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Revision of time distortion effects through new Doppler approaches

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Abstract

Time dilation is considered as a pure relativistic phenomenon not deduced from wavelength distortion. Alternatively, it is suggested here that time distortion originates from wavelength distortion in all contexts including special relativity. A link between the special relativistic time and the quantum of time is first established to propose that wavelengths are adjustable uncertainty parameters allowing to maintain the invariance of the light speed. The light clock of Einstein is then used to develop a Doppler analysis calculated in the frame of an external observer and transformed to cancel the false effect between comoving points. This approach yields a new Doppler formula with remarkable properties. Contrary to the previous Doppler equations characterized by their asymmetry, the new formula gives geometrically symmetrical Doppler effects in the whole space in front and behind the closest point from the source, whose center of gravity corresponds precisely to a global inter-frame dilatation factor $\frac{1}{\sqrt{1 - (v/c)^2}}$. The direct relation between wavelengths and time distortion effects is then examined in other fields unrelated to uniform motion. A new and concrete solution to the round trip (twin paradox) is proposed using the classical Doppler effect, that is not the accepted solution of special relativity. Finally, an unified energetic view of wavelength distortion is shown to apply to all the cases of time distortion, irrespective of whether they are related to motion, such as the special relativistic and the round trip dilation, or not, such as the gravitational redshift.

1 Introduction

Time dilation, the keystone of special relativity, is conceived as a pure relativistic phenomenon. The time between wave crests is just expected to comply with this time dilation [1]. The inverse view proposed here is that time distortion results from wavelength distortion. Special relativity does not escape the fundamental parallel between time and wavelengths which is much more general than special relativity. A new equation based on wave distortion and using the classical Doppler formula, is presented. It is very similar to the relativistic one and is obtained by cancelling the virtual Doppler effect generated by spacetime stretching. Then, the classical Doppler formula is successfully applied to differential ageing phenomena: the round-trip (better known as the twin paradox) and gravitational time dilation.

2 Slowing time by stretching the waves

The most striking evidence that distortions of wavelengths exactly correspond to time distortions has not been provided by a Doppler effect, but by the cosmological redshift, to which the relativistic Doppler formula does not apply because many recessional speeds exceed the speed of light. Supernovae are star explosions remaining extremely bright for a few weeks, a short duration at the cosmological scale. This time window is relatively constant for comparable supernovae, but astronomers made a remarkable discovery: the apparent time window of brightness depends on the distance of the supernova, in exactly the same proportion that their redshift. For instance, a distant supernova with a redshift of $\frac{\lambda_{app}}{\lambda} = 1.5$, has precisely a 1.5-fold longer duration of brightness [2]. This observation, supported by others, allows to define the first principle of this study:

Principle: Apparent time dilation corresponds to wavelength increase, or equivalently frequency frequency decrease, whereas apparent time contraction corresponds to wavelength shortening and frequency increase.

The decrease or increase of wavelengths is evaluated by comparison with their standard values measured in the comoving frame. Standard frequencies are for example given by atomic rays which are identical in all inertial frames, viewed by comoving inertial observers. Strangely, although the Doppler effect is clearly a phenomenon of wavelength distortion, the scientific community did not establish a clear relationship between time dilation and a Doppler effect, perhaps because time dilation depends only on relative speeds whereas a Doppler effect is orientation-dependent. Certain [3], but not all [4], authors stated that there is no necessary relation at all between the relativity theory and the Doppler ef-
fect. Using the light clock of Einstein, it will be shown relativistic time dilation is analogous to a generalized Doppler effect. While doing so, a new and thorough theoretical Doppler equation will be obtained. The famous light clock of Einstein is a universal clock that is not based on an atomic ray, but on the constancy of light velocity.

## 3 The light clock of Einstein

The beat of the light clock of Einstein is the rebound of a photon between facing mirrors. This clock is placed vertically in a wagon rolling at constant speed $v$. For an external observer, the light path appears oblique when the train moves, whereas for an observer located inside the train, it appears always vertical (Fig.1). The reference time interval $\Delta t$ corresponds to the frame comoving with the standard clock.

![Figure 1: The famous triangular time-space diagram of the moving Einstein's clock. The horizontal scale is artificially stretched relatively to the vertical one for better visualization.](image)

With respect to the angle $\theta$ of Fig.1, elementary trigonometry says

$$\sin \theta = \frac{c\Delta t}{c\Delta t_{\text{mov}}} = \frac{\Delta t}{\Delta t_{\text{mov}}}$$ (1a)

and

$$\cos \theta = \frac{v\Delta t}{c\Delta t_{\text{mov}}} = \frac{v}{c}$$ (1b)

which immediately gives, using the relationship $\sin^2 \theta + \cos^2 \theta = 1$,

$$\frac{\Delta t_{\text{mov}}}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$ (1c)

Another straightforward way to obtain this result is to use the powerful geometrical tool of special relativity: the Minkowski spacetime. The oblique (hypotenuse) and vertical paths of light are necessarily identical in the Minkowski spacetime because light starts from and arrives to the same points. This common spacetime interval should reconcile the point of view of an observer in the train, for whom the clock appears immobile, and that of an outside observer for whom there is an additional translation. Hence,

$$\Delta s^2 = (c\Delta t)^2 = (c\Delta t_{\text{mov}})^2 - \Delta v_{\text{mov}}^2$$ (2a)

giving

$$c^2 \left( \frac{\Delta t}{\Delta t_{\text{mov}}} \right)^2 = c^2 - \left( \frac{\Delta v_{\text{mov}}}{\Delta t_{\text{mov}}} \right)^2$$ (2b)

and finally

$$\frac{\Delta t_{\text{mov}}}{\Delta t} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$ (2c)

### Principle:

In spacetime diagrams, the proper time of the clock is obtained when the distance to be crossed by light appears minimal.

