Physical mesomechanics as a field theory

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The theoretical foundation of physical mesomechanics is viewed as a field theory analogous to electrodynamics. The electromagnetic field is reviewed as a compensation field necessary to make the Shrödinger equation invariant under local phase transformation on the wave function of a charged particle. The derivation of the Maxwell equation based on this view is reviewed, and the basic equation of physical mesomechanics is characterized as a field equation analogous to the Maxwell equation. The underlying mechanism for this compensation effect is argued in terms of the broad sense of the Lenz law and its physical interpretation is discussed.

1. Introduction

Since first reported in 1982 [1], physical mesomechanics has made significant contributions to the materials science and related communities. Introducing the concept of structural levels in solid-state media under deformation [2], physical mesomechanics [3, 4] has unified continuum mechanics and dislocation theory. Consequently, it has successfully explained various crystallographic phenomena on a sound physical basis, which previously was understood in rather empirical fashions. From this viewpoint, it can be said that physical mesomechanics has brought a new epoch in materials science and related fields. One very important aspect of physical mesomechanics that differentiates this theory from other conventional theories is its universality. The theory is basically applicable to any heterogeneous solid-state media, regardless of the type of the loading or scale size. In addition, it is capable of describing all the stages of deformation, including the fracturing stage, on the same theoretical basis. These features make the theory attractive for various engineering applications [4].

It should be emphasized that this universality of physical mesomechanics comes from the fact that the theory is formulated at the most fundamental level in science, i.e., the theoretical formulation is based on the principle of gauge symmetry [5]. From this standpoint, it can be said that physical mesomechanics is analogous to other gauge theories such as Maxwell's electrodynamics and Einstein's general relativity. In fact, various physical-mesomechanical interpretations can be rationalized through the analogy to electrodynamics. Good examples are the similarity between the plastic deformation wave and electromagnetic wave as the Poynting vectors carrying the field energy [6, 7], and the similarity between electric breakdown of gas media and fracture of solid-state media as the final stage of the energy dissipation process [8].

Previously, I have discussed the analogy between physical mesomechanics and electrodynamics for various observations and interpreted their physical meanings [6–8]. On this anniversary issue, I would like to focus on the fundamental similarity between the two theories. Rather than going into details of each individual observation, I will describe the big picture of my view of physical mesomechanics as a theory fundamentally analogous to electrodynamics, where the temporal and spatial variations of field variables compensate each other to achieve certain stability in the underlying dynamics. As the physical process to actualize this compensation mechanism, the basic equation governing the dynamics of field variables will be discussed from the viewpoint of the Lenz law. It will be shown that the two theories are not only formulaically but also physically analogous, which in turn rationalizes the universality of physical mesomechanics.

2. Electrodynamics and quantum dynamics

The analogy between physical mesomechanics and electrodynamics can be most conveniently described by considering electrodynamics in connection with quantum dynamics. Aitchson and Hey [9] describe this subject and related physics comprehensively. In short, it can be explained as follows. In quantum dynamics, the electromagnetic field can be viewed as a compensation field to gain the gauge (local) symmetry in the theory. Consider that the wave function of a charged particle is under a phase $U(1)$ transformation. The phase transformation basically represents a displacement in space-time (see below), and it is unnatural to postulate that all the charged particles in the universe are under the same transformation; thus we have to let each particle experience different transformation. Then a question arises. Is the transformation of each particle completely independent of each other? The answer is no. They are somehow related to each other, otherwise the dynamics of each particle cannot be described by the same form of the Shrödinger equation, i.e., the theory must be reformulated. In other words, the particle can no longer be a free particle, and must interact with each other through the potential term in the Shrödinger equation. This interaction can be represented by means of a compensation field, and the electric and magnetic field vectors are a vector representation of the field.

