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Abstract

The article presents a novel relay control design strategy for some classes of nonlinear systems. The main idea of this work is to use an Linear Parameter-Varying (LPV) modeling to design nonlinear switching surfaces. This modeling is motivated by the fact that numerical tools such as LMI may be used by exploiting the quasi-linear structure of the system. Here we propose LMI formulation providing criteria for the design of switching surfaces. This design is seen as the synthesis of Parameter Dependent Relay (PDR). The second contribution of this article is the implementation of this relay strategy for Permanent Magnet Synchronous Motors (PMSM). Since the LPV framework can encompass the PMSM model, the obtained relay control depends on the motor speed variations. Experimental results on a PMSM are presented.

Index Terms

Relay feedback control, Linear Matrix Inequalities, switched systems, PMSM.

I. INTRODUCTION

The design of switching controller for nonlinear systems with input constraints represents a challenging area in control theory [1], [2], [3], [4]. Among the class of systems with input constraints [5], [6], relay feedback control has been applied in a wide range of technical domains. It may be found for example in servos-mechanisms in mechanics or in hydraulic systems, in power-electronic actuators with Pulse-Width Modulation (PWM) etc. Such control systems have been used in variable structure control [1], [7], [2], [8], since they offer interesting robustness properties faced to particular perturbations. The design of relay control systems is a non-trivial issue: the control design procedure has to take into account complex phenomena such as zeno solutions [9], limit cycles [10], [11], [12] and sliding modes [8], [1], [13]. In fact the system dynamics are inherently hybrid since to each control value we may associate a discrete state variable. Therefore, the synthesis of a relay feedback can be related with the design of switching surfaces in hybrid systems [14], [15], [16], [17].

Although relay feedback has been studied for a long time, there are still many unsolved issues. For the moment very few numerical tools exist for designing switching surfaces while optimizing the system performances or the size of the domain of attraction. Sliding mode control theory is well developed for generic nonlinear switching surfaces. However, for relay control, the exiting numerical optimization approaches can only handle switching surfaces described by affine hyperplanes [18], [19].

The main idea of this work is to use a Linear Parameter-Varying (LPV) [20], [21], [22], [23] modeling in order to design nonlinear switching surfaces. The LPV formulation has a long history in the automatic control literature. This modeling enables the use of well known linear design approaches to the case of nonlinear systems. The plant dynamics are rewritten in such a manner that the system nonlinearities are embedded in linear models as time-varying parameters. Powerful numerical optimization tools (such as LMI solvers [24]) can be applied in the design procedure by exploiting the quasi-linear system structure. The main theoretical contribution of the article is to provide criteria for the design of nonlinear switching surfaces by using an LMI formulation. From the point of view of LPV systems, the design of nonlinear switching surfaces is seen as the synthesis of Parameter Dependent Relay (PDR). For the sake of generality we consider a wide class of relays which is not only restricted to the classical *sign* controller (vector taking $\pm$ one as elements), but may take any values in the vertices of a time-varying convex polytope. Such a control can also encompass the simplex type variable structure control [25], [26], [27] for which few LMI design procedure exist. From a practical point of view, the second contribution of the article is the illustration of the proposed methodology for the control of a Permanent Magnet Synchronous Motor (PMSM). Indeed, PMSM are usually controlled by relays and thus only a finite set of control values is available. However in most of classical control design methods the use of averaging and PWM ignores the relay nature of the actuator [28], [29]. Here we propose a direct relay control which may use the advantages of the switching actuator in power electronics. The LPV framework encompasses the PMSM model. The obtained switching surfaces depend in a nonlinear manner on the motor speed. Preliminary results have been proposed in the conference version [30].

The article is divided into five sections. Section II presents the motivation of the article. An LPV representation is derived from the PMSM model and the relay nature of the actuators is exhibited. Section III is dedicated to the design of the PDR control for generic model of LPV systems. In Section IV, an LMI criterion is proposed for the stabilization problem. This section also provides tools for the estimation of the domain of attraction. The theory presented in this article is illustrated through an academic example in Section V. The last section, Section VI, is devoted to the real implementation of the control design procedure on a PMSM.

Notations: The identity (or null) matrix with appropriate dimension is denoted by $\mathbf{I}$ (or $\mathbf{0}$). For a real square symmetric matrix M, $M > 0$ ($M < 0$) indicates that M is positive (negative) definite. For a full rank square symmetric matrix M, M^{-1} denotes the inverse of M. In a symmetric matrix $*$ denotes an element that may be inferred by symmetry.

For a matrix $A \in \mathbb{R}^{n \times n}$, we denote the Hermitian of A by: $\mathcal{H}e\{A\} = A + A^T$. We denote by $\mathcal{I}_N$ the set $\{1, 2, \ldots, N\}$. The symbol Δ_N denotes the unit simplex in $\mathbb{R}^N$ defined by:

$$\Delta_N = \left\{ \mu \in \mathbb{R}^N : \mu_i \geq 0, i \in \mathcal{I}_N, \sum_{i=1}^{N} \mu_i(t) = 1 \right\}.$$

For a given set $\mathcal{S} \in \mathbb{R}^n$ the symbol $\overline{\text{conv}}(\mathcal{S})$ denotes the closed convex hull of the set. For a symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$ and a positive scalar c we denote $\mathcal{E}(P, c) = \{x \in \mathbb{R}^n : x^T P x \leq c\}$. Let $\mathcal{B}(x, c)$ denotes the open ball centered on $x \in \mathbb{R}^n$ with radius $c > 0$, $\mathcal{B}(x, c) = \{y \in \mathbb{R}^n : |x - y| < c\}$.
Given a constant scalar $U > 0$, we define the finite set $\Psi_m(U) = \{u \in \mathbb{R}^m : u_i \in \{-U, U\}, i \in \mathcal{I}_m\}$.
Given a continuous function $f : \mathbb{R}^n \to \mathbb{R}$ and a compact set $\mathcal{S}$ we denote by $\underset{s \in \mathcal{S}}{\arg\min} f(s) = \{s \in \mathcal{S} : f(s) \leq f(r), \forall r \in \mathcal{S}\}$.
For a set $\mathcal{S} \subset \mathbb{R}^m, K \in \mathbb{R}^{m \times n}$, we denote $\mathcal{C}_{\mathcal{S}}(K) = \{x \in \mathbb{R}^n : Kx \in \overline{\text{conv}}(\mathcal{S})\}$.

