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Interpretation of mesomechanical behaviors of plastic deformation based on analogy to Maxwell electromagnetic theory

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Abstract

Physical mesomechanics has introduced a variety of new concepts. At the same time, it has introduced a number of new terms. In this paper, explanations are given to those new terms making use of the similarity to the Maxwell electromagnetic theory. The meaning of plastic deformation as a wave phenomenon representing stress relaxation, and fracture as energy dissipation associated with the stress relaxation will be discussed.

1. Introduction

Physical mesomechanics (PMM) [1] is a recent theory of plastic deformation (PD) and fracture based on the principle of gauge transformation [2]. The uniqueness of this theory, which is also its most important feature, is that the theory is solely based on fundamental principles in physics without relying on any empirical formalism. Because of this feature PMM is capable of describing all the stages of deformation including the fracturing stage on the same theoretical basis, as well as being applicable to any heterogeneous media universally. These advantages make PMM quite useful for various engineering applications, such as predictable nondestructive evaluation [3] and computer aided design of new materials [4].

In PMM, PD is described as locally defined linear transformation of line element. By requesting that the Lagrangian is invariant under this local transformation, PMM has derived a set of equations that describes the self-consistent relationship between the translational and rotational modes of deformation. Mathematically, the whole process of derivation of these equations is quite analogous to that of the Maxwell electromagnetic theory. Consequently, the mechanical field of a plastically deforming object possesses wave characteristics. In a more physical picture, the wave characteristics represent a flow of energy associated with the stress caused by an external force. From this standpoint, the whole process of PD can be viewed as stress relaxation in which the relaxation is manifested as the decaying nature of the wave. In this context, fracture can be explained as the stage of deformation where the only possible way of stress relaxation is the generation of a discontinuity [5].

While it is relatively easy to understand the above-mentioned theoretical basis mathematically, the physical meaning of the formulation is not necessarily straightforward. Previously a number of investigators described the concept using various mesomechanical terms, which is rather common terminology in the gauge theory but completely different from conventional theories of deformation. In my opinion, these complexities hamper many of potential uses of PMM, especially for engineering applications. One way to digest the concept relatively easily is to use the analogy to the Maxwell theory (the EM analogy). Previously, in various occasions [5, 6, 7] I gave interpretation on a number of experimental observations using the EM analogy. The purpose of this paper is to give comprehensive explanation on those somewhat vague "mesomechanical terms" based on the EM analogy.

2. Theoretical formulation

The theoretical basis of PMM is described in a number of references [1, 2, 8]. In short, PMM describes PD as a transformation of a locally defined line element η.

$$\eta_i^{\mu} = \beta_i^j \eta_j^{\mu} \qquad (1)$$

where i and j are the internal indices, μ is the external index, η_i is the i-th component of the vector representing the line element at μ, β is the matrix representing the transformation of η. The volume element

corresponding to the local frame is referred to as the deformation structural element (DSE). It is assumed that the medium is continuous within the DSE. Physically, various elements, such as a grain, block and cell can be a DSE.

The transformation represented by β is classified as a transformation group of GL(3, R), the general transformation group of three dimension. GL(3, R) is characterized by nine independent parameters of real number, which corresponds to the nine degrees of freedom associated with the dynamics; i.e., translational degrees of freedom, rotational degrees of freedom and degrees of freedom in the length change of the line element. The plastic regime can be characterized as the situation where β depends on the global coordinates. Under that situation, it becomes necessary to introduce a compensation field known as the gauge field and redefine the spatial derivative so that the Lagrangian is invariant under the transformation. The redefined derivative is known as the covariant derivative D [9]. The Lagrangian invariant under this transformation can be given as follows.

$$L = g^{ij} D_\mu \eta_i^\alpha D_\mu \eta_i^\alpha C_{\alpha\beta}^{\mu\nu} - \frac{l^2}{4} \sigma_{\mu\nu}^a \sigma^{a\mu\nu} \qquad (2)$$

where g_{ij} is the metric tensor of the inner space, C is the dimensionless elastic constant of the medium characterizing the external space, l is a parameter representing the size of DSE, σ is the stress tensor and a is the group index.