In the field theoretical term, the above situation can be stated as follows: "To make the Shrödinger equation invariant under the situation where the wave function of a charged particle experiences locally defined phase transformation, it is necessary to introduce interaction between this particle and other surrounding charged particles". In this situation, since the Shrödinger equation contains temporal and spatial derivatives of the wave function, it becomes necessary to redefine the operation of differentiation so that these derivatives transform in the same fashion as the wave function itself, i.e., it is necessary to replace the derivatives by covariant derivatives [10]. Otherwise the Shrödinger equation does not remain invariant under the phase transformation, as mentioned above. The covariant derivatives can be introduced by adding extra terms to the spatial and temporal partial differentiation as below [9]:

$$D \equiv \nabla - iqA, \tag{1.1}$$

$$D^0 \equiv \frac{\partial}{\partial t} + iq\phi, \tag{1.2}$$

where A and ϕ are the additional terms needed for the spatial and temporal derivatives, and q is the electric charge of the particle. To keep the Shrödinger equation invariant, it is further necessary that the compensation field obey a certain transformation. As shown below, this transformation depends on the phase that the wave function transforms (note that Eqs. (2.1)–(2.3) contain the same function χ). Thus for a given phase transformation, the request of gauge symmetry specifically determines the way the particle interacts with the field. In fact, the additional terms introduced in Eqs. (1.1) and (1.2) represent the vector and scalar potential of the electromagnetic field. The combination of the phase transformation (the medium field transformation) and the associated transformation of the compensation field (the gauge field transformation) is referred to as the gauge transformation, and it can be written in the following form.

$$\psi' = e^{iq\chi}\psi, \tag{2.1}$$

$$\vec{A}' = \vec{A} + \nabla\chi, \tag{2.2}$$

$$\phi' = \phi - \frac{\partial \chi}{\partial t}. \tag{2.3}$$

Here $\vec{A}$ and ϕ are the vector and scalar potential of the field, and ψ is the wave function. Equation (2.1) represents the medium field transformation, and Eqs. (2.2) and (2.3) the compensation field transformation.

For the purpose of comparing electrodynamics and physical mesomechanics, it is helpful to take some time to consider the meaning of the phase transformation [11]. Consider a wave packet describing the wave function of a charged particle is displaced spatially, temporally, or both. The space part of the wave function representing the states before and after the transformation can be written as follows

$$\psi(r - \delta) = U_r(\delta)\psi(r). \tag{3}$$

When the displacement is infinitesimally small, the left-hand side of the above equation can be Taylor-expanded as

$$\psi(r - \delta) = \psi(x, y, z) - \delta\frac{\partial}{\partial x}\psi(x, y, z) + + \frac{\delta^2}{2!}\frac{\partial^2}{\partial x^2}\psi(x, y, z) \ldots . \tag{4}$$

The right-hand side of Eq. (4) can be written in the form of $e^{-\delta(\partial/\partial x)}$, and extending the expression to three dimensions, the transformation in Eq. (3) can be written in the following form.

$$U_r(\delta) = e^{-\delta\nabla}\psi(r). \tag{5}$$

The next level of question is that we know that the medium field variable ψ obeys the Shrödinger equation but is there any equivalent equation that governs the compensation field variables? The answer to this question is yes, and it can be derived through the concept of least action principle. In the same sense as an equation of motion can be derived by postulating that when a state changes from one to another under transformation the variation should be zero, we can derive a basic equation that governs the compensation field variable. After some mathematical manipulation, the equation can be written in the following form, and this is nothing but the well-known Maxwell equation:

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_e}, \tag{6.1}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \tag{6.2}$$

$$\nabla \times \vec{B} = \varepsilon_e \mu_e \frac{\partial \vec{E}}{\partial t} + \mu_e \vec{j}, \tag{6.3}$$

$$\nabla \cdot \vec{B} = 0. \tag{6.4}$$

Needless to say, $\vec{E}$ is the electric field, ρ is the charge density, ε_e and μ_e are the electric permittivity and magnetic permeability of the medium, $\vec{B}$ is the magnetic field, and $\vec{j}$ is the current density.

