Introducing minimum energy tool path in 5-axis flank milling

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Abstract: This paper deals with a surface based-approach for 5-axis flank milling of complex surfaces. The surface representation of the tool trajectory allows globally minimizing the geometrical deviations between the machined surface and the designed surface. However, within the context of high-speed machining, the smoothness of the calculated tool trajectory is essential to ensure high performance machining. Indeed, oscillatory trajectories may penalize process efficiency. Taking advantages of the surface based approach, the smoothness of the trajectory can be controlled through the estimation of the energy of deformation of the tool path surface. Minimum energy tool paths lead to minimal machining time. As geometrical deviations are not minimised for minimum energy tool paths, a compromise must be done to find the best solution.

Keywords: Flank milling, minimum energy, High Speed Milling, Machining Surface

1 Introduction

Due to many advantages, such as high removal material rate, or better surface roughness, 5-axis flank milling has become very popular compared with point milling. In particular, this process is now widely used for the machining of slender complex parts like impellers or turbine blades. However, process advantages now motivate industrials to apply high-speed flank milling to any types of surfaces, leading up to new challenges for tool path calculation. Over the last decade, several positioning strategies for flank milling have been developed with the objective of minimizing geometrical deviations. Studies mainly focused on the machining of ruled surfaces. Most flank milling methods rely on a first tool positioning on the surface at one or more points, followed by the tool positioning optimization to reduce geometrical errors [Menzel et al., 2004]. Over the years, methods evolved by increasing the number of contact points with the objective of deviation minimization to the detriment of trajectory smoothness and computation time. Lui [Liu, 1995] developed two methods to machine non-developable ruled surfaces with hemispherical tools. The first one, the Simple Point Offset (SPO), consists in positioning the tool collinear to the surface rule passing through a point offset at the mid curve. Geometric errors are thus divided into overcut and undercut. The second method is the Double Point Offset (DPO) method. Two contact points located at the parametric values 0.25 and 0.75 of the rule are offset by a value equal to the too radius along the surface normal. Redonnet et al., [Redonnet et al., 1998] proposed a positioning strategy based on 3 contact points. The axis of a cylindrical tool is initially positioned collinear to the rule at an offset distance value equal to the tool radius. A rotation around the normal at the surface at the mid-curve is applied so that the tool becomes tangent to the two directrices while preserving a contact point with a rule. This leads to a system of seven transcendental equations that must be solved. Improvements of the method was proposed in [Monies et al., 2000] [Senatore et al., 2007]. The positioning method developed by Bedi et al, [Bedi et al., 2003] consists in positioning the tool tangent to the two boundary curves of the ruled surface. As contact points are limited to the boundaries, the maximum error is located at the mid-curve. To reduce such errors, an optimization is developed in [Menzel et al., 2004]. Gong developed the Three Point Offset method (TPO) for flank milling using a cylindrical tool [Gong et al., 2005]. The set of tool axes are interpolated to obtain the tool axis trajectory surface. A simple least square approximation scheme is developed to make the tool axis trajectory surface best fits the offset surface of the surface to be machined.

Although the control of geometrical deviations is still of major importance, the smoothness of the calculated tool trajectory is essential to ensure high performance machining. In previous works, we have proposed a surface based approach for tool positioning in flank milling [Lartigue et al., 2003]. The method relies on a first positioning of the tool according to a two contact point strategy. Extremity points of the set of tool axes are approximated by two curves, which thus define the directrices of a ruled surface, also called the machining surface. To improve tool positioning, the machining surface can be deformed. In this direction, the minimisation of the deviations between the envelope surface of the tool movement and the surface to be machined has been performed. This minimisation is simply carried out according to the least-square criterion, by the displacement of the control points of the d-curves (directrices of the ruled surface).

However, as for methods based on local tool positioning, resulting trajectories may be oscillatory even folded-in, leading to process inefficiency. The smoothness of the trajectory can be controlled through the calculation of the energy of deformation. Indeed, a trajectory which minimises its energy then minimises its curvature variations, and as a result acceleration and jerk variations are decreased during trajectory processing. Taking advantages of the surface representation, the trajectory smoothness can be easily controlled through the evaluation of the energy of deformation. Therefore, the issue of minimizing geometrical deviations between the envelope surface and the designed surface, coupled to the trajectory smoothing is investigated in this paper.