II. MOTIVATION

In this section the speed model of the PMSM is presented and the practical problem that motivated our research is introduced. The equations (1) give the standard PMSM model in the phase (or winding) variables [31]:

$$\begin{cases} L\dfrac{di_\alpha}{dt} &= v_\alpha - Ri_\alpha + K\Omega \sin(n_p\theta), \\[2mm] L\dfrac{di_\beta}{dt} &= v_\beta - Ri_\beta - K\Omega \cos(n_p\theta), \\[2mm] J\dfrac{d\Omega}{dt} &= K\left(i_\beta \cos(n_p\theta) - i_\alpha \sin(n_p\theta)\right) - f_v\Omega - \tau, \end{cases} \tag{1}$$

where v_α and v_β are the voltages applied to the two phases of the PMSM, i_α and i_β are the two phase currents, L is the inductance of a phase winding, R is the resistance of a phase winding, K is the back-EMF constant (and also the torque constant), n_p is the number of pole pairs (or rotor teeth), J is the moment of inertia of the rotor (including the load), f_v is the coefficient of viscous friction and τ represents the load torque. The variable θ is the angular position of the rotor, $\Omega = d\theta/dt$ is the angular velocity of the rotor. While for particular applications the variable θ can be included in the state vector, in this article we consider only the speed control, justifying the fact that θ is not in the state vector. We are interested in the stability of the velocity in a constant value. In this case the position θ is time varying. For this reason θ is not included in the state vector.

The non-linear state space representation of the system of equations (1) is given by:

$$\dot{x}_{\alpha\beta}(t) = f(x_{\alpha\beta}, t) + Bv_{\alpha\beta}(t) + D\varpi(t), \tag{2}$$

where $x_{\alpha\beta}^T = \begin{bmatrix} i_\alpha & i_\beta & \Omega \end{bmatrix}$, $v_{\alpha\beta}^T = \begin{bmatrix} v_\alpha & v_\beta \end{bmatrix}$ and $\varpi = \tau$. The function $f(x_{\alpha\beta}, t)$ is defined by:

$$f(x_{\alpha\beta}, t) = \begin{bmatrix} -\dfrac{R}{L}i_\alpha(t) + \dfrac{K}{L}\Omega(t)\sin(n_p\theta(t)) \\[3mm] -\dfrac{R}{L}i_\beta(t) - \dfrac{K}{L}\Omega(t)\cos(n_p\theta(t)) \\[3mm] \dfrac{K}{J}\left(i_\beta(t)\cos(n_p\theta(t)) - i_\alpha(t)\sin(n_p\theta(t))\right) - \dfrac{f_v}{J}\Omega(t) \end{bmatrix}, B = \begin{bmatrix} \dfrac{1}{L} & 0 \\[2mm] 0 & \dfrac{1}{L} \\[2mm] 0 & 0 \end{bmatrix} \text{ and } D = \begin{bmatrix} 0 \\[1mm] 0 \\[1mm] \dfrac{1}{J} \end{bmatrix}.$$

Considering that each motor phase is actuated via commutation, the control vector $v_{\alpha\beta}$ belongs to the set $\Psi_2(V)$, where V represents the maximal voltage.
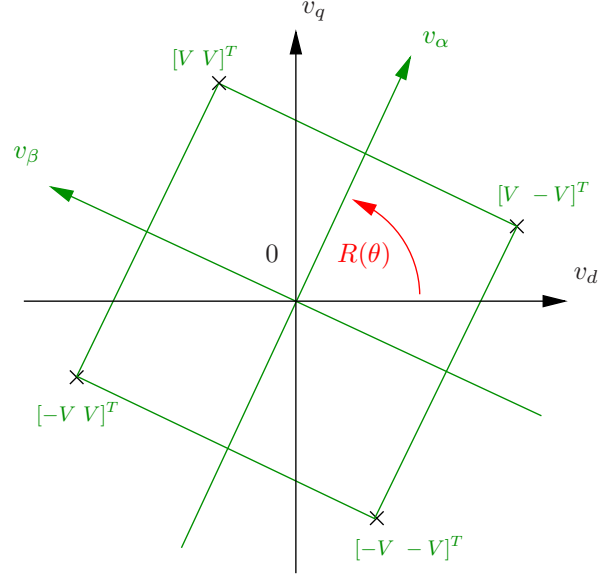
Fig. 1. Finite set of control in the fixed frame and in the rotating frame.

Model in the rotating frame $(d-q)$

In the phases frame the signals i_α and i_β vary at n_p times the frequency of rotation. This high frequency problem is alleviated by the use of the direct quadrature $(d-q)$ transformation, also known as the Park transformation [32]. This transformation changes the frame of reference from the fixed phase axes to axes moving with the rotor. Equation (3) gives the transformation performed to obtain the rotating frame:

$$R(\theta(t)) = \begin{bmatrix} \cos(n_p\theta(t)) & \sin(n_p\theta(t)) \\ -\sin(n_p\theta(t)) & \cos(n_p\theta(t)) \end{bmatrix}, \begin{bmatrix} i_d(t) \\ i_q(t) \end{bmatrix} = R(\theta(t)) \begin{bmatrix} i_\alpha(t) \\ i_\beta(t) \end{bmatrix} \text{ and } \begin{bmatrix} v_d(t) \\ v_q(t) \end{bmatrix} = R(\theta(t)) \begin{bmatrix} v_\alpha(t) \\ v_\beta(t) \end{bmatrix}. \tag{3}$$

The state space representation is then given by:

$$\dot{x}_{dq}(t) = A_{dq}(\Omega(t))x_{dq}(t) + Bv_{dq}(t) + D\varpi(t), \tag{4}$$

where $x_{dq}^T = \begin{bmatrix} i_d & i_q & \Omega \end{bmatrix}$, $v_{dq}^T = \begin{bmatrix} v_d & v_q \end{bmatrix}$, and,

$$A_{dq}(\Omega(t)) = \begin{bmatrix} -\dfrac{R}{L} & n_p\Omega(t) & 0 \\ -n_p\Omega(t) & -\dfrac{R}{L} & -\dfrac{K}{L} \\ 0 & \dfrac{K}{J} & -\dfrac{f_v}{J} \end{bmatrix}.$$

The matrices B and D remain unchanged. Consider that $\Omega(t)$ ranges between known extremal values $\Omega(t) \in [\underline{\Omega}, \overline{\Omega}]$. In this frame the PMSM can be described using an LPV state space representation. The state space representation of the system depends linearly on a vector of time-varying parameters: $\Omega(t)$. The model may be represented as follows:

$$\begin{cases} \dot{x}_{dq}(t) &= A(\alpha(t))x_{dq}(t) + Bv_{dq}(t) + D\varpi(t), \\ A(\alpha(t)) &= \displaystyle\sum_{i=1}^{N_A} \alpha_i(t)A_i, \ \forall i, \ \alpha_i(t) \geq 0, \sum_i \alpha_i(t) = 1, \end{cases} \tag{5}$$

where $N_A = 2$, with $A_1 = A_{dq}(\underline{\Omega})$, $A_2 = A_{dq}(\overline{\Omega})$. The controls $v_{dq}(t)$ are defined for all $\theta \in [0, 2\pi]$ by:

$$v_{dq}(t) = k(x_{dq}(t), \theta(t)), \forall k \in \mathbb{R}^n \times [0, 2\pi] \to \mathbb{R}^m \tag{6}$$

Note that the control $v_{dq}(t)$ is a PDR control which takes values in a finite set of vectors depending on θ: $\{u \in \mathbb{R}^2 : \exists v \in \Psi_2(V), u = R(\theta(t))v\}$. The input vector in the different frames is represented in Fig. 1. For a given V, the objective is to determine the switching surfaces in the state space, which ensure the closed loop stability of the system (5) with the control law (6). This represents one of the motivations for the theoretical developments in the paper.