3. Equivalent picture in physical mesomechanics

Now consider the equivalent picture in deformation. The gauge field associated with the dynamics of plastic deformation was originally derived in conjunction with the general transformation GL(R, 3) [1–3], which is basically a non Abelean transformation unlike the phase $U(1)$ transformation in electrodynamics. However, after summation over the group indices, the basic equation can be written in the same form as the Maxwell equation [1–5].

Let us begin with the consideration of the medium field transformation. Here, for simplicity, I will consider a two-dimensional model, but the model can be extended to three dimensions with the same argument. Consider in Fig. 1 that a point $P(x, y)$ and another point $Q(x+\eta^1, y+\eta^2)$ in the vicinity of P displace in unit time to points P' and Q', respectively. The position vectors associated with this transformation can be written as follows:

$$\overrightarrow{OP'} = \overrightarrow{OP} + \vec{V}(x, y), \tag{7.1}$$

$$\overrightarrow{OQ'} = \overrightarrow{OQ} + \vec{V}(x+\eta^1, y+\eta^2). \tag{7.2}$$

Here $\vec{V}$ is the displacement vector per unit time, and η^1 and η^2 are the x and y components of the line element vector $\overrightarrow{PQ}$. By subtracting Eq. (7.1) from Eq. (7.2), we can express the line element vectors before and after the transformation as

$$\overrightarrow{PQ'} = \overrightarrow{PQ} + [\vec{V}(x+\eta^1, y+\eta^2) - \vec{V}(x, y)].$$

By expressing the content of the bracket on the right-hand side of this expression in terms of spatial derivatives of $\vec{V}$ up to the first order and denoting vectors $\overrightarrow{PQ}$ and $\overrightarrow{PQ'}$ by (η^1, η^2) and (η'^1, η'^2), respectively, the transformation can be written in the following form:

$$\begin{bmatrix} \eta'^1 \\ \eta'^2 \end{bmatrix} = \begin{bmatrix} \eta^1 \\ \eta^2 \end{bmatrix} + \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{bmatrix} \begin{bmatrix} \eta^1 \\ \eta^2 \end{bmatrix} \tag{8.1}$$

which can be conveniently expressed as below

$$\vec{\eta}' = U\,\vec{\eta}, \tag{8.2}$$

$$U = I + \beta, \tag{8.3}$$

where I is the unit matrix and β is the matrix known as the distortion matrix

$$\beta = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} \\ \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} \end{bmatrix}, \tag{9}$$

where u and v are the x and y component of vector $\vec{V}$. By viewing the components of the distortion matrix as the first-order term of the infinite series of the expanded form of $[V(x+\eta^1, y+\eta^2) - V(x, y)]$, e.g., $\beta_{11} \Rightarrow (\partial u/\partial x)\eta^1 + (\partial^2 u/\partial x^2)(\eta^1)^2/2! + \ldots$, the transformation matrix U in Eq. (8.2) can be viewed as the first-order term of an exponential expression equivalent to Eq. (5). This together with the fact that the transformation in Eq. (5) represents quantum dynamical displacement and Eq. (8.2) literally represents classical dynamical displacement rationalizes referring to the transformation U as a phase transformation [1–4].

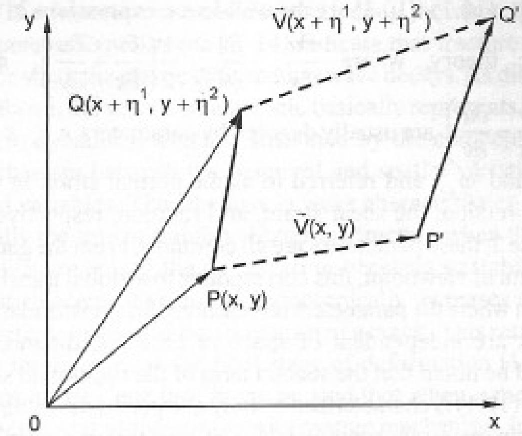

Fig. 1. Change in the length of line element represented as linear transformation

To further explore the physical meaning of the above transformation, consider the following three cases.