In triangular schemes like Fig.1, the proper time of the clock corresponds to the vertical path ($\Delta t$), whereas for the external observer moving at speed $v$ relatively to the clock this path appears stretched by $\Delta t_{\text{mov}}/\Delta t = 1/\sqrt{1 - (v/c)^2}$.

## 4 Linking the relativistic and the quantum times

### 4.1 The quantum of time

The apparent paradox of special relativity is that the hypotenuse and the vertical side of the triangle shown in Fig.1 are simultaneously crossed by light at the same speed $c$. To maintain this somewhat magical constancy, there is a very simple solution which consists in modifying the fundamental components of $c$. The speed of light can be written

$$c = \lambda \text{ meters/T seconds}$$ (3)

where the spatial unit is the wavelength $\lambda$ and the time unit is the period $T$. A quantum of time $\varepsilon$ can be defined, which is not an absolute value but depends on the energetic status of the system. It is the time necessary to cross the length unit below which successive configurations cannot be distinguished because of the uncertainty principle. For a thermodynamic system, the length unit is the thermal wavelength of de Broglie $\lambda$ and the mean particle velocity is obtained by averaging the Maxwell velocities distribution, which gives [5]:

$$\varepsilon = \frac{\lambda}{4k_B T} = \frac{\hbar}{4k_B T} = 4 \times 10^{-14} \text{ s at } T = 300 \text{ K}$$ (4)

The corresponding value for photons ($k_B T = h$), is very simple

$$\varepsilon = \frac{1}{4v} = \frac{\lambda}{4c} = \frac{T}{4}$$ (5)

The time quantum is a quarter of period because a period can be subdivided into 4 identical and indivisible motifs ($\pi/2$ windows). Furthermore, the 4 is imposed as a condition for recovering the undulatory behaviour of light from the definition of $\varepsilon$ as an “uncertainty window”. Indeed, the conception of $\varepsilon$ as the minimal time
interval during which no evolution can be perceived, suggests that its evolution obeys a delay differential equation of the form

$$\frac{dy(t)}{dt} = -ky(t - \varepsilon)$$  \hspace{1cm} (6a)

of which an undulatory solution is

$$k = \frac{\pi}{2\varepsilon}$$  \hspace{1cm} (6b)

and

$$y(t) = A \sin \left( \frac{\pi}{2\varepsilon} t \right)$$  \hspace{1cm} (6c)

In statistical mechanics,

$$\frac{\pi}{2\varepsilon} = \frac{k_B T}{\hbar}$$  \hspace{1cm} (6d)

(where T is the temperature) and for photons, if adding to this pulsation a uniform spatial translation along x at speed $\lambda/4\varepsilon$

$$y(x, t) = A \sin \left( \frac{\pi}{2\varepsilon} \left( \frac{4\varepsilon}{\lambda} x - t \right) \right)$$  \hspace{1cm} (6e)

which is the traditional light wave function

$$y(x, t) = A \sin 2\pi \left( \frac{x}{\lambda} - \frac{t}{T} \right)$$  \hspace{1cm} (6f)

(where $T$ is the period). Time and energy can be adjusted in time-energy boxes of uncertainty $\Delta t \Delta E \geq \hbar/4\pi$, provided maintaining invariants.

**Principle**: The velocity of light and the number of time units can be preserved from all viewpoints by compressing or dilating the waves.

### 4.2 Stretching the waves

The Einstein clock gives

$$\Delta t_{\text{mov}} > \Delta t$$  \hspace{1cm} (7a)

Since the two paths of the light are the same, the number of periods, say n, is preserved so that Eq.(7a) is equivalent to

$$nT_{\text{mov}} > nT$$  \hspace{1cm} (7b)

Hence

$$\nu_{\text{mov}} < \nu$$  \hspace{1cm} (7c)

a phenomenon of wave stretching is predicted to compensate spacetime dilation in such a way that (i) the speed of light and (ii) the number of periods, are both maintained identical (Fig.2). According to this scheme, relative times simply emerge from relative frequencies. It is interesting to note that the Doppler effect obtained here by stretching a fictitious space, somewhat resembles the cosmological redshift [6]. In this respect, special relativity appears as a phenomenon of virtual space expansion.

**4.3 Time dilation is a perspective effect**

In the experiment of the light clock inside a train described above, the distinction between moving and non-moving frames is irrelevant as they can be permuted. The wagon and the station platform are two equivalent systems of reference and the situation is simply inverted if the clock is put on the platform and if the observer inside the train considers that this is the platform that moves relative to the train in the opposite direction. A perspective effect is naturally reciprocal. The best known perspective effect is the apparent contraction of the size of a person standing far away from us. We are very accustomed to this familiar effect and easily understand that this person has exactly the inverse perception. The distance is a symmetrical notion, as is the uniform motion. The apparent size reduction effect is so well integrated in our mind that it is unconsciously corrected. Moreover, it is used inversely to estimate mentally the distance. The same operation can be adapted to time dilation to deduce the speed of the source from the degree of wavelength stretching. Eq.(2a) can be modified as follows: On the one hand, $c = \lambda/T$ and on the other hand, the time intervals can be replaced by a given number (n) of periods $\Delta t = n\lambda/c$. Since this number is invariant from any viewpoint, Eq.(2a) becomes

$$\Delta x_{\text{mov}} \Delta t_{\text{mov}} = v = c \sqrt{\frac{\lambda_{\text{mov}}}{\lambda} - \frac{\lambda}{\lambda_{\text{mov}}}}$$  \hspace{1cm} (8a)

or using frequencies

$$\frac{v}{c} = \sqrt{1 - \frac{\nu_{\text{mov}}^2}{\nu^2}}$$  \hspace{1cm} (8b)

which can be transformed into

$$\frac{\lambda_{\text{mov}}}{\lambda} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$  \hspace{1cm} (8c)

Eq.(8c) is the typical equation of a Doppler effect. Accordingly, we will see that the time dilation of special relativity can be derived from a virtual Doppler effect.
5 Analogy between time dilation and a Doppler effect.