2 Generation of optimised Flank milling trajectories

The method for Generation of optimised 5-axis Flank milling trajectories (Geo5XF) is an extension of the method developed in [Lartigue et al., 2003] and consists of 4 main steps (Fig.1): Initial tool positioning; Approximation of the tool axis extremities by two curves (defining the two directrices); Calculation of the deviations between the envelope surface and the designed surface; Positioning optimisation by deforming the machining surface (MS).

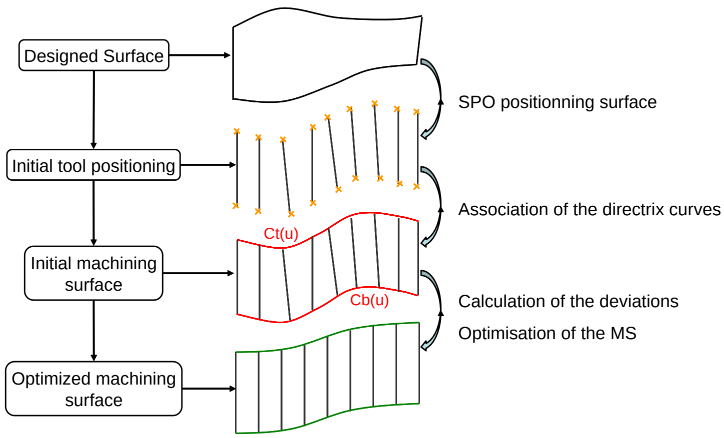

Figure 1: Geo5XF approach

The initial tool positioning is carried out using the SPO positioning method [Liu, 1995] for its robustness and simplicity to implement. Indeed, whatever the nature of the surface to be machined this method provides a result always exploitable in a minimum computation time. Cubic B-spline curves are then associated to the point extremities of the set of axes according to the least-square criterion. These two curves define the directrices of a ruled surface called the machining surface (Fig.2), the equation of which is:

$$MS(u, v) = (1 - v) \cdot Cb(u) + v \cdot Ct(u) \tag{1}$$

where $Cb(u)$ and $Ct(u)$ are the bottom curve and the top curve respectively.

Geometrical deviations are calculated according to the method proposed in [Lartigue et al., 2003]. The Machining Surface is sampled into a set of $A_k(u^*)$ points. Each point $A_k(u^*)$ is normally projected onto the designed surface, leading to $B_k(u^*)$ which is then projected onto the considered tool axis, giving the point $B_k^p(u^*)$. The last projection intersects the tool surface at a point $C_k(u^*)$. Therefore, the geometrical deviation $\xi_k(u^*)$, between the envelope surface and the designed surface at the point $B_k(u^*)$ is given by (Fig.3):

$$\xi_k(u^*) = \overrightarrow{C_k(u^*)B_k(u^*)} \cdot \overrightarrow{n}_k(k, u^*) \tag{2}$$

where $n_k(k, u^*)$ is the normal to the machining surface $MS(u, v)$.

The optimisation of the tool positioning consists in applying a deformation to the envelope surface so that deviations $e_k(u^*)$ between the envelope surface and the designed surface are minimised. To simplify the evaluation of the envelope surface only static instances of the tool movement are considered. Each static instance is defined by a value u^* of the u-parameter (Fig.2), and the optimisation of the tool positioning is obtained by applying a small displacement $D_{C_k(u^*)}$ to the tool axis [Bourdet et al., 1996]:

$$e_k(u^*) = \xi_k(u^*) - \overrightarrow{D}_{C_k(u^*)} \cdot \overrightarrow{n}_k(k, u^*) \tag{3}$$

Considering that $\overrightarrow{D}_{C_k(u^*)} = \overrightarrow{D}_{B^p_k(u^*)}$, and by expressing $\overrightarrow{D}_{B^p_k(u^*)}$ in function of the displacement of the control points of both directrices, this yields to:

$$\begin{aligned} e_k(u^*) = \xi_k(u^*) - \left(1 - \frac{k_p(u^*)}{h(u^*)}\right) \cdot \sum_{l=0}^{n1} \left[N_{l3}(u^*) \cdot \overrightarrow{\delta Cb_l}\right] \cdot \overrightarrow{n}_k(k, u^*) \\ + \left(\frac{k_p(u^*)}{h(u^*)}\right) \cdot \sum_{m=0}^{n2} \left[N_{m3}(u^*) \cdot \overrightarrow{\delta Ct_m}\right] \cdot \overrightarrow{n}_k(k, u^*) \end{aligned} \tag{4}$$