Case 1. Displacement vector $\vec{V}$ is a constant

This is the case where all the points of the medium displace for the same amount, and evidently the situation represents a rigid body motion. β in Eq. (8.3) is zero because all the derivatives in Eq. (9) is zero, and there is no deformation at all.

Case 2. Displacement vector $\vec{V}$ is a variable but distortion matrix β is a constant

This is the case where the medium evidently deforms because different points displace for different amounts. In this case, it is convenient to divide the distortion matrix β into the symmetry term ε and anti-symmetry term ω:

$$\beta = \varepsilon + \omega, \tag{10.1}$$

where

$$\varepsilon = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} + \dfrac{\partial u}{\partial y}\right) & \dfrac{\partial v}{\partial y} \end{bmatrix} \tag{10.2}$$

and

$$\omega = \begin{bmatrix} 0 & -\dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial v}{\partial x} - \dfrac{\partial u}{\partial y}\right) & 0 \end{bmatrix}. \tag{10.3}$$

Eqs. (10.1)–(10.3) are the well-kwon expressions in the elastic theory, where $\frac{\partial u}{\partial x}$, $\frac{\partial v}{\partial y}$, $\frac{1}{2}\left(\frac{\partial v}{\partial x}+\frac{\partial u}{\partial y}\right)$, and $\frac{1}{2}\left(\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right)$ are usually denoted by parameters ε_{xx}, ε_{yy}, ε_{xy}, and ω_z, and referred to as the normal strain in the $x(y)$ direction, the shear strain, and rotation, respectively. In Case 2, these parameters are all constants. From the gauge theoretical viewpoint, this corresponds to a global transformation where the parameters representing the transformation matrix are independent of space or time coordinates. It should be noted that the second term of the right-hand side of Eq. (10.1) is characterized by only one parameter ω. Since this term represents rotation and ω is a constant, the situation indicates that the rotation is common to the whole medium, i.e., it represents a rigid body rotation.

Case 3. Distortion matrix β depends on space coordinates

This is the case that physical mesomechanics defines as the plastic deformation. There are two important changes from Case 2. First, the parameters ε_{xx}, ε_{yy}, ε_{xy}, and ω_z are no more spatial independent, thus rotation becomes substantial in deformation, i.e., deformation has the rotational mode [1–4]. Second, since the first order derivatives ($\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$, etc.) depend on the space coordinate, it becomes necessary to introduce covariant derivatives equivalent to Eqs. (1). Then by making use of the variation method and other mathematical procedure in the same way as the electrodynamics case, equations corresponding to the Maxwell equations can be derived. After summation over the group indices, the field equations can be written in the following form:

$$\nabla\cdot\vec{V}=j^0, \tag{11.1}$$

$$\nabla\times\vec{V}=\frac{\partial\vec{\omega}}{\partial t}, \tag{11.2}$$

$$\nabla\times\vec{\omega}=-\varepsilon_p\frac{\partial\vec{V}}{\partial t}-\vec{j}/\mu_p, \tag{11.3}$$

$$\nabla\cdot\vec{\omega}=0. \tag{11.4}$$

Figure 2 pictorially represents the above situation where each segment experiences deformation associated with constant parameters, and the deformation of the whole body is represented by the spatial dependence of these parameters. Each segment is called the deformation structural element (DSE) [1–4]. It is important to note that the DSE has nothing to do with the physical dimension (size). All that matter is that the dynamics within a given DSE can be characterized by the same ε_{xx}, ε_{yy}, ε_{xy}, and ω_z. This is the key to the applicability of physical mesomechanics regardless of the scale, including the nano/microscale.