In spite of its apparent simplicity, this original section is completely heterodox because the relativistic Doppler formula is assumed to be not a matter of compression or elongation of wavelengths \[1,3\]. Conversely, it is suggested here that the phenomenon of time dilation of special relativity is simply based on a wavelength distortion which can be recovered using the classical Doppler formula. A Doppler effect is caused by the velocity of a wave source relatively to an observer. The classical doppler effect shortens the apparent wavelength \(\lambda\) of an object approaching at speed \(v\) such that \(\lambda_{\text{mov}} = cT - vT\) where \(T\) is the period, giving \(\lambda_{\text{mov}}/\lambda = 1 - \frac{v}{c}\). Conversely it stretches the apparent wavelength of a receding object such that \(\lambda_{\text{mov}}/\lambda = 1 + \frac{v}{c}\). In the general case, when the velocity vector is not strictly collinear with the line of sight, these equations should be modified by replacing \(v\) by a smaller value. When using radial velocities which are the orthogonal projection of the velocity vector on the source-observer line \((v \cos \theta)\), there is no transverse effect because when the source is at the closest point from the observer, \(\theta = \pi/2\) and the radial speed is zero. But let us develop a new Doppler approach using the classical Doppler formula but in which radial velocities defined with angles, are replaced by time-dependent Doppler-generating speeds. Since the light speed is finite and invariant, this new approach is completely reciprocal between the sources and the observer, contrary to the angles which are subject to aberration effects. The resulting Doppler formula will prove surprisingly elegant.

5.1 Doppler effect in the light clock of Einstein

Classical Doppler effects mix longitudinal and transverse effects and range between the two asymptotes \(1 - \frac{v}{c}\) and \(1 + \frac{v}{c}\). The intermediate values are currently defined using the angle \(\theta\) between the motion line and the receiver. In fact, since \(\theta\) varies with time and generates certain problems such as aberration effects, it seems more rational to skip it and to calculate the Doppler effect directly as a function of time. A Doppler-generating speed \(h\) can be defined using the Pythagorean theorem which holds in the Minkowski space. When a moving source at distance \(H_0\) from the observer reaches at speed \(v\) the closest point from this observer, the triangle shown in Fig.3 evolves such that the hypotenuse reduces from \(H_0\) to \(D\) while the source path reduces from \(L_0\) to \(0\), with speeds \(h\) and \(v\) respectively, related to each other with a couple of simple equations.

\[
H_0^2 = D^2 + L_0^2 \quad \text{(9a)}
\]

and

\[
(H_0 - ht)^2 = D^2 + (L_0 - vt)^2 \quad \text{(9b)}
\]

whose substraction allows to eliminate \(D\) and yields

\[
h t = H_0 - \sqrt{H_0^2 + (vt)^2 - 2L_0vt} \quad \text{(9c)}
\]

Eq.(9c) also holds in the particular case where \(H_0\) and \(L_0\) are precisely adjusted such that the wave front reaches the receiver when the source reaches the closest point, following a time delay \(\Delta t\). In this case, \(H_0\) and \(L_0\) can be replaced by \(c\Delta t\) and \(v\Delta t\) respectively and Eq.(9c) becomes

\[
h = \left(\frac{c\Delta t - \sqrt{(c\Delta t)^2 + (vt)^2 - 2v^2t\Delta t}}{t}\right) \quad \text{(10)}
\]

\[\text{Figure 3. A source moving at constant speed } v \text{ starts from a distance } H_0 \text{ from the immobile observer. The shortest distance between the source and the observer is } D.\]

When inserted in the classical Doppler formula, the speeds calculated with Eq.(10) give the results presented in Fig.4. The signal received when the observer is at right angle to the motion line was ejected towards the observer at rate \(h = v^2/c\). This value is calculated as the limit of a series expansion of Eq.(10) that is not defined at this point. The resulting Doppler effect is \(\lambda_{\text{mov}}/\lambda = 1 - \frac{v}{c}\). Of course, this effect is artificially generated and should now be cancelled to impose the expected absence of Doppler effect between the comoving mirrors of the light clock of Einstein (Fig.1). This cancellation yields a corrected Doppler equation with very interesting properties.
Figure 4. Evolution of the Doppler-generating speed calculated using Eq.(10). The time unit $\Delta t$ is the travel time of the signal reaching the receiver when the source is the closest to it. The origin of time $t = 0$ is centered at this closest position (corresponding to $-\Delta t$ in Eq.(10))

6 Comparison of the different Doppler approaches

6.1 Normalization of the different Doppler formulas with respect to time

To compare the different Doppler formulas, the new and existing equations should be comparable for any relative configuration of the source and the observer. The comparison with the relativistic equation [7] is delicate because several equations are possible depending on the angle used: either the original angle between the velocity vector and the source-observer connection line ($\theta$) or the reception angle ($\theta'$).

\[
\lambda_{\text{mov}} / \lambda = 1 + \frac{v}{c} \cos \theta = \frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{v}{c} \cos \theta'}, \quad (11a)
\]

the two angles of this identity are related to each other through the so-called aberration formula [7]

\[
\cos \theta = \frac{\cos \theta' - \frac{v}{c}}{1 - \frac{v}{c} \cos \theta'} \quad (11b)
\]

Table 1: The aberration effect in special relativity is related to the time points of wave emission.