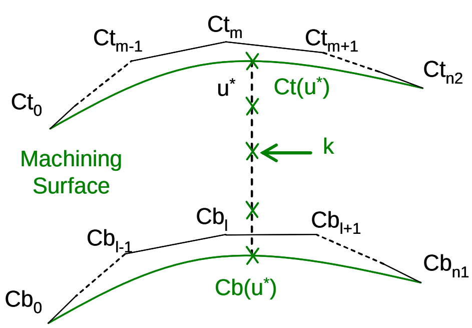

Figure 2: Directrices of the MS

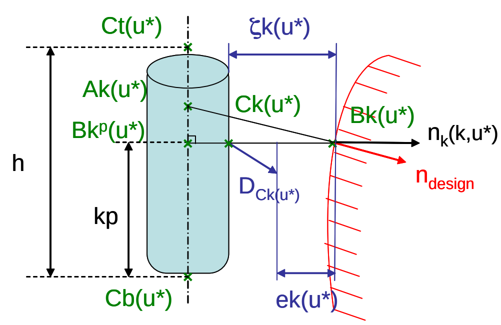

Figure 3: Geometrical deviations

As the objective is to find the optimised machining surface MS_o so that the envelope surface best fits the set of $B_k(u^*)$ points (designed surface), the least-square criterion is used, leading to the following optimization scheme:
Find $\overrightarrow{\delta Cb_l}$ and $\overrightarrow{\delta Ct_m}$, control point displacements of the MS directrices so that $W = \sum_{u^*} \sum_k e_k^2(u^*)$ is minimised.
This leads to solve a large linear system, $A \cdot x = b$ with $3 \cdot (n1 + n2)$ equations, with n_1 and n_2 the number of control points of the bottom curve and the top curve respectively.

3 Minimum energy trajectories

To solve the linear system, the Singular Value Decomposition (SVD) is used leading to an estimate of the solution. SVD gives an estimate of a given matrix by a lower rank matrix of same dimensions. The method consists in the decomposition of a rectangular matrix into a product of three matrices, two orthogonal matrices and a diagonal matrix:

$$A = U \cdot \Sigma \cdot V^T \tag{5}$$

If A is a positive semi-definite matrix, the pseudo-inverse of A is obtained from the eigenvalues λ_i of A as follows:

$$\hat{A}^{-1} = V \cdot \hat{\Sigma}^{-1} \cdot U^T \quad \text{avec} \quad \hat{\Sigma}^{-1} = \begin{bmatrix} \frac{1}{\lambda_1} & & & & & \\ & \ddots & & & & \\ & & \frac{1}{\lambda_k} & & & \\ & & & 0 & & \\ & & & & \ddots & \\ & & & & & 0 \end{bmatrix} \tag{6}$$

Therefore, the estimate of the solution depends on the rank k of the pseudo-inverse matrix (limit value of the eigenvalues). The rank determines the level of approximation.

As mentioned previously, the smoothness of the trajectory is an important criterion for high performance machining within the context of HSM. Machine tool solicitations are decreased for trajectories at least continuous in curvature [Dugas et al., 2003]. Within the framework of curve and surface fittings, many authors use the energy of deformation as a criterion to control element smoothness [Faux and Pratt, 1979]. The work reported in [Wang et al., 1997] shows in particular that the energy is the most essential criterion when fitting curves and surfaces to clouds of points. As the tool trajectory is represented as a surface, the evaluation of the smoothness through the energy seems to be relevant. For this purpose, the calculation of the energy of deformation as proposed in [Wang et al., 1997] is adopted in the present work.

$$E = \iint_D \left(\left|\frac{\partial^2 S}{\partial u^2}\right|^2 + 2 \cdot \left|\frac{\partial^2 S}{\partial u \partial v}\right|^2 + \left|\frac{\partial^2 S}{\partial v^2}\right|^2 \right) dudv \tag{7}$$

To each estimate of the solution, the energy of deformation is calculated with the objective of finding a solution that gives minimal geometrical deviations while preserving a correct smoothness of the trajectory. This point is investigated through various examples in the next section.

4 Experimental investigations

Various surfaces are proposed in literature to compare flank milling positioning algorithms. Among these surfaces, two test surfaces are selected, both non developable ruled surfaces; the first one is the Two flipped surface defined in [Menzel et al., 2004], and the second one is named Liu as it was introduced by Liu in [Liu, 1995](Fig.4) (Fig.5). The efficiency of our approach is also illustrated through the machining of an impeller, which is characteristic of the slender complex surfaces industrials attempt to flank mill. For each test surface, minimal and maximal deviations obtained using the different methods are given and compared to the optimal solution given by Geo5XF. In addition, the kinematical behaviour during machining is also investigated through the relative tool/surface velocity.