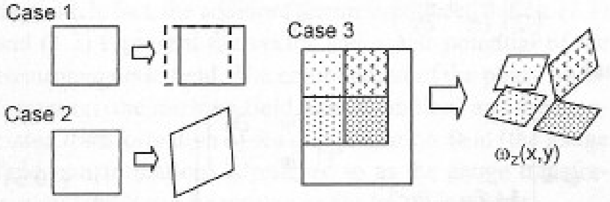

Fig. 2. Pictorial representation of transformation describing deformation under three different conditions

It is interesting to note that the elastic theory uses constant ε_{xx}, ε_{yy}, ε_{xy}, and ω_z and therefore describes the dynamics of individual DSE (Case 2). From this view and in analogy to the gravitational field, the elastic theory corresponds to the theory of special relativity and physical mesomechanics corresponds to general relativity, as Utiyama [12] explains the gravitational theory on the same idea.

4. The Lenz law and wave characteristics

Equations 6 and 11 yield wave solutions, known as electromagnetic and plastic deformation waves [3, 4, 6, 7], respectively. The wave solution is basically a stable solution, where the temporal and spatial variations compensate each other via the mutually opposite signs in front of the spatial and temporal secondary derivatives in the wave equation. From the physical viewpoint, this stability is associated with the Lenz law. To explore this further, let us first consider the electrodynamic case known as the Faraday–Lenz law. Imagine that a positive ion starts to move rightward under the influence of some external agent. This causes a displacement current to flow. Via Eq. (6.3), the displacement current generates a magnetic field in the following fashion (Fig. 3). At a point in front of the positive ion on its path, the rightward electric field increases as the ion approaches. Therefore, the displacement current $\partial E/\partial t$ is rightward, or in the same direction as E itself. Consequently, a magnetic field is induced in such a way that it goes into the page below the

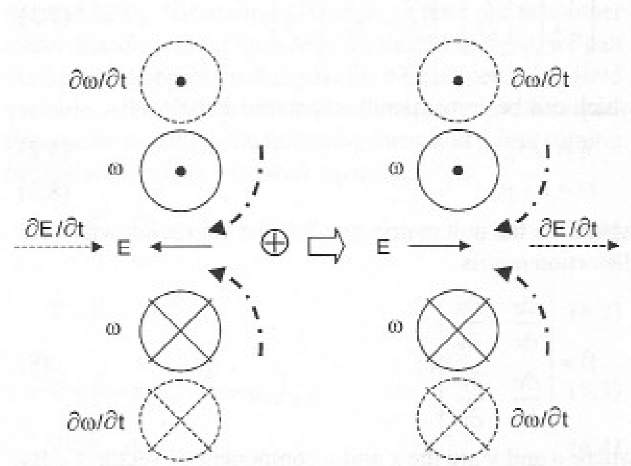

Fig. 3. The Faraday–Lenz law. Dashed lines represent $\partial\omega/\partial t$, and dash-dotted lines represent induced electric fields. The initial electric field generated by the positive ion is represented by solid lines, while the temporal change in the electric field due to the motion of the ion is represented by dashed lines

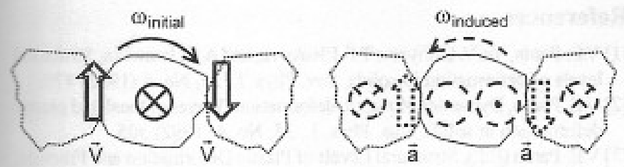

Fig. 4. The Lenz law in physical mesomechanics. Solid arrows represent velocity caused by the initial rotation due to an external torque and dashed arrows represent induced acceleration

ion path and comes out of the page above the path. Before the ion starts to move, there was no magnetic field at this point. So, the motion of the ion causes $\partial\omega/\partial t$ to be into the page below the path and out of the page above the path, or $\partial\omega/\partial t$ is in the same direction as ω itself. Via Eq. (6.2), this temporal change in the magnetic field induces a leftward electric field on the path of the positive ion, as indicated at the right end of Fig. 3. Evidently, this electric field counteracts on the initial motion of the ion. A point behind the positive ion where the electric field due to the ion is leftward, a similar thing happens. As the ion moves farther away from the point, the electric field strength decreases and therefore $\partial E/\partial t$ is opposite to E itself, or the displacement current flows rightward. Thus, the induced magnetic field and its temporal change is the same as in front of the ion, and consequently, an electric field counteracting on the initial movement of the ion is induced.