<table>
<thead>
<tr>
<th>$t$</th>
<th>$\cos \theta$</th>
<th>$\cos \theta'$</th>
<th>$\lambda_{\text{mov}} / \lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\Delta t$</td>
<td>$-\frac{v}{c}$</td>
<td>0</td>
<td>$\sqrt{1 - \frac{v^2}{c^2}}$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>$\frac{v}{c}$</td>
<td>$1/\sqrt{1 - \frac{v^2}{c^2}}$</td>
</tr>
</tbody>
</table>

This situation is somewhat confusing because if one assumes that the transverse effect is obtained when the cosine is 0, the first formula of Eq.(11a) predicts a wave dilation, whereas the second formula gives the inverse wave contraction. The former solution is the right
one in special relativity \([8]\). In fact, \(\theta\) and \(\theta'\) cannot be simultaneously equal to \(\pi/2\). This subtlety is a matter of delay of wave travel \(\Delta t\) (Table.1). The relativistic Doppler formula contains two variables: the speed \(v\) and an angle. This angle varies along the wave path and can be expressed as a function of time, such that \(\theta(t) = \tan^{-1}(D/\nu t)\). Hence, on the one hand \(\cos \theta(t) = 1/\sqrt{1 + (D/\nu t)^2}\), and on the other hand the distance \(D\) can itself be defined as a function of \(\Delta t\) \((D = \Delta t \sqrt{c^2 - v^2})\), thereby allowing to make the relativistic formula a function of time only. There are ambiguities in the literature about the sign of the velocity \((-v\) and \(+v\)) in Doppler equations. To eliminate this source of confusion, all the equations can be composed to make this speed always positive, irrespective of the relative location of the observer, by transferring the sign to the time \(t\) ranging from \(-\infty\) and \(+\infty\). To “synchronize” the formulas at the time points of wave emission, in the new formula, \(t\) should be replaced by \(t + \Delta t\). Finally, a dimensionless normalized time holding for all source paths relatively to a receiver is defined as \(\bar{t} = t/\Delta t\). A little algebra satisfying all these requirements gives the equations compiled in Table.2.

Table 2: Doppler effects generated by a wave emitted at the normalized time \(\bar{t}\) and calculated using the different formulas. The new formula uses the classical Doppler framework \(1 - v/c\), in which \(v\) is replaced by the Doppler generating speed \(h\) calculated previously. \(t = 0\) is the time point at which the distance between the source and the observer is minimal and \(\Delta t\) is defined as the time-of-flight of the wave reaching the observer when located at the closest point from the source.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Doppler effect</th>
<th>(\bar{t} = -\infty)</th>
<th>(\bar{t} = -1)</th>
<th>(\bar{t} = 0)</th>
<th>(\bar{t} = +1)</th>
<th>(\bar{t} = +\infty)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classical</td>
<td>(\lambda_{\text{mov}} = 1 + \frac{v^2}{c^2} \frac{\bar{t}}{\Theta})</td>
<td>(1 - \frac{v}{c})</td>
<td>(1 - \frac{v^2}{c^2})</td>
<td>(1)</td>
<td>(1 + \frac{v^2}{c^2})</td>
<td>(1 + \frac{v}{c})</td>
</tr>
<tr>
<td>Relativistic</td>
<td>(\lambda_{\text{mov}} = \sqrt{1 - \frac{v^2}{c^2}})</td>
<td>(\frac{1 - \frac{v}{c}}{1 + \frac{v^2}{c^2}})</td>
<td>(\sqrt{1 - \frac{v^2}{c^2}})</td>
<td>(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}})</td>
<td>(\frac{1 + \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}})</td>
<td>(\frac{1 + \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}})</td>
</tr>
<tr>
<td>Virtual</td>
<td>(\lambda_{\text{mov}} = \frac{\Theta + \bar{t}}{1 + \bar{t}})</td>
<td>(1 - \frac{v}{c})</td>
<td>(1 - \frac{v^2}{c^2})</td>
<td>(\sqrt{1 - \frac{v^2}{c^2}})</td>
<td>(1)</td>
<td>(1 + \frac{v}{c})</td>
</tr>
<tr>
<td>Corrected ((\lambda))</td>
<td>(\lambda_{\text{mov}} = \frac{\Theta + \bar{t}}{1 + \bar{t}})</td>
<td>(1 - \frac{v}{c})</td>
<td>(1 - \frac{v^2}{c^2})</td>
<td>(\sqrt{1 - \frac{v^2}{c^2}})</td>
<td>(1)</td>
<td>(1 + \frac{v}{c})</td>
</tr>
<tr>
<td>Corrected ((\nu))</td>
<td>(\nu_{\text{mov}} = \frac{\lambda}{\lambda_{\text{mov}}})</td>
<td>(1 + \frac{v}{c})</td>
<td>(1)</td>
<td>(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}})</td>
<td>(1)</td>
<td>(\frac{1 - \frac{v}{c}}{\sqrt{1 - \frac{v^2}{c^2}}})</td>
</tr>
</tbody>
</table>

For the time point \(\bar{t} = -1\) where the new function is not defined, the Doppler effect takes the limit value \(1 - \frac{v^2}{c^2}\). These different Doppler equations describe general combinations of longitudinal and transverse Doppler effects for any relative position of the source and the observer. As these normalized equations can now be compared, their profiles are superposed for visualization in Fig.5, for the same arbitrary values of \(c\) and \(v\) and using \(\bar{t}\) units.
6.2 Comparative symmetry of the different Doppler formulas

The mean values of the Doppler effects generated at time points symmetrically located on both sides of the closest point \( t = 0 \) depend on the modes of averaging, which are, when expressed using wavelengths,

- for the arithmetic mean: \( \frac{1}{2} \left( \frac{\lambda_{\text{mov}}(\bar{t} - t)}{\lambda} + \frac{\lambda_{\text{mov}}(\bar{t} + t)}{\lambda} \right) \)
- and for the geometric mean: \( \sqrt{\frac{\lambda_{\text{mov}}(\bar{t} - t)}{\lambda} \frac{\lambda_{\text{mov}}(\bar{t} + t)}{\lambda}} \), respectively.