4.1 Surfaces ”Liu” and ”Two flipped”

Geometric deviations obtained using the various positioning algorithms are reported in table1. Concerning Geo5XF, deviations are the result of a N-Buffer simulation of the trajectory that minimises the squared deviations between the designed surface and the machined surface. It can be noticed that Geo5XF method gives very satisfactory results for both test surfaces. Results are close to the optimal values obtained with Menzel' method. This solution obtained with all the eigenvalues is optimal as regards geometrical deviations.

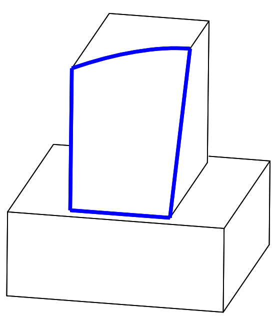

Figure 4: Surface ”Liu”

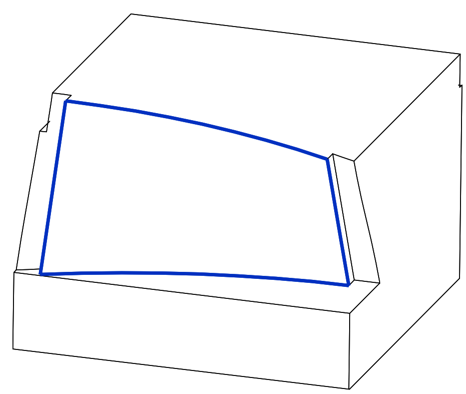

Figure 5: Surface ”Two flipped”

Test surface	”Liu”				”Two flipped”		
Algorithms	Liu	Bedi	Menzel	Geo5XF	Bedi	Menzel	Geo5XF
Undercut (mm)	0.582	2.2393	0.2644	0.12	0.2876	0.0061	0.0086
Overcut (mm)	0.585	0	0.2114	0.27	0	0.0091	0.016

Table 1: Geometrical deviations

Test surface	"Liu"			"Two flipped"		
Eigenvalues	λ_1	λ_2	λ_3	λ_1	λ_2	λ_3
Undercut (mm)	1.7	0.12	0.12	0.57	0.0086	0.0086
Overcut (mm)	0.19	0.32	0.27	0.035	0.016	0.016
Energy of deformation (10^{-6})	56.1	207.2	233.3	137.2	227.8	721.9

Table 2: Geo5XF results

To study the influence of the energy of deformation on kinematical performances three Machining surfaces are tested, corresponding to three estimates of the solution (3 pseudo solution for 3 different eigenvalues); the minimum energy solution (corresponding to λ_1), the minimum deviation solution (corresponding to λ_3) and a compromise between energy and deviation, which locally minimises the energy of deformation (corresponding to λ_2). In table 2, maximal and minimal deviations as well as the energy of deformation are reported for the 3 cases.

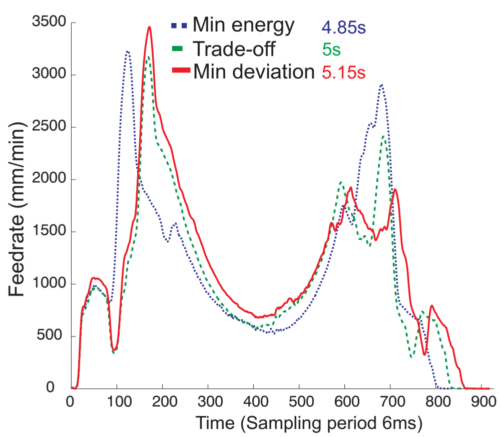

Figure 6: Effective feedrate (Two F.)