An equivalent picture can be drawn in physical mesomechanics. Imagine that a certain DSE rotates clockwise under the influence of an external agent (by an external torque). As Fig. 4 pictorially represents, this induces an upward velocity at the boundary with the left neighboring DSE and a downward velocity at the boundary with the right neighboring DSE. This is represented by Eq. (11.2). Since the velocity was initially zero at these boundaries, this means that the acceleration field is as shown in the right picture of Fig. 4. $\partial\vec{V}/\partial t$ in the first term of the right-hand side of Eq. (11.3) represents acceleration. Thus via Eq. (11.3) (i.e., via $\nabla\times\vec{\omega}$), ω is induced in such a way that it goes into the page at the right side of the right acceleration and at the left side of the left acceleration, and it comes out of the page at the left side of the right acceleration and right side of the left acceleration. Evidently, this induced rotation counteracts the initial rotation due to the external force. Note that since the medium constant ε_p has a dimension of density (kg/m^3) [13], the first term on the right-hand side of Eq. (11.3) has a dimension of force per unit volume, and the counter effect represented by the left-hand side of Eq. (11.3) can be interpreted as a counter force per unit volume.

One distinction in the physical mesomechanics case from the electrodynamics case is that the counter effect is due to the negative sign of the third equation (Eq. 11.3), rather than the second equation (Eq. (11.2)). From this standpoint, in the case of physical mesomechanics, the third equation (Eq. (11.3)) rather than the second equation represents the Lenz law.

The fracture can be understood along the same line of argument. Experiments [7, 14] indicate that fracture takes place when the plastic deformation wave decays. As discussed above, the wave characteristic basically represents stability in dynamics, which is sustained by the compensation mechanism between the temporal and spatial variation of field variables. Thus the loss in wave characteristics is basically the loss in stability. Materials fracture when the dynamics governing the deformation becomes unstable and the displacement vector $\vec{V}$ monotonically increases in one direction, leading to the formation of a crack. This rationalizes the fracture as the final stage of deformation [14]. In terms of the Lenz law, it can be said that when a medium loses the spatial-temporal compensating mechanism, it loses the capability of maintaining its state as a continuous solid-state, and this event is the fracture. It is interesting to note that from this viewpoint, the fracture is analogous to the electric breakdown where the dielectric medium becomes conductive and the electromagnetic wave decays [8].

5. Application to nano/microscience

Finally, I would like to briefly discuss the applicability of physical mesomechanics to nano/microscience, one of the most important emerging fields of recent science. Note that no comprehensive theory is available to describe many aspects of material properties and interactions at nano/microscale levels, and this causes practical problems such as the life of microsystems is substantially shorter than expected. Detailed analysis on the applicability of physical mesomechanics to these scale level is out of the scope of this paper, and I would just like to raise two questions: (a) if the theoretical basis of physical mesomechanics is valid in the nano/microscale levels, and (b) if the theory relies on a macroscopic parameter. As for the first point, physical mesomechanics is based on the symmetry in physics, which is scale-insensitive. The deformation structural element is defined independent of the material structure. It is thus evident that the theoretical basis is valid at the nano/microscale levels without modification.