The appropriate tool to evaluate the symmetry on both sides of the closest point is logically the geometric mean, because it is the only one which holds for both periods and frequencies such that \( \langle T(\bar{t} - t), T(\bar{t} + t) \rangle = \frac{1}{\langle \nu(\bar{t} - t), \nu(\bar{t} + t) \rangle} \). The Doppler equations based on radial velocities, classical and relativistic, are characterized by their absence of symmetry whereas the corrected formula shown here displays a perfect geometric symmetry, in such a way that the geometric mean of all these sources which distribute equally in front of and behind their mutual closest points relative to the moving point. With the new Doppler formula, the mean interframe Doppler effect is \( \langle \frac{\lambda_{\text{mov}}}{\lambda} \rangle = 1/\sqrt{1 - \frac{v^2}{c^2}} \). This result suggests a correspondence between the time dilation effect and a generalized interframe Doppler effect described by the theoretical Doppler equation generated here.

7 A new putative relativistic Doppler equation

The corrected virtual Doppler equation built here appears remarkably elegant in many aspects. (i) As shown above, it is completely symmetric and predicts a global interframe effect equal to the Lorentz factor. (ii) By construction, its inversion point between the blue and red shifts is simply obtained at \( \bar{t} = -1 \), which seems to be the only expectable value based on the light clock device of Fig.1. In this respect, it is of interest to compare the inversion points of the formulas.
7.1 Switching points

The 3 different Doppler equations compared in Fig.4, have different switching points between contracted and dilated waves:

- for the classical formula, at $\bar{t} = t = 0$,
- for the new corrected formula at $\bar{t} = -1$ or $t = -\Delta t$,
- and finally, the most complicated result is obtained for the traditional relativistic formula, for which the switching point is close to the transverse line, at

$$\bar{t} = -\frac{c}{v\sqrt{2}} \sqrt{1 - \frac{v^2}{c^2}} \left( 1 - \sqrt{1 - \frac{v^2}{c^2}} \right)$$

or

$$t = -\frac{D}{v\sqrt{2}} \sqrt{\frac{1}{1 - \frac{v^2}{c^2}}} - 1$$

where $D$ is the minimal distance between the source and the observer.

Table 3: Arithmetic and geometric means of Doppler effects expressed using either wavelengths or frequencies. The geometric means of the new formulas are time-independent for wavelengths as well as frequencies. $\bar{t}$ and $\Theta$ are defined in Table.2.

<table>
<thead>
<tr>
<th>Mean</th>
<th>$\lambda$ vs $\nu$</th>
<th>Classical</th>
<th>Relativistic</th>
<th>Virtual</th>
<th>Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda$</td>
<td>$1$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\nu$</td>
<td>$\Theta^2$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$</td>
<td>$\frac{\Theta^2}{1 - \bar{t}^2}$</td>
<td>$(\frac{\Theta - \bar{t}^2}{1 - \bar{t}^2}) / (1 - \frac{v^2}{c^2})$</td>
</tr>
</tbody>
</table>

|            | $\lambda$           | $\frac{1}{\Theta} \sqrt{\frac{1}{1 - \frac{v^2}{c^2}} \left( 1 + \frac{v^2}{c^2} \bar{t}^2 \right)}$ | $\frac{1}{\Theta} \sqrt{\frac{1 + \frac{v^2}{c^2} \bar{t}^2}{\bar{t}^2}}$ | $\frac{\sqrt{1 - \frac{v^2}{c^2}}}{1 - \bar{t}^2}$ | $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ |
|            | $\nu$               | $\Theta$ | $\Theta^2$ | $1$ | $\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ | $\sqrt{\frac{1 - \frac{v^2}{c^2}}{\bar{t}^2}}$ |

7.2 Test of the relativistic Doppler equations

Many experiments have been conducted to test the theory of special relativity through verifying the relativistic Doppler formula of Einstein [9], through

- **Transverse effects.** Following a suggestion of Einstein himself [10], the existence of the transverse Doppler effect has been experimentally observed in [11]. But this test is of no help to distinguish between the equation of Einstein and the corrected one presented here, which predicts the same transverse effect.

- **Longitudinal effects.** The celebrated experiment of Ives and Stilwell [12] and its descendents [13] focused on the longitudinal Doppler effect. Ives and Stilwell recovered the time dilation factor by measuring the arithmetic means of the shifted wavelengths in front and behind moving atoms [12]. This result is not obtained for the new formula presented here, except for small values of $\bar{t}$. Note that Ives and Stilwell did not use frequencies although their study would have been easier, perhaps because their arithmetic mean is not simple and is
time-dependent (Table 3). Beyond their common features, the traditional and new relativistic equations present some differences. These equations could be probed through their different time-course profiles (Fig. 4).

7.3 Extension to cosmological expansion

The relativistic Doppler equation is little used in astrophysics because recessional velocities are often superluminal [14]. To justify the disqualification of this formula, authors invoke the fact that galaxies in mutual recession can belong to the same inertial frame. Alternatively, the two interpretations of the cosmological redshift, Doppler effect and wave stretching, have been considered equivalent [15]. This equivalence will be tested with the different Doppler equations presented here for the exponential mode of expansion that is assumed to hold today.