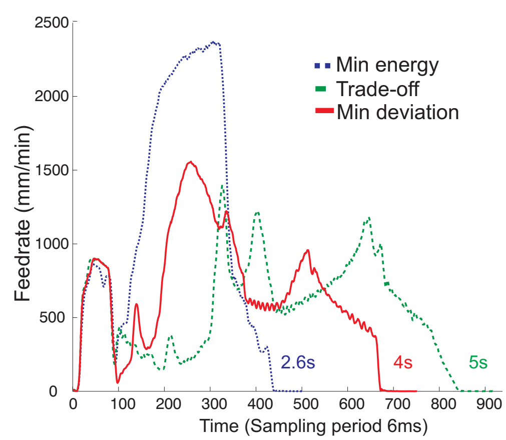

Figure 7: Effective feedrate (Liu)

The tool path is carried out using a cylindrical tool with a tool radius equal to 20mm, a programmed feedrate of 5m/min and a spindle speed of 18000tr/min. Effective feedrates are measured during actual machining and are reported in Fig.6 and Fig.7. Whatever the test surface, graphs show that machining time is minimal for the minimum energy solution. Minimum energy tool paths lead to high performance process. As the "Two Flipped" surface is concerned, machining time for the compromise is less than machining time for the minimum deviation solution, which is consistent with the energy criterion. Energy of deformation seems to be relevant to characterize trajectory smoothness which affects machining time. Nevertheless, this criterion only characterizes the smoothness in the part coordinate system and as a result, does not take into account changes of solutions during the inverse kinematical transformation (IKT). This is highlighted with the example of the Liu's surface (Fig.7). Indeed, the evolution of machining time is no more consistent with the evolution of the energy : despite a lower energy, machining time for the compromise is higher than with the minimum deviation solution. Smoothness in the part coordinate system does not ensure smoothness of the effective trajectory processed in the machine coordinate system.

4.2 Impeller

The next application concerns an indutrial case: the flank milling of an impeller blade surface which is made of several patches (Fig.8). Table 3 gathers results obtained with the Geo5XF appraoch. Note that, the calculated tool trajectory allows the machining of both sides of the part.

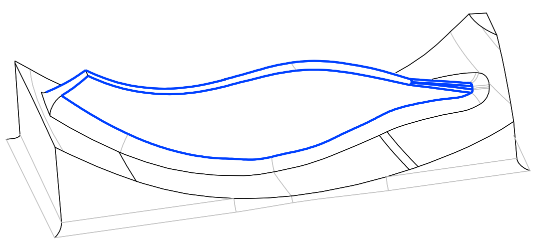

Figure 8: Surface impeler

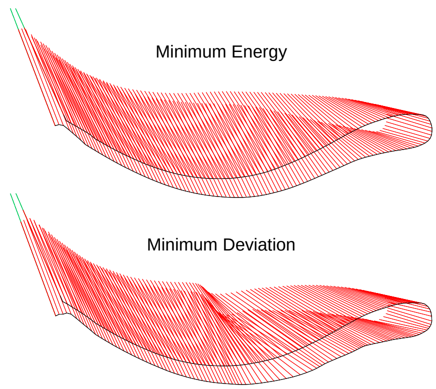

Figure 9: Tool path

Test surface	”Impeler”		
Eigenvalues	λ_1	λ_2	λ_3
Undercut (mm)	0.63	0.19	0.2
Overcut (mm)	0.8	0.42	0.39
Energy of deformation (10^{-3})	142.7	384.6	693.3

Table 3: Geo5XF results

The tool trajectory is calculated (Fig.9) with a 6mm cylindrical tool, a programmed feedrate of 5m/min and a spindle speed of 18000tr/min. Fig.10 shows effective feedrates for three different tool paths: the tool path that minimises the geometrical deviations (in red), the one that minimises the energy of deformation (in blue) and a tool path resulting from a trade-off between deviations and energy of deformation. Since the IKT does not introduce space of solution swapping, results are consistent with expectations. Indeed, the fastest tool path is the smoother and vice et versa. The area for which the feedrates are the lowest is located at the leading edge when passing from the internal face to the other which is also consistent with well-known results. The three machined surfaces are presented (Fig.11) to visualize the influence of the energy on the tool path. Marks on the part confirm the smoothness effect of the energy parameter.

