Exploiting User Movement for Position Detection

The Dang Huynh, Chung Shue Chen, Siu-Wai Ho

Exploiting User Movement for Position Detection

The Dang Huynh*, Chung Shue Chen*, and Siu-Wai Ho†
*Alcatel-Lucent Bell Labs, Centre de Villarceaux, 91620 Nozay, France
†Institute for Telecommunications Research, University of South Australia, Australia
Email: {The_Dang.Huynh, cs.chen}@alcatel-lucent.com, siuwai.ho@unisa.edu.au

***Abstract*—The major issue of indoor localization system is the trade-off between implementation cost and accuracy. A low-cost system which demands only few hardware devices could save the cost but often it turns out to be less reliable. Aiming at improving classical triangulation method that requires several reference points, this paper proposes a new method, called Two-Step Movement (2SM), which requires only one reference point (RP) by exploiting useful information given by the position change of a mobile terminal (MT), or the user movement. This method can minimize the number of reference points required in a localization system or navigation service and reduce system implementation cost. Analytical result shows that the user position can be thus derived and given in simple closed-form expression. Finally, simulation is conducted to demonstrate its effectiveness under noisy environment.**

***Index Terms*—Positioning system, localization algorithm, user movement, mobile device, smart applications.**

I. Introduction

Positioning systems are crucial to today's digital society. They provide geographic information about devices that facilitates many human activities. For instance, vehicle navigation systems are indispensable for drivers in big cities. Some location-based services are deployed in commercial malls so that customers can get navigation while walking in complex environment and can receive promotion advertisement from shops. The market of indoor and outdoor location-based services has grown rapidly in the last decade.

Global positioning system (GPS) is very popular and widely used for user localization. When line-of-sight to at least four GPS satellites is available, location (latitude, longitude, and elevation) and timing information can be obtained. Although GPS is very convenient in outdoors, its quality is susceptible to weather conditions, for example when sky view is poor due to fog, rain, cloud, etc., or being blocked by tall buildings in urban areas. These issues can significantly degrade the accuracy. As expected, GPS is not for indoors due to the lack of line-of-sight. There also exist cellular-based positioning systems [1] which are built on measuring signal strength from three or more base stations for tracking mobile user's location. However, these solutions also do not work well for indoors.

Various indoor positioning systems have been developed, see e.g., [2]–[4]. They can be categorized into network-based or non-network-based solutions. The network-based approach, which takes advantages of existing network infrastructure such as wireless local area networks (WLANs), without demanding new infrastructure, can maintain low deployment cost. The non-network-based approach uses dedicated positioning infrastructure and often can provide higher reliability but with extra cost. For example, solutions based on ultrasound and infrared have high deployment cost. One may also consider simple proximity-based solution like iBeacon [5] which however is only able to offer an approximate location. Some systems consider using visible light to construct an indoor positioning system with high accuracy [6]. A good positioning system should be cost-effective and also be able to offer high accuracy.

Constructing an efficient and simple positioning system is always challenging. Technically, it would depend on: 1) the number of available reference points (RPs); 2) the technologies used (e.g., RF-based, ultrasound, infrared, etc.) and; 3) the characteristics of the environment. In this study, we propose a geometry-based positioning method which can determine user position by only using one RP and exploiting user's simple movement, for instance walking or waving user's hand-held device, and some simple information. As the solution requires only one RP and can provide either exact result in noiseless environment or accurate positioning in noisy condition, our approach brings competitive advantages compared to other methods, thanks to its simplicity and effectiveness. Meanwhile, the method is interesting and may have a high potential to improve today's technology or existing solutions.

II. Related Work

Indoor positioning problem has attracted a lot of interest over years [7], [8]. Studies have been done extensively and many possible solutions have been proposed. There are mainly four major approaches to solve this problem: triangulation, fingerprinting, scene analysis, and proximity.

Triangulation is used to estimate the position of a user or mobile terminal (MT) if the geographical coordinates of the RPs are known and assume that the MT is capable of measuring the distance between itself and the RPs. A priori, this method requires three RPs to construct a distinct geometric intersection of three circles, which indicates the position of the MT. The principle is illustrated in Fig. 1(a) (or see e.g., [2]). Note that not all schemes based on triangulation requires three circles (see e.g., Fig. 1(b) and (c)). For instance, given angle-of-arrival (AoA) information, using only one RP is sufficient to locate the MT.