7.3.1 The cosmological redshift

Even before the publication of Hubble [19], Lemaître had shown that wavelengths should follow expansion [21]. For an interval of universe

$$ds^2 = dt^2 - a(t)^2 d\sigma^2$$

(12)

where $d\sigma$ is the element length of a space of radius equal to 1, the equation of a light beam is

$$\sigma_2 - \sigma_1 = \int_{t_1}^{t_2} \frac{dt}{a}$$

(13)

where $\sigma_1$ and $\sigma_2$ are the coordinates of a source and an observer. A beam emitted later at $t_1 + \Delta t_1$ and arriving at $t_2 + \Delta t_2$ undergoes a shift such that

$$\frac{\Delta t_2}{a_2} - \frac{\Delta t_1}{a_1} = 0$$

(14)

giving

$$z = \frac{\Delta t_2}{\Delta t_1} - 1 = \frac{a_2}{a_1} - 1$$

(15)

where $\Delta t_1$ and $\Delta t_2$ can be considered as the periods at emission and reception respectively [21]. If a procession of walkers regularly spaced crosses a stretching rubber band, on arrival their spacing will obviously be stretched in the same ratio as the rubber band. The same reasoning applies to a series of wave crests. In his article, Lemaître called this effect a Doppler effect [21]. This term is acceptable if broadly defining the Doppler effect as a wave distortion, but this is not the classical Doppler effect related to the speed of the source. The ratio between the reception and emission wavelengths simply follows the increase of the distance $D$ between the source and the receiver, which took place during the travel of light:

$$\frac{\lambda_{app}}{\lambda} = \frac{D_{reception}}{D_{emission}}$$

(16)

This redshift is exclusively a phenomenon of wave distortion, holding even if the sources and the receiver belong to the same inertial frame. The association between redshift and duration [2], is the ultimate proof of the connection between time and wavelength distortion. The different Doppler equations can now be compared to test the proposal of [15] of an equivalence between the redshift and a Doppler effect.

7.3.2 The exponential mode of expansion

The proportionality between the recession velocity and the distance (Hubble law) can be written $dD/dt = HD(t)$, where $H$ is the slope of the straight Hubble diagram. As the slope of a straight line is a constant, the unique solution of the Hubble law is $D(t) = D(0) e^{Ht}$. This expansion often called a model (of de Sitter) is not only a model but just the only mathematically possible solution of $V_{rec} = HD$. It is supposed to be the present mode of expansion driven by vacuum energy since we entered the “dark energy-dominated era” in which $H$ is a constant ($H_0 = \sqrt{\Lambda/3}$, where $\Lambda$ is the cosmological constant). Let us calculate the redshift under two hypotheses: (i) the accepted hypothesis of wave stretching of Lemaître [21] and (ii) the hypothesis of a Doppler effect, using the different Doppler equations.

Once light is emitted at the cosmic date $t_e$ in the direction of a receiver, a race begins between the light beam and the recession of the receiver. During the travel of light, the distance between the source and the telescope (which did not exist yet at the time of emission!) stretches according to $D(t) = D_e e^{Ht}$. The issue of this race is determined by the relative speed of light

$$\frac{dx(t)}{dt} = \frac{c}{D_e e^{Ht}}$$

(17a)

whose integration gives a dimensionless relative position of the wave front

$$x(t) = \frac{c}{H D_e} (1 - e^{-Ht})$$

(17b)

This function has a maximum of $c/H D_e$, showing that light will never reach receivers which are farther than $c/H$ from the source at the date of emission. If $D_e < c/H$, light will finally reach the receiver at the date $t_e$ precisely satisfying $x(t_e) = 1$, which allows to determine the total distance $D_e$ crossed by light before its capture by a telescope.

7.3.3 Redshift envisioned as a wave stretching effect (official theory)

The wavelengths are stretched in the same ratio that the distance crossed by light

$$\frac{\lambda_{app}}{\lambda} = \frac{D_e}{D_e} = e^{H \Delta t}$$

(18)
which can be calculated when \( x(t) = 1 \). The coincidence between the state of space stretching and the time point of light arrival reads

\[
x(t) = \frac{c}{HD_k} (e^{H\Delta t} - 1) = 1
\]

which implies

\[
e^{H\Delta t} = 1 + \frac{HD_k}{c}
\]

and gives a redshift

\[
z = \frac{\lambda_{\text{app}}}{\lambda} - 1 = \frac{HD_k}{c}
\]

This is the currently used redshift formula.

### 7.3.4 Redshift envisioned as a classical Doppler effect

The speed of the source at the time of light emission was

\[
\frac{dD_k}{dt} = HD_k
\]

where the initial distance can be deduced from the final distance \( D_k = D_k e^{-H\Delta t} \), allowing to express the initial recession velocity as

\[
\frac{dD_k}{dt} = HD_k e^{-H\Delta t}
\]

which gives, using the previous calculation,

\[
z = \frac{HD_k}{c} = \frac{c}{1 + \frac{HD_k}{c}}
\]

This value is (i) different from that obtained by wave stretching and (ii) always lower than 1, which is clearly contradicted by observations.

### 7.3.5 Redshift envisioned as a Doppler effect using the relativistic formula

The relativistic equation \( \sqrt{(c+v)/(c-v)} \) gives \( z = \sqrt{1 + 2HD_k/c} - 1 \), which is not the result obtained from the wave stretching hypothesis.

### 7.3.6 Redshift envisioned as a Doppler effect using the new Doppler formula

The equation given for \( \ell = +\infty \) in Table 2 is \( \lambda_{\text{mov}}/\lambda = 1/(1 - v/c) \). It gives

\[
z = \frac{1}{1 - \frac{HD_k}{c}} - 1 = \frac{HD_k}{c}
\]

which is exactly the result obtained with the wave stretching interpretation of the redshift. The proposal of [15] is right in this case.

It is shown in the first part of this study that special relativity does not escape the fundamental relation between time and wave distortion. This relation has then been extended to the cosmological redshift. Examples from other fields will now be examined, which demonstrate that this relation is universal. The first one is the famous twin paradox. Contrary to a widespread belief, this problem does not relate to special relativity that is restricted to uniform motion, but it can be solved in a purely classical manner if accepting the correspondence established above between wavelength and time distortion.