With the help of the least action principle applied to the gauge field, a filed equation can be derived. After summation over the group indices, the field equation can be written in the following form.

$$div\, \boldsymbol{V} = j^0 \qquad (3)$$

$$rot\, \boldsymbol{V} = \frac{\partial \boldsymbol{\omega}}{\partial t} \qquad (4)$$

$$rot\, \boldsymbol{\omega} = -\frac{1}{c^2} \frac{\partial \boldsymbol{V}}{\partial t} - \boldsymbol{J} \qquad (5)$$

$$div\, \boldsymbol{\omega} = 0 \qquad (6)$$

where $\boldsymbol{V}$ is the vector representing the rate of displacement, j^0 is the temporal component of the four vector associated with the symmetry, $\boldsymbol{\omega}$ is the vector representing the angle of the line element and $\boldsymbol{J}$ is the spatial component of the four vector. Eqs. (3) – (6), referred to as the basic equation hereafter, describe the self-consistent relationship between the translational filed $\boldsymbol{V}$ and rotational field $\boldsymbol{\omega}$.

3. Meanings of mesomechanical concepts of PD

3.1 Shear instability

PMM states that a material enters the plastic regime when it loses its shear stability [10]. It also states that PD has rotational mode of deformation. Mathematically, these statements are commonly equivalent to the above-mentioned statement that PD is characterized by the situation where β depends on the global coordinates. In general, the transformation matrix describing deformation can be written as $\beta=E+\delta$, where E is the unit matrix and δ is the distortion tensor. δ can be further decomposed to the symmetric part ε and the anti-symmetric part Ω, where the former is usually called the strain tensor and the latter is called the rotation tensor. When the material is in the elastic regime, both ε and Ω can be expressed by constants, ε_n, ε_s, and ω_r, known as the normal strain, shear strain and rotation, respectively. When the material enters the plastic regime, β becomes dependent on the spatial coordinates. Consequently, different parts of the same object rotate differently. PMM states this situation as "the material loses shear stability", or "the deformation has the rotational mode." Note that when the material is in the elastic regime, the object can have rotation but since it is common to the whole object it is not deformation, i.e., is a rigid body rotation.

On pursuing the consideration of PD as local transformation, it will be helpful to view the Maxwell theory from the gauge theoretical viewpoint. Consider the Maxwell equation written in a similar form as the basic equations of PMM.

$$div\,\boldsymbol{E} = \frac{q}{\varepsilon_e} \tag{7}$$

$$rot\,\boldsymbol{E} = -\mu \frac{\partial \boldsymbol{H}}{\partial t} \tag{8}$$

$$rot\,\boldsymbol{H} = \varepsilon_e \frac{\partial \boldsymbol{E}}{\partial t} + \boldsymbol{i} \tag{9}$$

$$div\,\boldsymbol{H} = 0 \tag{10}$$

where $\boldsymbol{E}$ is the electric field, q is the charge density, ε_e is the dielectric constant, $\boldsymbol{H}$ is the magnetic field, μ is the magnetic permeability, and $\boldsymbol{i}$ is the density of conduction current. In this analogy, η corresponds to the wave function of a charged particle, β corresponds to *U(1)* transformation, and the electromagnetic field is the gauge field. Let us consider for a moment the Maxwell theory from the gauge theoretical viewpoint [11]. The transformation *U(1)* describes a phase transformation. When *U(1)* depends on the spatial coordinates, it is necessary to introduce a gauge field. Under the influence of the gauge field, the charged particle is not a free particle any more, i.e., it interacts with the field, or equivalently speaking, it feels a force exerted by the field. Accordingly, the Schrödinger equation describes the dynamics of the particle under the influence of the interaction with the field, or equivalently, the Lagrangian has an interaction term. Let us consider the equivalent concept for the mechanical field. When a material deforms plastically and therefore the matrix β depends on the spatial coordinates, it becomes necessary to introduce a gauge field to define the covariant derivative. Then the Lagrangian comes to have an interaction term, which is the first term on the right-hand side of eq. (2). This means that there arises a field of force and deformation proceeds under the influence of the force. The basic equations describe the dynamics associated with this interaction. Note that in the elastic regime, since the introduction of a covariant derivative is not necessary, there is no gauge field, nor field equation.