III. PARAMETER DEPENDENT RELAY (PDR) CONTROL

In this section we will present a generic model of LPV systems. This LPV framework encompasses the PMSM model. Local stabilization results will be presented for this model.

A. Generic problem formulation

We consider the class of Linear Parameter-Varying (LPV) systems with the state-space realization:

$$\dot{x} = A(\mu)x + B(\mu)u, \tag{7}$$

where $x \in \mathbb{R}^n$ is the state vector and $u \in \mathbb{R}^m$ is the control vector, the matrices $A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}$ are polytopic matrices with the following form:

$$A(\mu(t)) = \sum_{i=1}^{N} \mu_i(t)A_i, B(\mu(t)) = \sum_{i=1}^{N} \mu_i(t)B_i, \tag{8}$$

with $A_1, \ldots, A_N, B_1, \ldots, B_N$ being known constant matrices. The vector $\mu(t) = \begin{bmatrix} \mu_1(t) & \ldots & \mu_N(t) \end{bmatrix}^T$ is a vector of real and known parameters which evolves piecewise continuously in the unit simplex Δ_N. Such models are interesting since they can be useful for absorbing locally the behavior of more complex affine nonlinear systems [33], [34]. Here we consider the LPV representation as a starting point. From now, the manner in which it is obtained from a nonlinear model is neglected (see [34] for details).

We assume that for each $\mu \in \Delta_N$ the control u may only take values in a finite set which depends on the parameter μ. We define this set of finite values $\mathcal{V}_\mu$ by:

$$\mathcal{V}_\mu = \{v_i(\mu), i \in \mathcal{I}_k\}, v_i : \Delta_N \to \mathbb{R}^m, \forall i \in \mathcal{I}_k. \tag{9}$$

We consider that $\overline{\text{conv}}(\mathcal{V}_\mu)$ is a non empty bounded set containing the origin in its interior for any $\mu \in \Delta_N$. The objective is to find a PDR control $u = K^d(x, \mu)$ which locally stabilizes the system (7):

$$K^d : \mathbb{R}^n \times \Delta_N \to \mathcal{V}_\mu. \tag{10}$$

Given that u is discontinuous, the closed loop vector field is discontinuous. System solutions are considered in the sense of Filippov [35]. We recall the definition of local asymptotic stability. One poses:

$$\dot{x} = f(t, x(t)), \text{ where } f(t, x(t)) = A(\mu(t))x(t) + B(\mu(t))K^d(x(t), \mu(t)). \tag{11}$$

Definition 1. *[36] The equilibrium point $x = 0$ of (11) is*
- *stable if, for each $\epsilon > 0$, there is $\delta = \delta(\epsilon, t_0) > 0$ such that $||x(t_0)|| < \delta \Rightarrow ||x(t)|| < \epsilon, \forall t \geq t_0 \geq 0$,*
- *locally asymptotically stable at the origin if it is stable and there is $c = c(t_0) > 0$ such that $x(t) \to 0$ as $t \to \infty$, for all $||x(t_0)|| < c$.*

B. Stabilization by PDR control

In this subsection we present a method for the construction of the PDR control (10). The main idea is to use the existence of any stabilizing control law (possibly continuous) in order to redesign a locally stabilizing relay control. The following proposition allows to use a previously designed control law to find a relay based one.

Proposition 1. Consider system (7) with the description (8). Consider $\Gamma \subset \mathbb{R}^n$ be a domain containing $x = 0$. Assume that there exists a control $u = K(x, \mu)$, with $K : \mathbb{R}^n \times \Delta_N \to \mathbb{R}^m$ such that $K(x, \mu) \in \overline{\text{conv}}(\mathcal{V}_\mu), \forall \mu \in \Delta_N, x \in \Gamma \setminus \{0\}$. Let $V(x), V : \Gamma \to \mathbb{R}$, be a continuously differentiable function such that

$$W_1(x) \leq V(x) \leq W_2(x), \ \frac{\partial V}{\partial x}\left(A(\mu)x + B(\mu)K(x, \mu)\right) < -W_3(x), \forall \mu \in \Delta_N, x \in \Gamma \setminus \{0\}, \tag{12}$$

where $W_1(x), W_2(x)$ and $W_3(x)$ are continuous positive definite functions on Γ. Then system (7) with the control law:

$$u = K^d(x, \mu) \in \underset{v \in \mathcal{V}_\mu}{\text{argmin}} \frac{\partial V}{\partial x}B(\mu)v, x \in \Gamma, \tag{13}$$

is locally asymptotically stable when solutions are understood in the sense of Filippov. Furthermore, for any level set $\mathcal{L}_V(c) = \{x \in \mathbb{R}^n : V(x) \leq c\}$ such that $\mathcal{L}_V(c) \subseteq \Gamma$, the following relation is satisfied for any Filippov solution $x(t)$ originating from the initial condition x_0:

$$x_0 \in \mathcal{L}_V(c) \Rightarrow \lim_{t \to \infty} ||x(t)|| = 0, \tag{14}$$

i.e. $\mathcal{L}_V(c)$ is an inner estimation of the domain of attraction.

Proof: In order to show the local stability of the closed loop system when solution are understood in the sense of Filippov [35] we consider the differential inclusion $\dot{x} \in F(x,t)$, where

$$F(x,t) = \overline{\text{conv}} \left(\left\{ A(\mu(t))x + B(\mu(t))\tilde{u}, \tilde{u} \in \underset{v \in \mathcal{V}_\mu}{\text{argmin}} \frac{\partial V}{\partial x} B(\mu(t))v \right\} \right), \mu(t) \in \Delta_N, \tag{15}$$

and show that:

$$\max_{y \in F(x,t)} \frac{\partial V}{\partial x} y < -W_3(x), \forall x \in \Gamma \setminus \{0\}, y \in F(x,t). \tag{16}$$

This is a sufficient condition for local stabilization [35], since $V(x)$ acts as a Lyapunov function. Furthermore (16) implies that the level sets of V contained in Γ provide an inner estimation of the domain of attraction since they are invariant sets.