### 8 The twin paradox

Contrary to special relativity in which all the frames are inertial, in the famous twin paradox of Langevin, the symmetry of special relativity is broken by the moving twin, even if flying at constant speed. As shown below, the result of this experiment does not require the tools of special relativity and can be solved with the classical Doppler equation. Twin brothers are separated; one of them remains inert while the other crosses a distance \( dl \) before joining his brother. The wrong treatment of this situation would be to use special relativity tools and to arbitrarily decide which travels and which remains at rest. This can be readily shown: After their reunion, the paths of the twins in Minkowski’s space (\( ds \)) are the same, as they started from the same point and arrived to the same point of spacetime. If one decides that it is the twin whose proper time is labelled \( t' \) who travels

\[
d^2s = (cdt)^2 = (cdt')^2 - dl^2
\]

with \( v = dl/dt' \), one obtains \( dt/dt' = \sqrt{1 - \frac{v^2}{c^2}} \), and conversely if it is the other twin (of proper time \( t \)) who travels,

\[
d^2s = (cdt)^2 - dt^2 = (cdt')^2
\]

with \( v = dl/dt, dt'/dt = \sqrt{1 - \frac{v^2}{c^2}} \). Hence, it is impossible to break the symmetry. No physical experiment can measure the absolute velocity of a frame with a uniform motion in which it is conducted, so that one can not assign the speed \( v \) specifically to one twin. In fact, as pointed by several authors, the moving twin does not remain in a single inertial frame but breaks its uniform motion. Although many interpretations of this actively debated paradox have already been described, a new one is proposed below. To simplify the treatment, let us consider a collinear round trip (starting from a spatial station to eliminate a role for gravity). When located at a distance \( D \) from his sedentary brother, the travelling twin makes an about-turn at constant speed and infinite acceleration (like a frontal elastic collision). The twins continuously exchange light pulses with the same
table 4: mean wavelength averaged over the whole round trip, perceived by the resting (R) and travelling (T) twins. The classical Doppler formula appears superior in this situation.

<table>
<thead>
<tr>
<th>Point of view</th>
<th>Classical</th>
<th>Relativistic</th>
<th>Corrected</th>
</tr>
</thead>
<tbody>
<tr>
<td>Traveller (\lambda_{RT}^{app}/\lambda)</td>
<td>(1/2 \left[\left(1 + \frac{v}{c}\right) + \left(1 - \frac{v}{c}\right)\right] = 1)</td>
<td>(1)</td>
<td>(1)</td>
</tr>
<tr>
<td>Inert (\lambda_{TR}^{app}/\lambda)</td>
<td>(1/2 \left[\left(1 + \frac{v}{c}\right)^2 + \left(1 - \frac{v}{c}\right)^2\right] = 1 + \frac{v^2}{c^2})</td>
<td>(1 + \frac{v^2}{c^2})</td>
<td>(1 + \frac{v^2}{c^2})</td>
</tr>
<tr>
<td>Ratio (\lambda_{TR}^{app}/\lambda_{RT}^{app})</td>
<td>(1 + \frac{v^2}{c^2})</td>
<td>(1 + \frac{v^2}{c^2})</td>
<td>(1 + \frac{v^2}{c^2})</td>
</tr>
</tbody>
</table>

9 shortcut verifications with energetic considerations

The mass of clocks (travelling inside rockets) will be supposed conserved in the following treatments to avoid an internal contradiction in special relativity pointed by de Broglie. A change of frame predicts a change of energy, from \(E_0 = m_0c^2\) to \(E_{mov} = m_0c^2/\sqrt{1 - \frac{v^2}{c^2}}\) and then to a change of frequency \(\nu_{mov} = E_{mov}/h = m_0c^2/\sqrt{1 - \frac{v^2}{c^2}}/\nu_0 = \nu_0/\sqrt{1 - \frac{v^2}{c^2}}\). This result is inverse to that expected from a time dilation perspective. To plug this breach, de Broglie developed a complex theory called "harmony of the phases" with phase speeds higher than \(c\). As they have not been evidenced yet, mass effects will be prudently omitted here.
9.2 The uniform motion of special relativity

The total energy of an inert clock is its resting energy \( E = mc^2 \). Relatively to this frame, a clock moving in an other inertial frame at speed \( v \), has an additional kinetic energy such that \( E_{\text{mov}} = mc^2 + \frac{1}{2}mv^2 \).

\[
\frac{E_{\text{mov}}}{E} = \frac{mc^2 + \frac{1}{2}mv^2}{mc^2} = 1 + \frac{1}{2} \frac{v^2}{c^2}
\]

Since

\[
1 + \frac{1}{2} \frac{v^2}{c^2} \sim \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

the time dilation of special relativity is recovered:

\[
\lambda_{\text{mov}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (27)
\]

This value is given by the Newtonian rules of uniform acceleration:

\[
v_B = v_A + \gamma t \quad (33)
\]

and

\[
d = v_A t + \frac{1}{2} \gamma t^2 \quad (34)
\]

By eliminating the variable \( t \) between these two equations, one obtains

\[
v_B^2 - v_A^2 = 2\gamma d \quad (35)
\]

As it is impossible to get out of the space-wide acceleration field, the farthest object \( A \) will be arbitrarily considered as the reference one and Eq(31) finally gives

\[
\frac{\nu_B}{\nu_A} = \frac{mc^2 - m\gamma d}{mc^2} = 1 - \frac{\gamma d}{c^2} \quad (36)
\]

This treatment is sometime adapted to the gravitational redshift by considering \( \gamma \) as a gravitational acceleration \( \gamma = g = GM/r^2 \) where \( G \) is the gravitational constant, \( r \) is the radial distance between the source and the center of a graviting body of mass \( M \). But even with the approximation \( h \ll r \), this treatment misses the fact that a gravitational free fall is not uniform but is an accelerated acceleration, as detailed later.