3.2 PD wave and translational-rotational synergetic interaction

Let us go back to the Maxwell theory and consider the meaning of the wave characteristics. In eqs. (7)-(10), suppose the $\boldsymbol{E}$ field is static. Then the first term on the right-hand side of eq. (9) is zero, and the second term is constant with respect to time (because if there is a temporal variation then the $\boldsymbol{E}$ field is no more static). If, for some reason, the $\boldsymbol{E}$ field starts to have a temporal variation, then via eq. (9) the magnetic field varies. This in turn induces an electric field via eq. (8) in such a way that the induced electric field reduces the original temporal variation in the $\boldsymbol{E}$ field. This is how the field possesses wave characteristics. The whole process can be interpreted as a natural feed back process in which a change in the electric field is compensated by a magnetic field, or vice versa. Now consider the basic equation of PMM from the same viewpoint. If the translational displacement $\boldsymbol{V}$ varies as a function of time, the rotational mode ($\boldsymbol{\omega}$) starts to have rotation (curl) in such a way that it reduces the translational motion. As an example, let us consider a simple two-dimensional case. Suppose the horizontal component of $\boldsymbol{V}$, u, starts to increase in a plane sample shown in Fig. 1 (note that since u has a dimension of velocity, this increase corresponds to an acceleration). Then according to eq. (5), the x-component of $rot\,\boldsymbol{\omega}$ decreases, which means in this particular case where ω_z is the only component of $\boldsymbol{\omega}$, $\partial\omega_z/\partial y<0$, i.e., above the location where $\boldsymbol{V}$ increases the material tends to rotate clockwise and below this location the material tends to rotate counterclockwise (see Fig. 1). These rotations apparently reduce the original change in u, and this is how the displacement field has the wave characteristics. PMM refers to this mechanism as the synergetic effect or the translational-rotational interaction [10].

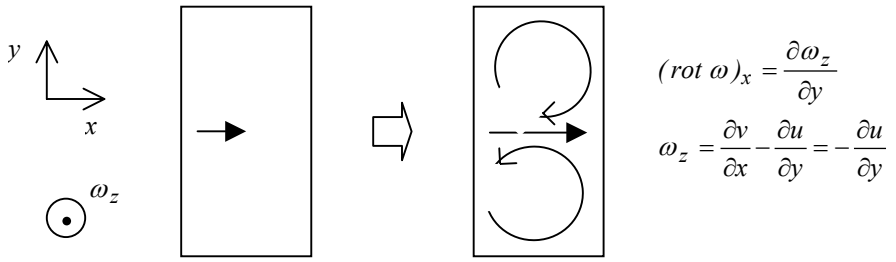


Fig.1 Rotational mode of deformation is induced so that the original change in the translational deformation is reduced. Note that ω_z is coming out the paper perpendicularly.

Along this argument, the dynamics associated with the synergetic interaction can be represented by a simple mechanical model. Details on this model will be described elsewhere [12]. In short, in this model neighboring DSEs are connected with each other with a spring, where the displacement of the DSEs from the equilibrium positions corresponds to the stretch or compression of the spring. Hence, the force exerted from a neighboring DSE is a recovery force represented by the spring constant. In this way, the translational motion transferred from a neighboring DSE is first converted to rotational motion of the DSE and subsequently, it is transferred to the next DSE as translational motion. This mechanism causes the wave characteristics in the translational motion and rotational motion, respectively. From the viewpoint of energy conservation, it can be said that the change in the translational deformation is stored in the spring as the potential energy. Note that the longitudinal effect (the vertical force as a function of the vertical coordinate) does not contribute to the wave characteristics, in the same sense as the Coulomb's force does not contribute to the generation of electromagnetic waves. Fig. 2 illustrates such a model schematically for a two-dimensional (2D) case. Note that the same arguments hold for a three-dimensional case.

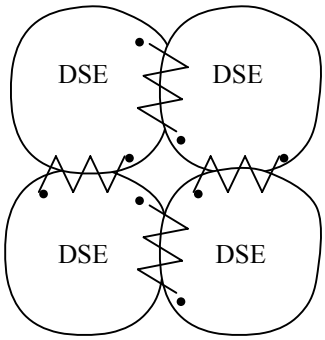


Fig.2 DSE spring model (2D). One end of each spring is attached to a DSE and the other end is attached to the neighboring DSE.

3.4 Charge and charge conservation

In the Maxwell theory the charge is characterized as the source of $\boldsymbol{E}$ field; via eq. (7) an $\boldsymbol{E}$ field is generated by an electric charge. Similarly, in PMM it can be said that a $\boldsymbol{V}$ field is generated by eq. (3), and from that standpoint, it is natural to call j^0 a charge. Considering that the left-hand side of eq. (3) represents spatial non-uniformity of the rate of $\boldsymbol{V}$, this equation can be interpreted as representing the situation where charge j^0 causes the change in velocity along the spatial coordinates, or equivalently saying, acceleration is generated along the spatial axis. Regarding j^0 is as a constant, this acceleration has linear dependence on the displacement. From this viewpoint, it can be further said that the whole equation equivalently describes a spring-mass system represented by j^0 [7]. In the above model, this spring corresponds to a spring placed at

the boundary of DSEs. From the material scientific viewpoint, this can be interpreted as stress concentration operating at the boundary of DSEs.