Suppose that there exists a feedback $u = K(x,\mu)$, such that (12) holds, *i.e.* there exists a state feedback such that the equilibrium point $x = 0$ is locally asymptotically stable (see Theorem 4.9 p.152 [37]). Then one has:

$$\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)K(x,\mu)) < -W_3(x), \forall x \neq 0, \forall \mu \in \Delta_N. \tag{17}$$

Since $\forall x \in \Gamma, \mu \in \Delta_N \Rightarrow K(x,\mu) \in \overline{\text{conv}}(\mathcal{V}_\mu)$, then for any $x \in \Gamma$ and $\mu \in \Delta_N$ there exist k scalars $\alpha_i(x,\mu) \geq 0, i \in \mathcal{I}_k$, with $\sum_{i=1}^k \alpha_i(x,\mu) = 1$ such that:

$$K(x,\mu) = \sum_{i=1}^k \alpha_i(x,\mu)v_i(\mu), \tag{18}$$

holds. From (17) and (18) we have:

$$\sum_{i=1}^k \alpha_i(x,\mu) \left(\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)v_i(\mu)) + W_3(x) \right) < 0, \forall \mu \in \Delta_N \text{ and } \forall x \in \Gamma \setminus \{0\}. \tag{19}$$

Considering $\alpha_i(x,\mu) > 0$, $\mu \in \Delta_N$ and $i \in \mathcal{I}_k$, then for any $x \in \Gamma \setminus \{0\}$ there must be at least one, $x-$dependent index, $i = i(x) \in \mathcal{I}_k$ such that:

$$\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)v_i(\mu)) < -W_3(x), \forall \mu \in \Delta_N. \tag{20}$$

Consider the subset of control vectors in $\mathcal{V}_\mu$ minimizing the expression (20):

$$\mathcal{V}_{min}(x,\mu) = \underset{v \in \mathcal{V}_\mu}{\text{argmin}} \frac{\partial V}{\partial x} (A(\mu)x + B(\mu)v). \tag{21}$$

Note that $A(\mu)x$ does not depend on v_i, thus for a given x the subset $\mathcal{V}_{min}(x,\mu)$ can be rewritten as

$$\mathcal{V}_{min}(x,\mu) = \underset{v \in \mathcal{V}_\mu}{\text{argmin}} \frac{\partial V}{\partial x} (B(\mu)v). \tag{22}$$

Since there exists at least one index such that (20) holds, then for all $x \in \Gamma \setminus \{0\}$ there exist at least one control $v \in \mathcal{V}_{min}(x,\mu) \subset \mathcal{V}_\mu$ such that

$$\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)v) < -W_3(x), \forall \mu \in \Delta_N. \tag{23}$$

When for some $x \in \Gamma \setminus \{0\}$ there are several minimizers in $\mathcal{V}_{min}(x,\mu)$ (that is the cardinal of $\mathcal{V}_{min}(x,\mu)$ is greater than 1), then for any control vector $\tilde{u} \in \mathcal{V}_{min}(x,\mu)$

$$\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)\tilde{u}) < -W_3(x), \forall \mu \in \Delta_N. \tag{24}$$

In this case, given any two minimizers $\forall \tilde{u}_a, \tilde{u}_b \in \mathcal{V}_{min}(x,\mu), \forall x \in \Gamma \setminus \{0\}$ one has

$$\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)\tilde{u}_a) < -W_3(x) \text{ and } \frac{\partial V}{\partial x} (A(\mu)x + B(\mu)\tilde{u}_b) < -W_3(x). \tag{25}$$

Therefore $\forall x \in \Gamma \setminus \{0\}, \forall \mu \in \Delta_N$ and $\forall \rho \in [0,1]$ one has:

$$\begin{aligned} &\rho \frac{\partial V}{\partial x} (A(\mu)x + B(\mu)\tilde{u}_a) < -\rho W_3(x) \\ &\text{and } (1-\rho)\frac{\partial V}{\partial x} (A(\mu)x + B(\mu)\tilde{u}_b) < -(1-\rho)W_3(x). \end{aligned} \tag{26}$$

By summing the two inequalities in (26) $\forall x \in \Gamma \setminus \{0\}$ and $\forall \mu \in \Delta_N$ one has

$$\frac{\partial V}{\partial x}(A(\mu)x + B(\mu)\tilde{u}) < -W_3(x), \forall \tilde{u} \in \overline{\text{conv}}(\mathcal{V}_{min}(x, \mu)), \forall \mu \in \Delta_N. \tag{27}$$

Considering the differential inclusion (15) then from (27) we have that

$$\frac{\partial V}{\partial x}y < -W_3(x), \forall x \in \Gamma \setminus \{0\}, \forall y \in F(x, t), \tag{28}$$

which using (16) is sufficient for the local stability with the control function (13) when solutions are understood in the sense of Filippov. ∎

Remark 1. The previous theorem uses the existence of any stabilizer $K(x, \mu)$ (possibly continuous) in order to redesign a PDR control $K^d(x, \mu)$ which takes values only in the set $\mathcal{V}_\mu(x, \mu)$. Comparing (17) with (28) we have shown that the PDR has at least the same guaranteed decay of the Lyapunov function as $K(x, \mu)$. The result provides a general theoretical framework for the design of PDR. In the following section we will show how this result can be used in a constructive manner.

IV. LMI DESIGN CONDITION

In this section the LMI framework [24] is exploited to provide tractable and constructive conditions for the design of a PDR control. An estimation of the domain of attraction with the proposed control law is equally provided. Generally, when using a relay control law, only local stability may be ensured. Therefore, it is of interest to provide an estimation of the domain of attraction.

Considering that for all $\mu \in \mathcal{V}_\mu$, $\overline{\text{conv}}(\mathcal{V}_\mu)$ is non empty and contains the origin in its interior, remark that there exists a polytopic region:

$$\mathcal{Q} = \overline{\text{conv}}(\{q_1, q_2, \ldots, q_p\}) = \{z \in \mathbb{R}^m : h_i z \leq 1, i \in \mathcal{I}_{N_h}\}, \quad \text{such that } \mathcal{Q} \subset \overline{\text{conv}}(\mathcal{V}_\mu), \forall \mu \in \Delta_N \text{ and } 0 \in \text{Int}\{\mathcal{Q}\}. \tag{29}$$

Using the polytope $\mathcal{Q}$ one can adjust the design conditions to include an LMI based optimization of the domain of attraction.

Proposition 2. Consider system (7). Consider $\Gamma \subset \mathbb{R}^n$ be a domain containing $x = 0$. Assume that there exists $Q = Q^T > 0$, $Y_i \in \mathbb{R}^{m \times n}$, $i \in \mathcal{I}_N$ and a positive scalar δ such that:

$$\mathcal{H}e\{(A_i + A_j)Q + B_i Y_j + B_j Y_i\} < -2\delta Q, \; i, j \in \mathcal{I}_N, \tag{30a}$$

$$\begin{bmatrix} 1 & h_i Y_j \\ * & Q \end{bmatrix} > 0, i \in \mathcal{I}_{N_h}, j \in \mathcal{I}_N, \tag{30b}$$

$$\begin{bmatrix} eI & I \\ * & Q \end{bmatrix} > 0. \tag{30c}$$

Let

$$u = K^d(x, \mu) \in \underset{v \in \mathcal{V}_\mu}{\text{argmin}} \; x^T Q^{-1} B(\mu)v, x \in \Gamma. \tag{31}$$

Then the equilibrium point $x = 0$ of the closed-loop system (7)-(31) is locally asymptotically stable. An estimation of the domain of attraction is provided by the ellipsoid $\mathcal{E}(Q^{-1}, 1)$ containing the ball $\mathcal{B}(0, \sqrt{\epsilon})$ with $\epsilon = \frac{1}{e}$, i.e. $\forall x(0) \in \mathcal{E}(Q^{-1}, 1), \lim_{t \to \infty} ||x(t)|| = 0$.