9.3 The round trip (rt)

In the twin paradox experiment, one twin remains at rest while the other one makes a round trip. He first jumps in a new inertial frame moving at speed \( v \) relative to the starting one, and then jumps back to his initial frame.

\[
\nu_{\text{rt}} = \frac{E_{\text{rt}}}{E} = \frac{E - \Delta E_{\text{jumps}}}{E} \quad (30a)
\]

with

\[
\Delta E_{\text{jumps}} = \frac{1}{2}mv^2 + \frac{1}{2}m(-v)^2 = mv^2
\]

which gives

\[
\nu_{\text{rt}} = \frac{mc^2 - mv^2}{mc^2} = 1 - \frac{v^2}{c^2} \quad (30c)
\]

so that

\[
\lambda_{\text{rt}} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \approx 1 + \frac{v^2}{c^2} \quad (30d)
\]

This is the result given here.

9.4 Uniform acceleration

Suppose that two objects \( A \) and \( B \) of identical masses \( m \), are submitted to a uniform acceleration field \( \gamma \) shaping the whole space and that \( B \) is shifted forward at distance \( d \) from \( A \). The potential energy of \( B \) has decreased compared to that of \( A \)

\[
\frac{\nu_B}{\nu_A} = \frac{E_B}{E_A} = \frac{E_A - \Delta E}{E_A} \quad (31)
\]

with

\[
\Delta E = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_A^2 = \frac{1}{2}m(v_B^2 - v_A^2) \quad (32)
\]

This is sometimes adapted to the gravitational redshift by considering \( \gamma \) as a gravitational acceleration \( \gamma = g = GM/r^2 \) where \( G \) is the gravitational constant, \( r \) is the radial distance between the source and the center of a graviting body of mass \( M \). But even with the approximation \( h \ll r \), this treatment misses the fact that a gravitational free fall is not uniform but is an accelerated acceleration, as detailed later.

9.5 The gravitational redshift

9.5.1 The loss of gravitational potential energy

An object trapped in a gravitational well should consume energy to escape it, because of its loss of potential gravitational energy \( (GMm/r) \), compared to an identical mass located very far from the well,

\[
\frac{\nu_{\text{gr}}}{\nu} = \frac{E - \Delta E_{\text{grav}}}{E} \quad (37a)
\]

and

\[
\frac{\nu_{\text{grav}}}{\nu} = \frac{mc^2 - mGM/r}{mc^2} = 1 - \frac{GM}{rc^2} \quad (37b)
\]

Using series expansions near zero,

\[
\lambda_{\text{grav}} \approx 1 + \frac{GM}{rc^2} \approx \frac{1}{\sqrt{1 - \frac{2GM}{rc^2}}} \quad (37c)
\]

known as the redshift of Einstein.
9.5.2 The accelerated acceleration of free gravitational falls

Gravity (g) is not an uniform acceleration but is inversely proportional to the square of the distance r from the center of the gravitating mass M (g = GM/r²). Hence, a free fall in vacuum towards a massive body is an "accelerated acceleration". The distance x(t) crossed by an inert body jumping at t₀ = 0 over a star from an altitude a (v₀ = 0), follows

\[ \frac{dx}{dt} = GM \frac{t}{(a-x)^2} \]

with the initial condition x(0) = 0. Based on this equation, the distance between the jumper and the center of gravity r = a - x, decreases with time according to

\[ r(t) = \sqrt{a^2 - \frac{3}{2}GMt^2} \]

and when r reaches the altitude b (a ≥ b), the speed is

\[ v_b = \frac{1}{b^2} \sqrt{\frac{2}{3}GM(a^3 - b^3)} \]

which implies a drop of potential energy of the same amount, generating a redshift of a source B (at altitude b) perceived by a receiver A (at altitude a), of

\[ \frac{\nu^B}{\nu^A} = 1 - \frac{GM}{3c^2} \frac{a^3 - b^3}{b^4} \]

This gravitational redshift between two objects in the same well is not an absolute value but depends on the relative altitude of the source and the receiver. If both are at the same distance from the massive object (a = b), no shift is detected. The usual treatments of the gravitational redshift using Eq.(36) with \( \gamma = GM/r^2 \), yields different results. The comparison with the present treatment gives the following value to r:

\[ r = b^2 \sqrt{\frac{a-b}{a^3-b^3}} \]

which shows that the two approaches are equivalent only for r = a = b, that is to say without jump. The virtual radius r is in fact lower than b, because the free fall is an accelerated acceleration. This overlooked property forbids coupling Eq.(36) to \( \gamma = GM/r^2 \).

Principle: Wavelength/time distortion effects are proportional to the loss or gain of energy.

10 Conclusion

Contrary to the relativistic Doppler equation built independently of optical considerations, a genuine wave Doppler approach is presented here suggesting that the interframe time dilation is a matter of wave stretching, in full agreement with the parallel between time and redshift observed for supernovae [2]. While doing so, a putative Doppler formula emerged, which has several interesting properties. In particular, it includes a perfect geometric symmetry with a global mean value of \( \sqrt{1 - \frac{\gamma^2}{\gamma^2}} \), which corresponds exactly to a global interframe time dilation factor. Strangely, the popular twin paradox is considered as a verification of time dilation in special relativity, whereas special relativity deals only with inertial frames. Doppler analyses predict different time distortion effects depending on whether the moving source only passes near the receiver or starts from the receiver frame and returns to its point of origin.

The relativistic Doppler formula is in fact rarely used. It is not used for analyzing gravitational redshifts and the cosmological redshift. It is suggested here that is not more suitable for analysing round-trip or twin paradoxes. In turn, the classical Doppler formula, combined to an energetic view of the Doppler shifts, allows to recover (i) the reciprocal special relativistic contraction and non-reciprocal effects: (ii) the round trip dilation of and (iii) the gravitational dilation.

References