From the gauge theoretical viewpoint, an electric charge has a more significant meaning; it is a conserved quantity associated with the invariance of Lagrangian. Mathematically, this charge conservation can be described by eqs. (7) and (9); taking divergence of both hand side of eq. (9) and using the general, mathematical relationship of *div rot* $\boldsymbol{A}$ *=0* where $\boldsymbol{A}$ is a given vector, it is easy to show the charge conservation law as $dq/dt+div\ \boldsymbol{i}=0$. In PMM, the same mathematical argument holds and you can easily verify that the charge j^0 conserves by substituting eq. (3) to eq. (5) and using the relationship *div rot* $\boldsymbol{A}$ *=0*. This means that stress concentration caused by an external force can move in the form of current but not disappear [7].

3.5 PD as stress relaxation

An electromagnetic wave carries electromagnetic power in the form of Poynting vector. Let us consider an equivalent quantity in the mechanical field.

$$div(\boldsymbol{V}\times\omega)=\omega\cdot rot\boldsymbol{V}-\boldsymbol{V}\cdot rot\omega=\frac{1}{2}\frac{\partial}{\partial t}(\omega\cdot\omega)+\frac{1}{2c^2}\frac{\partial}{\partial t}(\boldsymbol{V}\cdot\boldsymbol{V})+\boldsymbol{V}\cdot\boldsymbol{J} \tag{11}$$

A little consideration on a harmonic oscillator based on a spring-mass system reveals that the first term on the right-hand side of eq. (11) represents the potential energy stored in the spring [6]. Apparently the second term represents the kinetic energy of the mass connected to the spring. From the EM analogy, the third term can be interpreted as power carried by a moving charge, i.e., by a conduction current.

Eq. (11) indicates that in the mechanical field of a plastically deforming object the mechanical power associated with the deformation can propagate either in the form of a plastic deformation wave or a moving charge. As mentioned above, charge j^0 represents a stress concentration and it generates translational displacement field $\boldsymbol{V}$. From these arguments it is naturally interpreted as follows. When a material has a stress concentration, translational displacement field $\boldsymbol{V}$ is generated and its temporal variation induces rotational displacement in a synergetic fashion. This synergetic interaction generates a wave that propagates through the material with a phase velocity c, carrying the mechanical power. This mechanical power can be interpreted as the flow of the stress energy generated by the stress concentration. Thus the wave characteristics can be interpreted as stress relaxation, and this is why the PD wave is referred to as a relaxation wave. The third term in eq (11) indicates that the flow of j^0, i.e. $\boldsymbol{J}$ is an alternative way to relax stress energy. The meaning of this second way of stress relaxation will be discussed in the next section.

3.6 Decay of PD wave and charge

PMM explains that the PD wave is a decaying wave and when it decays completely the material fractures [5]. It is well known that when an electromagnetic wave propagates in a conductive medium, it decays exponentially and charges, if exist, flow. Let us discuss the decay of PD wave based on the EM analogy. The decaying electric field can be generally written as follows.

$$E(t)=E_0\,e^{-\kappa t}\,sin(2\pi f' t+\phi) \tag{12}$$

where κ represents the conductivity, f' is the frequency of the decaying wave and ϕ is the phase delay associated with the decay. Since the medium is conductive, a conduction current can flow and if it flows, part of the energy is dissipated. The exponential term at the beginning of eq. (12) represents the decay in the $\boldsymbol{E}$ field as the wave, and is related to conduction current by the Ohm's law. The associated energy dissipation is the Ohmic loss.

From the mathematical similarity, the mechanical dissipation should be given in a similar way. In the mechanical picture, the exponential decay corresponding to that in eq. (12) represents viscous damping. Therefore, it makes more sense to describe the dynamics in the field of displacement, rather than the rate of displacement. Following the EM case, let us put an exponential term at the beginning and express the displacement wave as follows [6].

$$X(t)=\frac{\nu_0}{\nu'}X_0\,e^{-\gamma t}\,sin(2\pi\nu' t+\varphi)\,sin(\frac{2\pi}{\lambda}y) \tag{13}$$

where X is the displacement from the equilibrium position along x direction, γ is the damping coefficient representing the viscosity of the medium, ν_0 and ν' are the frequencies of the non-decaying and decaying wave, respectively, φ is the phase delay and λ is the wavelength. Here eq. (13) takes the form of a standing wave, because according to PMM the PD wave becomes a standing wave before the material fractures and therefore it is more convenient to use the standing wave form when fracture is discussed [5]. Note that the non-decaying wave in this context means the wave solution expressed by eq. (13) without the exponential term. From the displacement wave given by eq. (13), the corresponding u wave can be derived as follows.