Proof: Assume that the set of LMI conditions (30) holds for system (7) with the description (8). Multiplying the set of LMIs (30a) by $\mu_i, \mu_j \in \Delta_N$, $i, j \in \mathcal{I}_N$ and summing, followed by the multiplication of both side by Q^{-1} yields

$$2x^T Q^{-1} \left(\sum_{i=1}^{N} \sum_{j=1}^{N} \mu_i \mu_j \frac{A_i + B_i K_j + A_j + B_j K_i}{2} \right) < -2\delta x^T Q^{-1} x, \forall x \neq 0, \tag{32}$$

with $K_i = Y_i Q^{-1}$. Then the function $V(x) = x^T Q^{-1} x$, satisfies

$$\frac{\partial V}{\partial x}(A(\mu)x + B(\mu)K(x, \mu)) < -2\delta V(x), \forall x \in \mathbb{R}^n \setminus \{0\}, \forall \mu \in \Delta_N, \tag{33}$$

with

$$K(x, \mu) = \sum_{i=1}^{N} \mu_i Y_i Q^{-1} x. \tag{34}$$

Then $u = K(x, \mu)$ is a global stabilizer for system (7). It will be used for re-designing a locally stabilizing PDR based on Proposition 1.

First we compute a domain Γ where $K(x,\mu) \subset \overline{\text{conv}}(\mathcal{V}_\mu), \forall \mu \in \Delta_N$. The set of LMI constraints (30b) ensures that

$$\forall x \in \mathcal{E}(Q^{-1},1) \Rightarrow K(x,\mu) \in \mathcal{Q} \subset \overline{\text{conv}}(\mathcal{V}_\mu), \forall \mu \in \Delta_N. \tag{35}$$

To prove this we use the Constrained Quadratic Lemma (given in the Appendix) as follows. Using (29) with the definition of $K(x,\mu)$ in (34), one can note that for the condition (35) to be true, it is sufficient that none of the hyperplanes $h_i Y_j Q^{-1} x = 1, i \in \mathcal{I}_{N_h}, j \in \mathcal{I}_N$, crosses the ellipsoid $\mathcal{E}(Q^{-1},1)$. With Lemma 1, it leads to:

$$1 \;\leq\; \min_{h_i Y_j Q^{-1} x = 1} x^T Q^{-1} x = \min_{i \in \mathcal{I}_{N_h}} \left(h_i Y_j Q^{-1} Q (Y_j Q^{-1})^T (h_i)^T \right)^{-1}. \tag{36}$$

Using the Schur complement Lemma, the latter leads to (30b).

To guarantee that $\mathcal{B}(0,\sqrt{\epsilon})$ is included in the domain of attraction $\mathcal{E}(Q^{-1},1)$, we add the constraints

$$x^T x < \epsilon \Rightarrow x^T Q^{-1} x < 1. \tag{37}$$

If $x^T Q^{-1} x < \dfrac{1}{\epsilon} x^T x$ then (37) holds, with $e = \dfrac{1}{\epsilon}$ the LMI problem becomes (30c). Therefore if (30b) holds we obtain that for $K(x,\mu) \in \mathcal{Q} \subset \overline{\text{conv}}(\mathcal{V}_\mu), \forall x \in \mathcal{E}(Q^{-1},1), \mu \in \Delta_N$. Applying Proposition 1, with $\Gamma = \mathcal{L}_v(1) = \mathcal{E}(Q^{-1},1)$, $K(x,\mu)$ defined in (34) and satisfying the constraint (35), allows to show that the control law (31) ensures the local stabilization in $\mathcal{E}(Q^{-1},1)$. Furthermore, using (37) we show that the domain of attraction contains the ball $\mathcal{B}(0,\sqrt{\epsilon})$. ∎

Remark 2. Proposition 2 is inspired by convex optimization arguments from [38], [39], [40]. The feasibility of the LMI optimization problem (30) with some matrix Q, guarantees that any solutions of the LPV system (7) with the control (13) originating from the invariant ellipsoid $\mathcal{E}(Q^{-1},1)$ is converging to the origin with a decay rate δ. By minimizing e, we increase the radius $\sqrt{\epsilon}$ of the ball $\mathcal{B}(0,\sqrt{\epsilon})$ included in $\mathcal{E}(Q^{-1},1)$. This allows to provide an inner ellipsoidal approximation $\mathcal{E}(Q^{-1},1)$ of the domain of attraction.

V. ACADEMIC EXAMPLE

In order to illustrate the results presented in this section, we propose to show simulations through a simple second order system so that the trajectories of the system can be plotted in a two dimensions phase portrait. We consider the system:

$$\dot{x}(t) = A_0 x(t) + B_0(x_1(t)) u(t), \tag{38}$$

with $x = [x_1 \quad x_2]^T$ in $\mathbb{R}^2$, $u \in \mathbb{R}^2$, $A_0 \in \mathbb{R}^{2\times 2}$ and $B_0 \in \mathbb{R}^{2\times 2}$ defined by

$$A_0 = \begin{bmatrix} 0 & 3 \\ 1 & 1 \end{bmatrix}, B_0(x_1(t)) = \begin{bmatrix} 1 + 0.5\sin(x_1(t)) & 0 \\ 0 & 1 + 0.5\sin(x_1(t)) \end{bmatrix}.$$

For each $x_1(t)$, the control $u(t)$ is constrained to switch among four different values in the set $\{R(x_1(t))\rho, \rho \in \Psi_2(V)\}$. The matrix $R(x_1(t))$ is the rotation matrix defined by

$$R(x_1(t)) = \begin{bmatrix} \cos(x_1(t)) & \sin(x_1(t)) \\ -\sin(x_1(t)) & \cos(x_1(t)) \end{bmatrix}. \tag{39}$$

The relay value V is equal to 10, leading to two inputs each taking values in $\{-V,V\}$ which is a square of side length $2 \times V$.

Considering as bounded time-varying parameter $\sin(x_1), \cos(x_1)$, the system (38) may be rewritten as an LPV system of the form (7) defined by:

$$\dot{x}(t) = Ax(t) + B(\mu(t))u(t) \tag{40}$$

with $A = A_0$ and $B(\mu(t)) = \displaystyle\sum_{i=1}^{2} \mu_i(t) B_i = B_0(x_1(t))$, where $\mu_1(t) = \dfrac{1 - \sin(x_1(t))}{2}$, $\mu_2(t) = \dfrac{1 + \sin(x_1(t))}{2}$ and

$$B_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix}.$$

The control u takes values in the finite set (9) defined by

$$\mathcal{V}_{\mu(t)} = \{v_i(\mu(t)), i \in 1, \ldots, 4\} = \{R(x_1(t))\rho, \rho \in \Psi_2(V)\}. \tag{41}$$

In order apply Proposition 2 we need to construct a polytopic region $\mathcal{Q}$ such that equation (29) is satisfied.