The answer to the second question is not straightforward. There are two medium dependent constants, ε_p and μ_p, in the basic equation of physical mesomechanics (Eq. (11)). Of these two, ε_p is the density of the medium, and can be defined in the nano/microscale levels in the same way as the macroscale [6]. The other parameter μ_p is interpreted as related to the stiffness [6]. The concept of stiffness is based on the approximation where the restoring force (generally known as the spring force) is proportional to the displacement. In the macroscale level, this is a good approximation when the displacement is small. It is not clear to me in the nano/microscale level if this approximation is good, or, if it is good, what is the range of the small displacement where the approximation is valid with respect to the object size. However, considering the likeliness that the restoring force is still a function of the displacement in the short distance limit and that the function is Taylor-expandable, there ought to be a range of displacement where the linear approximation is valid and within that range a stiffness can be defined. It should be noted that the resultant μ_p in this range could be substantially different from that in the macroscale level; it is essential to determine the nano/microscopic stiffness experimentally. Also be noted is that the situation is fundamentally different from the use of a statistical parameter such as a diffusion coefficient for a nanoscale system, which is not an unusual practice in other theories, and in my opinion, is incorrect.

6. Conclusion

Physical mesomechanics has been viewed as a field theory analogous to electrodynamics. The basic equation governing the translational and rotational displacement has been derived in the same fashion as the Maxwell equation is derived in connection with the gauge symmetry associated with the phase transformation of the wave function of a charged particle. The underlying physics has been discussed from the viewpoint of the Lenz law. Through these discussions, the interpretation of fracture of solid-state media as the final stage of deformation has been rationalized. The applicability of physical mesomechanics to nano/micro systems has been briefly examined.

References

[1] V.E. Panin, Yu.V. Grinyaev, T.F. Elsukova, and A.G. Ivanchin, Structural levels of deformation in solids, Sov. Phys. J., 25, No. 6 (1982) 479.

[2] V.E. Panin, Dynamics of plastic deformation: Waves of localized plastic deformation in solids, Rus. Phys. J., 35, No. 4 (1992) 305.

[3] V.E. Panin (Ed.), Structural Levels of Plastic Deformation and Fracture, Nauka, Novosibirsk, 1990.

[4] V.E. Panin (Ed.), Physical Mesomechanics of Heterogeneous Media and Computer-Aided Design of Materials, Cambridge Interscience Publishing, Cambridge, 1998.

[5] V.E. Panin, Yu.V. Grinyaev, V.E. Egorushkin, I.L. Bukhbinder and S.N. Kul'kov, Spectrum of excited states and the rotational mechanical field in a deformed crystal, Sov. Phys. J., 30, No. 1 (1987) 24.

[6] S. Yoshida, Interpretation of mesomechanical behaviors of plastic deformation based on analogy to Maxwell electromagnetic theory, Phys. Mesomech., 4, No. 3 (2001) 29.

[7] S. Yoshida, Observation of plastic deformation wave in a tensile-loaded aluminum-alloy, Phys. Lett. A, 251 (1999) 54.

[8] S. Yoshida, Consideration on fracture of solid-state materials, Phys. Lett. A, 270 (2000) 320.

[9] I.J.R. Aitchison and A.J.G. Hey, Gauge Theories in Particle Physics, IOP Publishing, Ltd., Bristol, Philadelpia, 1989, Chap 2.

[10] I.R. Kenyon, General Relativity, Oxford Science Publication, Oxford, 1989, Chap 6.

[11] L.I. Schiff, Quantum Mechanics, McGraw Hill, Tokyo, 1968, Chap 7.

[12] R. Utiyama, Invariant theoretical interpretation of inertia, Phys. Rev., 101 (1956) 1597.

[13] S. Yoshida, Dynamics of plastic deformation and plastic deformation charge, Phys. Mesomech., 6, No. 4 (2003) 33.

[14] V.E. Panin et al., Method and Apparatus for Nondestructive Testing of the Mechanical Behavior of Objects under Loading Utilizing Wave Theory of Plastic Deformation, United State Patent, 5,508,801 (1996).