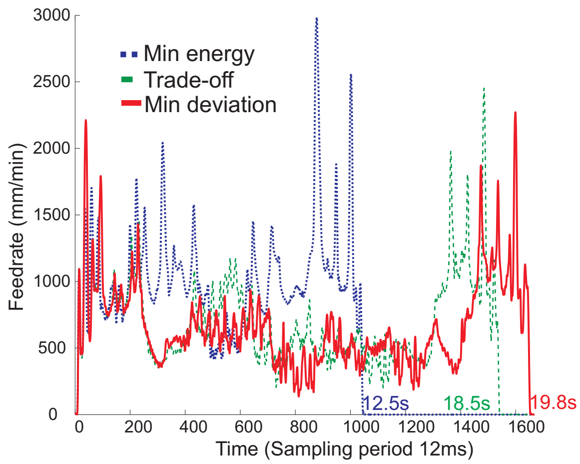

Figure 10: Effective feedrate (impeler)

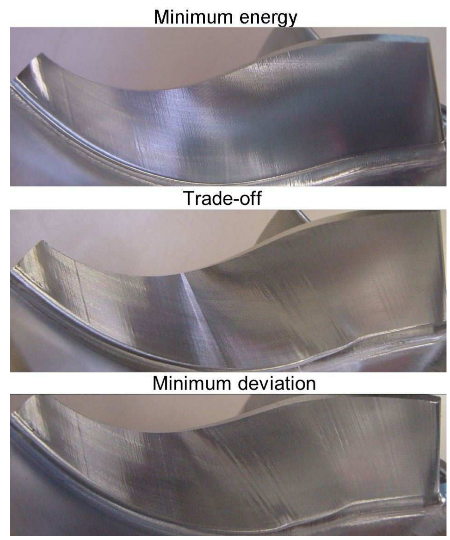

Figure 11: machined surfaces

5 Conclusion

In this paper, we proposed to integrate the smoothness of the tool path in 5-axis flank milling. Indeed, local tool positioning based methods may introduce oscillations in the tool path, leading to process inefficiency. With the surface approach proposed by the Geo5XF method we have developed, we can control the smoothness of the tool path which is a key parameter as regards the trajectory processing by the numerical controller during machining. Smoothness is evaluated through the energy of deformation of the machining surface, the ruled surface containing the tool axes. Energy of deformation can be modified by determining approximate solutions of the system that minimises the squared differences between the envelope surface and the machined surface. These solutions are compromises between geometrical deviations and smoothness of the tool trajectory. Results show that the smoothest paths are the fastest, which is consistent with results proposed in literature. Indeed, curve that minimises its deformation energy minimises its variations of curvature, thus the variations of acceleration and jerk. We can now look forward to deal with the smoothness in the joint coordinate system of the machine taking into account the various types of solutions and singularities.

6 Acknowledgements

This work is funded by the company ”Missler Software” global supplier of CAD, CAM and ERP solutions.

References

Bedi, S., Mann, S., Menzel, C. Flank milling with flat end milling cutters, Computer-Aided Design 35, 293 – 300, 2003.

Bourdet, P., Mathieu, L., Lartigue, C., Ballu, A. The concept of small displacement torsor in metrology, Advanced Mathematical Tools in Metrology II, Series on advances in mathematics for applied sciences 40, 22 – 100, 1996.

Dugas, A., Lee, J., Hascoet, J.-Y. An enhanced machining simulator with error analysis, Int. Journal of Manufacturing System 21(6), 451–564, 2003.

Faux, I., Pratt, M. Computational geometry for design and manufacture, 1979.

Gong, H., Cao, L.-X., Liu, J. Improved positioning of cylindrical cutter for flank milling ruled surfaces, Computer-Aided Design 37, 1205– 1213, 2005.

Lartigue, C., Duc, E., Affouard, A. Tool path deformation in 5-axis milling using envelope surface, Computer-Aided Design 35, 375– 382, 2003.

Liu, W.-W. Five-axis nc cylindrical milling of sculptured surfaces, Computer-Aided Design 27 (12), 887 – 894, 1995.

Menzel, C., Bedi, S., Mann, S. Triple tangent flank milling of ruled surfaces, Computer-Aided Design 36, 389– 296, 2004.

Monies, F., Redonnet, J.-M., Rubio, W., Lagarrigue, P. Improved positioning of a conical mill for machining ruled surfaces: application to turbine blades, Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture 214/7, 625–634, 2000.

Redonnet, J., Rubio, W., Dessein, G. Side milling of ruled surfacesoptimum positioning of the milling cutter and calculation of interference, The International Journal of Advanced Manufacturing Technology 14 (7), 1998.

Senatore, J., Monies, F., J-M.Redonnet, Rubio, W. Improved positioning for side milling of ruled surfaces: Analysis of the rotation axis's influence on machining error, International Journal of Machine Tools and Manufacture 47/6, 934–945, 2007.

Wang, X., Cheng, F., Barsky, B. Energy and b-spline interproximation, Computer-Aided Design 29 (7), 485 – 496, 1997.