Note that all squares defined by $\overline{conv}(\mathcal{V}_{\mu(t)})$ are centered at 0 and have the same size thus the disc of radius V centered at 0 belong to all $\overline{conv}(\mathcal{V}_{\mu(t)})$ (see Fig. 2). This disc can be approximated by the polytope $\mathcal{Q}$ represented in brown Fig. 2 for
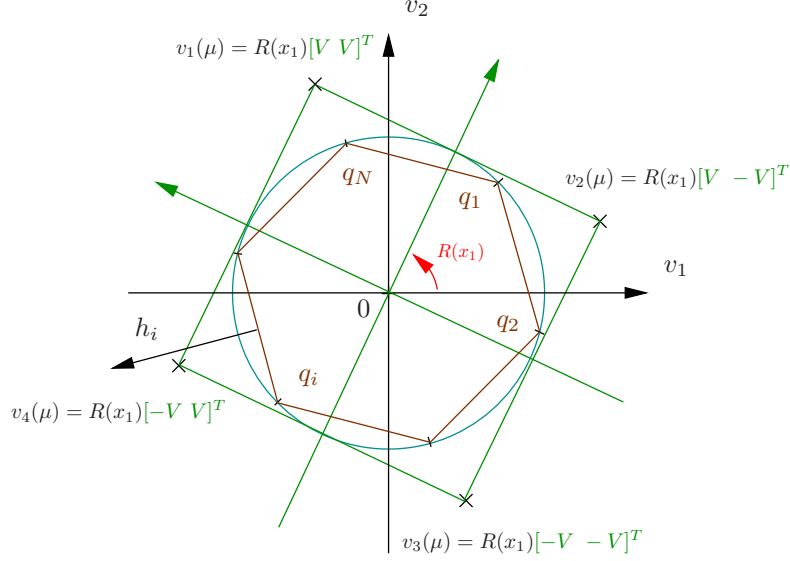
Fig. 2. Representation of control sets $\mathcal{V}_{mu}$ for the system in the simulation example.

which the vertices q_i are given by

$$q_{i+1} = V \begin{bmatrix} \cos\left(\dfrac{2i\pi}{p}\right) \\ \sin\left(\dfrac{2i\pi}{p}\right) \end{bmatrix}, i = 0, \ldots, p-1. \tag{42}$$

Each face of the polytope can be characterized by its normal:

$$h_{i+1} = \frac{q_i + q_{i+1}}{1 + \cos\left(\frac{2\pi}{p}\right)}, i = 0, \ldots, p-1. \tag{43}$$

The simulations are realized using Matlab/Simulink and the LMI in the Proposition 2 are solved using SeDuMi ([41]). In order to approximate the inscribed disc by the polytope $\mathcal{Q}$ we take $p = 15$. Choosing a decay rate $\delta = 4$ and applying Proposition 2, the LMI solver returns the matrices Q and $Y_i, i \in \mathcal{I}_2$ matrices:

$$Q = \begin{bmatrix} 43.17 & -18.86 \\ -18.86 & 9.77 \end{bmatrix}, Y_1 = \begin{bmatrix} -59.53 & 21.82 \\ 21.82 & -20.88 \end{bmatrix}, Y_2 = \begin{bmatrix} -21.70 & 7.66 \\ 7.66 & -8.17 \end{bmatrix}. \tag{44}$$

The Q matrix defines the parameter dependent relay control (31) and thus the switching regions. These regions are plotted Fig. 3 as function of the states x_1 and x_2. On this figure r_1 is the region for which the argument of the minimum is given for the control input $v_1\left(\mu(t)\right) = R\left(x_1(t)\right)\left[V \quad V\right]^T$, r_2 for $v_2\left(\mu(t)\right) = R\left(x_1(t)\right)\left[V \quad -V\right]^T$, r_3 for $v_3\left(\mu(t)\right) = R\left(x_1(t)\right)\left[-V \quad V\right]^T$, r_4 for $v_4\left(\mu(t)\right) = R\left(x_1(t)\right)\left[-V \quad -V\right]^T$.

To illustrate the theoretical results, one proposes to compare the Continuous State Feedback (CSF) control law (34) with the PDR control (31). In the continuous case, the control input applied to the system denoted by ρ in the description of the system is in $\mathbb{R}^2$ but it has elements saturated in the interval $[-V, V]$. The phase portrait of the states x_1 and x_2 for both cases are plotted Fig. 4. On theses figures, we have plotted in red the ellipsoid $\mathcal{E}(Q^{-1}, 1)$, characterizing the domain of attraction of the system. The brown lines represent the hyperplanes $h_i Y_j Q^{-1} = 1$. In accordance with condition (35) it shows that none of the hyperplanes crosses the domain of attraction.

The first simulation is executed while taking initial conditions outside the attractive ellipsoid. On the figure, the initial condition is denoted by $x_{0,1}$. One observes that outside the attraction domain, the closed-loop system does not converge to the origin. The second simulation is realized with the initial condition $x_{0,2}$, near the domain of attraction, but outside. The figures show that in this case, the trajectories are converging to the origin. Finally, the initial condition $x_{0,3}$ is taken inside the domain of attraction. In this case the trajectories also converge to the origin. For this example the attractive ellipsoid contains the ball $|x| < \epsilon$ with $\epsilon = 1.28$ and the initial condition $x_{0,3}$ with $|x_{0,3}| = 4.14$. Note that $|x_{0,1}| = 7.07$ and $|x_{0,2}| = 4.24$, this gives an idea about the conservatism introduced in the estimation of the domain of attraction. In comparison with the static feedback proposed in [30], the same settings lead to ϵ equal to 0.58, the PDR enlarges considerably the ellipsoid estimation of the domain of attraction.
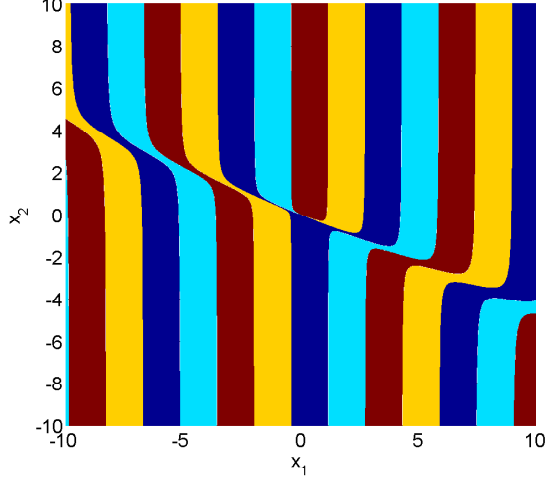
Fig. 3. Representation of the switching region, (the region r_1 in dark blue, r_2 in light blue, r_3 in yellow and r_4 in red).
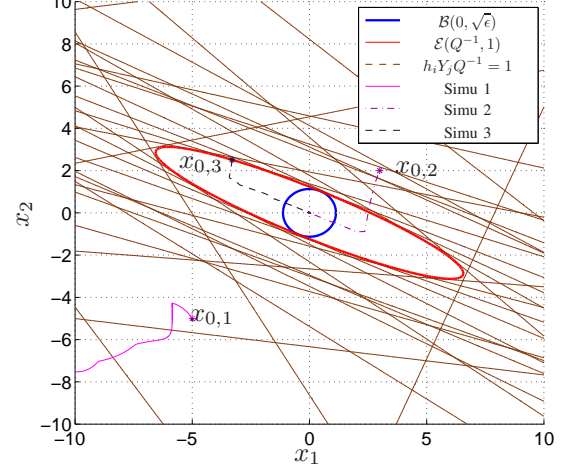

