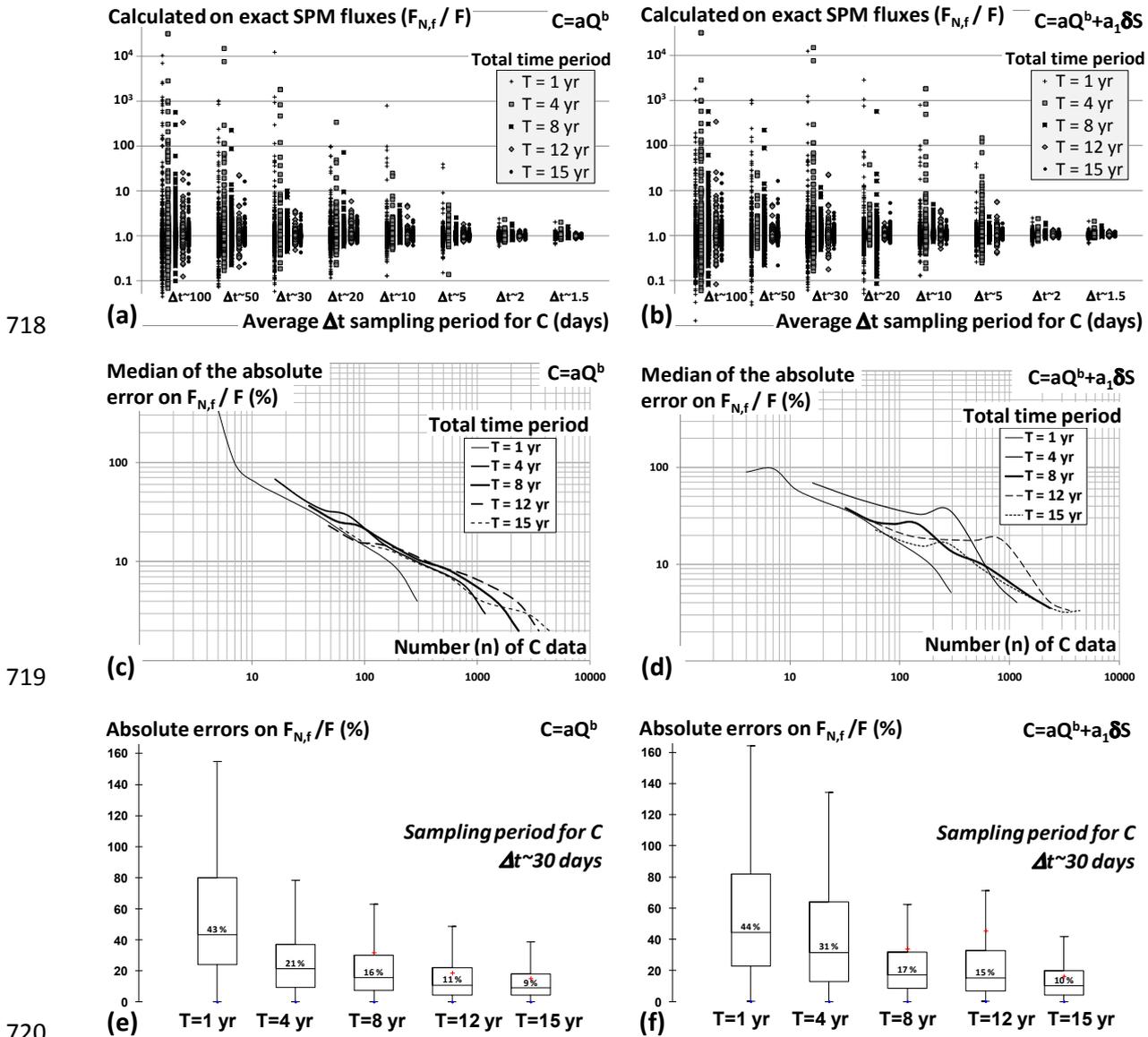
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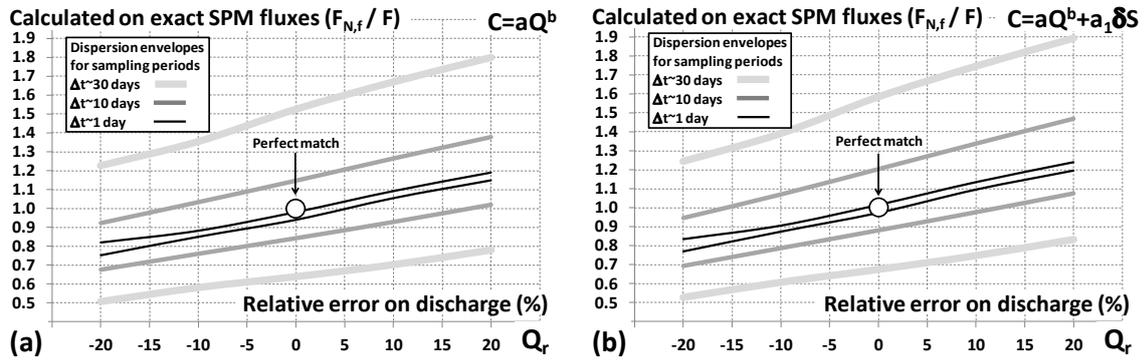
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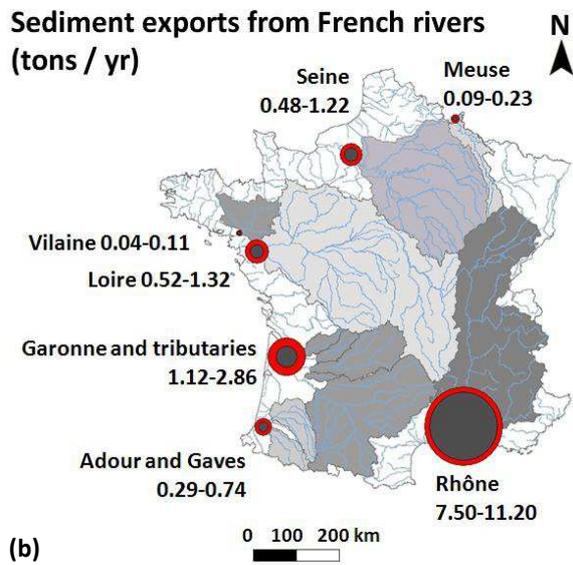
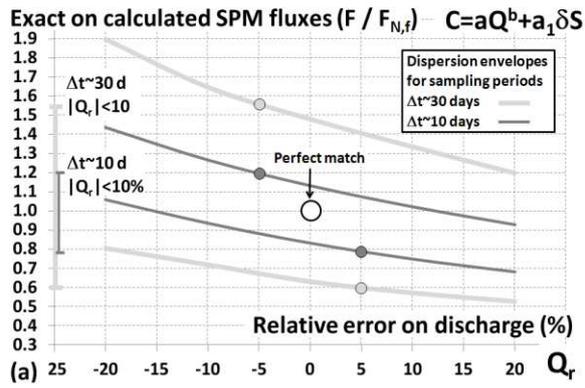
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723 Figure 6:  
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728 Figure 7:  
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