Fingerprinting [9] is to estimate device position by using pre-measured location-related data. This method consists of two phases: an offline training phase and an online position estimation phase. In the offline phase, location-related data is collected at different positions in the area. During the online

	Number of RPs required	Distance measurement	Additional requirement(s)
Triangulation	3	Yes	No
	2	Yes	Last position tracked or motion prediction
	1	Yes	Angle of arrival (i.e., using array of antennas)
2-Step Movement (2SM)	1	Yes	Measurement of user movement

TABLE I: Requirement comparison between triangulation and proposed 2-Step Movement (2SM) method.

position determination phase, real-time location-related data is measured and then matched with the set of data gathered during the offline phase to estimate the device's location.

Scene analysis [7] is a localization method based on a set of images or scenes received by one or multiple cameras. This approach in principle does not require user (to be tracked) to carry any extra device. However, the solution is usually expensive because it requires one or many cameras to perform tracking and may prone to a high computation cost due to image or video processing.

Proximity detects if a MT is nearby or for example in the coverage area of a RP. However, it is hard to provide accurate position with high reliability.

Each of the above method also has some variants or hybrid schemes. Our proposed geometry-based solution is built on triangulation. We will explain and discuss in comparison other methods stemmed from this branch. The cost and accuracy of triangulation method primarily rely on the number of RPs required. Traditionally, one would need at least three RPs to determine the position of the MT.

Figure 1(b) shows a variant of traditional triangulation method, which requires two RPs and the last estimated previous position of the mobile terminal so as to eliminate one of the two intersection points of the two circles constructed by the two RPs. In such case, the location closer to the last estimated position would be selected. Or, the system has to be able to predict user mobility pattern in order to select one. Note that this method still requires more than one RP. A variant of the above triangulation method is to use only one RP but requires the information of angle-of-arrival (AoA) provided by an array of antennas either implemented in the user terminal (MT) or at the RP [10], see Fig. 1(c). However, such an array of antennas is often costly and cumbersome.

III. System design

Here, we propose a new method called "Two-Step Movement (2SM)". It aims to improve the classical triangulation approach and requires only one RP. It makes use of the changes in the position of the MT relative to the RP. The changes is caused by either active movement (e.g., a user may wave his/her MT to assist) or natural movement (e.g., the user is walking or moving). Therefore, 2SM turns out to have a competitively low deployment cost and without extra or expensive tracking hardware such as antenna array and is able to determine user position in exact closed-form solution. The simplicity and effectiveness would highly facilitate practical indoor positioning systems. Table I gives a comparison of the above methods and outlines their key difference. In our proposed 2SM method, the MT is assumed be capable to measure its movement using its embedded sensors and software.

A. One-Step Movement (1SM)

Our method exploits useful information generated by user movement. For the sake of simplicity, the 2SM is presented as a combination of two One-Step Movements.

One-Step Movement (1SM) relies on one position change (one move) to identify the two possible locations (position candidates) of the MT with the following assumptions:

- The position of the RP is known.
- The MT is capable of measuring the distance between itself and the RP.
- The MT is capable of measuring the distance and the angle (direction) of its movement.

Figure 2 illustrates the system design, where

- A is the RP with a known position (x_A, y_A).
- B is the initial position of the MT that is unknown and yet to be found. It is denoted by coordinates (x_B, y_B).
- C is the position of MT right after the first movement, (x_C, y_C), which is also unknown.
- MT is capable of measuring the distance between itself and the RP. That is, the distances AB and AC are given for example by measuring the received signal strength or standard techniques.
- MT is capable of measuring the distance and the angle of its movement, thus BC and the angle $\alpha \in (0, 2\pi]$ (with respect to the positive x-axis) are also measurable.