Fig. 4. Characterization of the domain of attraction and simulation results of the PDR.
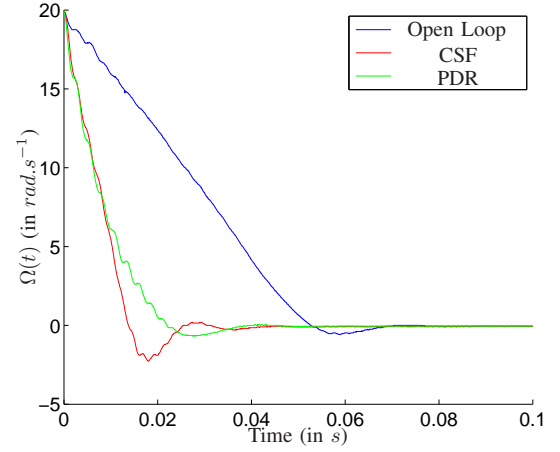

Fig. 5. PMSM test-bench.

Fig. 6. Comparison between the CSF and PDR control of the motor velocity stabilization.

VI. PMSM APPLICATION

This section is devoted to the experimental application on the PMSM, to illustrate the proposed control design methodology on a concrete case.

A. Test-bench description

The experiments are realized using a stepper motor bench developed in LAGIS at École Centrale de Lille (see Fig. 5). The parameters of the motor with coils in series have been identified using the offline procedure described in [42], leading to $L = 9mH$, $R = 3.01\Omega$, $K = 0.27N.m.A^{-1}$ and $J = 3.18.10^{-4}kg.m^2$. The number of pole pairs is $n_p = 50$. The input voltages v_a and v_b of each coil are delivered by two D/A outputs of the dSpace card and amplified by two linear power amplifiers (this means that the controls are directly applied to the coils without PWM implementation). The currents i_a and i_b are measured using Hall effect sensors with a precision of 1% of the nominal current $I_n = 3A$. The power supply provides a maximum voltage $v_{max} = 20V$ and $i_{max} = 3A$. The sampling period for this experiment is constant and equals to $10^{-4}s$ for the control.

B. Experimental results

We design a control law where we consider that only four control inputs are available. In this article, the control design is considered with the assumption that there is no external torque (*i.e.* $\tau = 0$). In the section tracking, an unknown torque will still be applied to the motor to show the disturbance rejection property of the new PDR control law. An integral action

is implemented with ζ the ouput of the integrator ($\zeta(0) = 0$)) [43] to ensure tracking performance with respect to a reference Ω_{ref}. The integral action given by:

$$\dot{\zeta} = \Omega - \Omega_{ref} = Cx - \Omega_{ref}, C = [0 \quad 0 \quad 1], \tag{45}$$

where ζ is the output of the integrator ($\zeta(0) = 0$). The combination of the state space representation (4) and the integral action without torque can be re-written as:

$$\underbrace{\begin{bmatrix} \dot{x}_{dq} \\ \dot{\zeta} \end{bmatrix}}_{\dot{z}} = \underbrace{\begin{bmatrix} A_{dq}(\Omega) & 0 \\ C & 0 \end{bmatrix}}_{\overline{A}(\Omega)} \underbrace{\begin{bmatrix} x_{dq} \\ \zeta \end{bmatrix}}_{z} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\overline{B}} u - \underbrace{\begin{bmatrix} 0 \\ I \end{bmatrix}}_{\overline{I}} \Omega_{ref} \tag{46}$$

where u is constrained to switch among four different values in the set $\{R(\theta)\rho, \rho \in \Psi_2(V)\}$. The matrix $R(\theta)$ is defined by equation (3). In this section, two different control strategies are proposed to show the experimental behavior of the PDR control applied to PMSM. Firstly, we are interested in the motor stabilization starting from non-zero initial conditions, next a velocity tracking strategy is proposed.

1) Stabilization to the origin: In the same way as the simulations, the stabilization is realized on the PMSM starting from different initial conditions to the origin. Here the PDR control is proposed based on Proposition 2 applied to model (46) with $\Omega_{ref} = 0$. It leads to a control law of the form $v_{\alpha\beta} = \underset{\rho \in \Psi_2(V)}{\mathrm{argmin}} \, z^T Q^{-1} R(-\theta)\rho$ where

$$Q = \begin{bmatrix} 29.6 & -4.8 & 9.4 & -0.012 \\ -4.8 & 26.6 & -15.9 & 0.038 \\ 9.4 & -16.0 & 208.4 & -2.8 \\ -0.012 & 0.038 & -2.8 & 0.069 \end{bmatrix}.$$

To compare the experimental behavior of the PDR control with the classical Continuous State Feedback (CSF) control, $v_{\alpha\beta} = R(-\theta)Y(\Omega)Q^{-1}z$, (obtained from (34)) starting from non null initial velocity, we have plotted in Fig. 6 the velocity evolution for three different cases (Open Loop, CSF and PDR). This control law is applied to the system by using linear amplifiers, without any PWM module. Knowing that the PMSM is a stable system, it is important to show that the stabilization performance are better than the open loop performance. For this reason, the blue curve represents the open-loop stabilization. The red line represents the CSF while the green one represents the PDR. The figure shows that the closed loop performances are better than the open loop performances (better settling-times and transient responses). The closed loop strategies show similar settling time given that the PDR uses only 4 inputs control values.

2) Velocity tracking: In this section we compare the behavior of the CSF and PDR for the tracking of a slowly varying velocity, although the proposed theoretical developments do not cover this case. The velocity profile is chosen according to industrial test trajectories [44]. The robustness of the proposed approach is also tested by applying an external torque to the motor produced by a Electromagnetic Particle Brake.