$$\begin{aligned} u &= -\frac{(2\pi\nu_0)^2}{2\pi\nu'} X_0\, e^{-\gamma t} \sin(2\pi\nu' t) \sin(\frac{2\pi}{\lambda} y) \\ &= -(2\pi)^2 \frac{k}{m} \frac{X_0\, e^{-\gamma t}}{\sqrt{(2\pi)^2 \frac{k}{m} - \gamma^2}} \sin(2\pi\nu' t) \sin(\frac{2\pi}{\lambda} y) \end{aligned} \tag{14}$$

where

$$\nu_0 = \sqrt{\frac{k}{m}} \tag{15}$$

and in going to the second line in eq. (14), the analogy to the spring-mass model is used [6]. Eq. (14) explicitly describes the situation where the displacement wave decays because of the damping term represented γ by .

Substitution of eq. (14) into eq. (5) for the two dimensional case indicates that current J_x has the following form, where J_x is the x component of current $\boldsymbol{J}$.

$$J_x = \frac{2\gamma}{c^2} u + \frac{\gamma^2}{c^2} X \tag{16}$$

Note that as γ increases current J_x grows. This indicates that with an increase in the damping coefficient more current flows causing higher energy dissipation. In the EM analogy, this corresponds to the situation where as the conductivity increases the conduction current becomes higher, and consequently, more electric power is dissipated by means of the Ohmic loss. In PMM, as mentioned above, current $\boldsymbol{J}$ is the flow of charge j^0 and j^0 represents a whole spring-mass system. Therefore, a motion of j^0 means that the whole spring-mass moves, not the mass alone moves relative to the equilibrium position, and therefore the kinetic energy of the mass is not stored as the potential energy of the spring. This is the physical picture of current $\boldsymbol{J}$ in connection with the energy dissipation.

Detailed dissipation processes causing the exponential decay of PD wave is not clear at this point. However, there are experimental observations consistent with the above-mentioned viscosity model [7]. Shown in Figs. 3 and 4 are displacement waves observed in a series of tensile experiment on an aluminum alloy in the direction perpendicular to the tensile axis. These figures indicate that PD wave decays either oscillatory or exponentially; it is well known that depending on the relative magnitude of k and m, harmonic oscillation in a spring-mass system decays oscillatory (ringing down) or exponentially (over-damping). One possible explanation for those decay characteristics is that whatever the dissipation mechanism involved in PD may be, the displacement decays through a process similar to viscosity.

4. Conclusion

Various mesomechanical concepts have been discussed using the analogy to the Maxwell electromagnetic theory. The basic mechanism causing the wave characteristics of PD, which results from the mathematical necessity of introducing a gauge field, has been considered in more phenomenological pictures. The PD wave and mesomechanical current have been formulated in the same mathematical forms as the EM counterparts, respectively, and their meanings as a relaxation wave and energy dissipation mechanics have been explained.

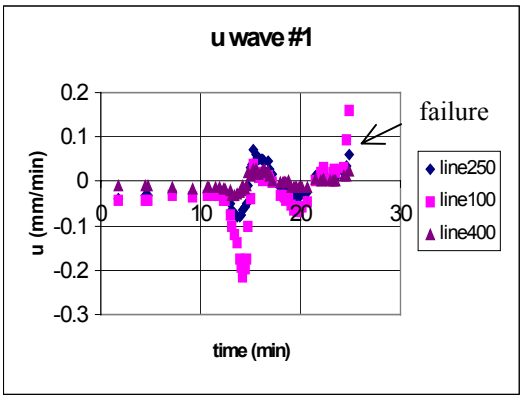


Fig. 3 Observed *u*-wave (damping oscillatory case)

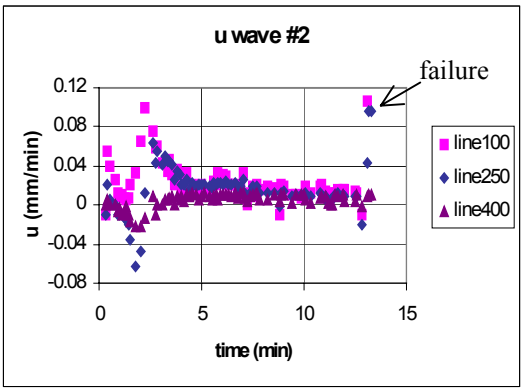


Fig. 4 Observed *u*-wave (over damping case)

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