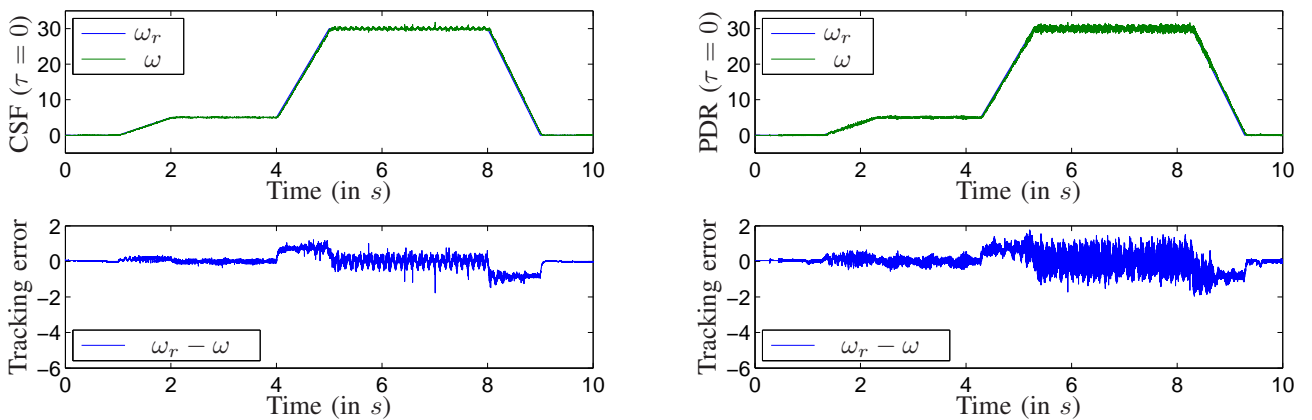Figure 7 exhibits the comparison between the CSF and the PDR when no external torque is applied to the motor.

Fig. 7. CSF and PDR experimental results wihtout pertubations.

Without additional torque the velocity tracking is accurate in both cases: it shows that at steady state the desired trajectory is tracked with a precision around $1 rad.s^{-1}$ for the PDR control. It must be noted that chattering phenomena appear in the PDR case leading to a sightly higher tracking error. However, in this case only four control inputs are used for the control.

Figure 8 shows the experimental result of the velocity tracking similarly to the previous figure. At time $t = 7s$ an unknown external torque is applied to the motor using an Electromagnetic Particle Brake. On this figure, the plot of the tracking errors shows that in the presence of external torque, the PDR is more robust to disturbances than CSF. Indeed the perturbation is rejected only by the PDR control. This results is more clearly illustrated on Figure 9, where a focus on both trajectories
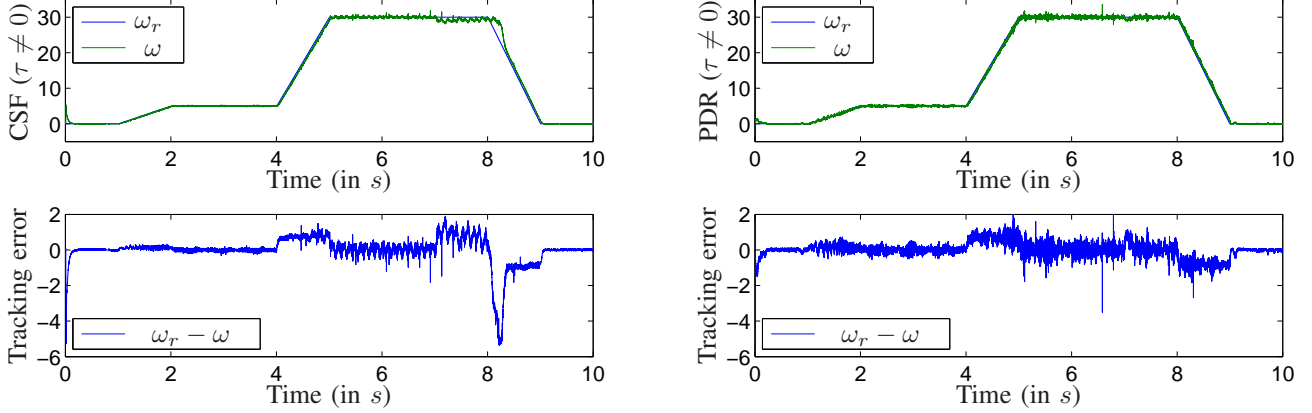

Fig. 8.   CSF and PDR experimental results wiht pertubations.

tracking is represented. We can see that the PDR control (represented in red) provides a better velocity tracking performance. Moreover, for the CSF case, when the load torque is applied, the power amplifiers are in saturation.

Finally, the last figure, Figure 10 shows the PDR inputs control applied to the motor on a small time window. The figure shows that indeed, the PDR do not behave as a PWM with regular switching instant but it switches only when needed.

VII. CONCLUSION

In this article we have proposed a novel relay control strategy. The main idea of this work was to use LPV modeling to design nonlinear switching surfaces. This modeling has been motivated by the fact that powerful numerical tools such as LMI could be used. Here we have proposed an LMI formulation to provide criteria for the design of nonlinear switching surfaces. This design was referred as Parameter Dependent Relay (PDR). The second contribution of this article was the implementation of this relay strategy for Permanent Magnet Synchronous Motors (PMSM). Since the LPV framework encompasses the PMSM model, experimental results on a PMSM have been proposed. These results have shown that stabilization and good trajectory tracking was possible despite the finite set of control inputs. The load torque introduction has shown the good disturbance rejection properties of the PDR control. The analysis of disturbance rejection represents an interesting future work.

APPENDIX

Lemma 1. ([39]). Let $V(x) = x^T Q^{-1} x$, where $Q = Q^T > 0$, C be a row vector in $\mathbb{R}^n$ and r be a nonzero scalar. Then the minimum of V along the hyperplane $\{x|Cx = r\}$ is given by $\alpha_r = \dfrac{r^2}{CQC^T}$
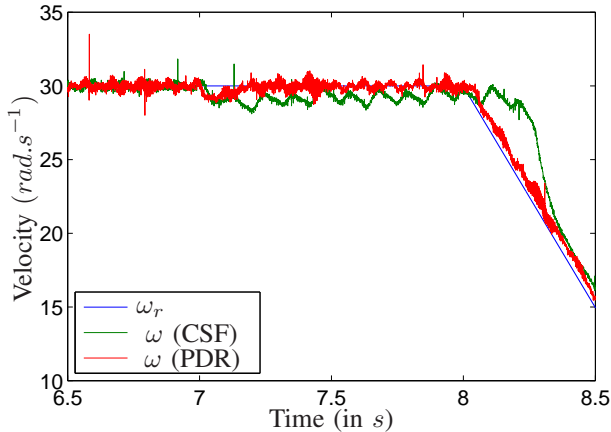

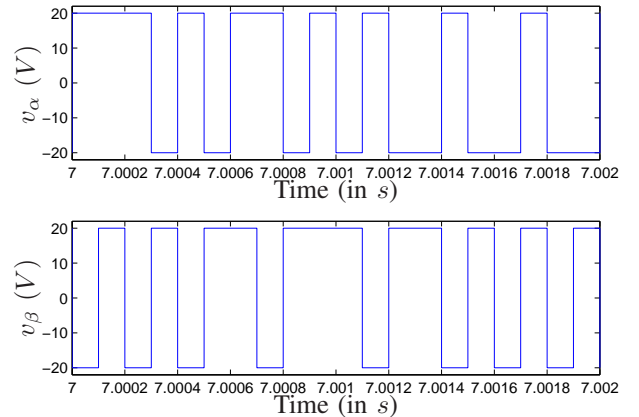Fig. 9.   Velocity zoom during the perturbation.

Fig. 10.   Relay input.

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