Theorem 1 *Suppose that $A(x_A, y_A), AB, BC, AC$, and α are known, the One-Step Movement (1SM) will give two estimated locations, denoted by generic point $B(x_B, y_B)$, whose x and y coordinates satisfy:*

$$\begin{aligned} x_B \cos\alpha + y_B \sin\alpha &= x_A \cos\alpha + y_A \sin\alpha \\ &- \frac{(AB^2 + BC^2 - AC^2)}{2BC}. \end{aligned} \tag{1}$$

Proof: Using Fig. 2, from the two measured distances AB and AC, the equations of the two circles centered at $A(x_A, y_A)$ on which the MT probably lies can be expressed as

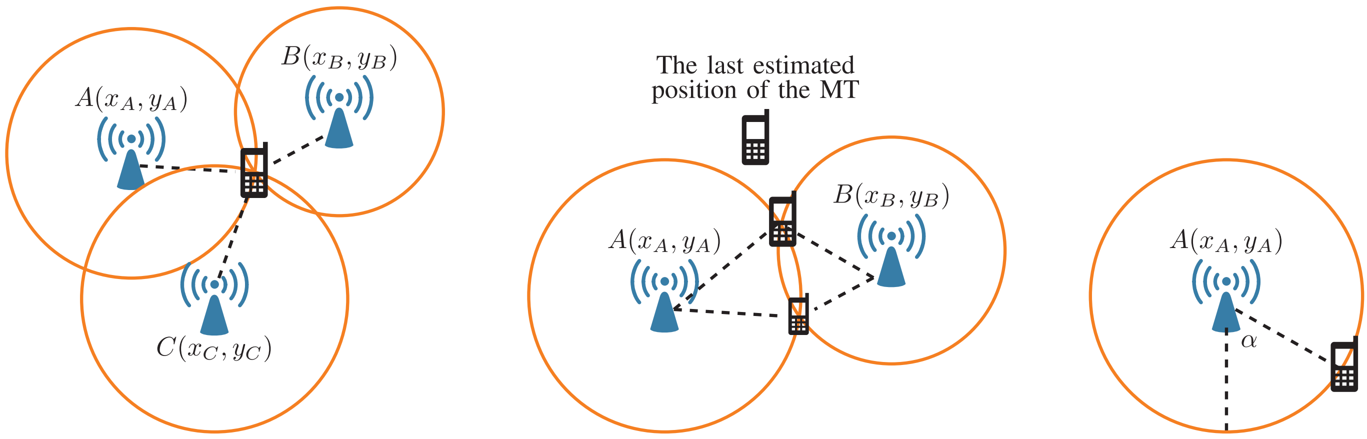
$$\begin{aligned} (x_B - x_A)^2 + (y_B - y_A)^2 &= AB^2 \\ (x_C - x_A)^2 + (y_C - y_A)^2 &= AC^2 \end{aligned} \tag{2}$$

where

$$\begin{aligned} x_C &= x_B + BC \cos\alpha, \\ y_C &= y_B + BC \sin\alpha. \end{aligned} \tag{3}$$

From (2), we have

$$\begin{aligned} AB^2 - AC^2 &= (x_B - x_C)(x_B + x_C - 2x_A) \\ &+ (y_B - y_C)(y_B + y_C - 2y_A). \end{aligned} \tag{4}$$



(a) Three RPs

(b) Two RPs, plus the last estimated position tracked

(c) One RP, plus information of angle-of-arrival α

Fig. 1: Positioning techniques using different number of reference points (RPs).

Substitute x_C and y_C in (3) to (4), we can have

$$\begin{aligned} AB^2 - AC^2 &= -BC\cos\alpha(2x_B + BC\cos\alpha - 2x_A) \\ &\quad -BC\sin\alpha(2y_B + BC\sin\alpha - 2y_A) \end{aligned}$$

which can be re-written as

$$\begin{aligned} AB^2 + BC^2 - AC^2 &= -2BC(x_B\cos\alpha - x_A\cos\alpha \\ &\quad + y_B\sin\alpha - y_A\sin\alpha). \end{aligned}$$

Hence,

$$\begin{aligned} x_B\cos\alpha + y_B\sin\alpha = x_A\cos\alpha + y_A\sin\alpha \\ - \frac{(AB^2 + BC^2 - AC^2)}{2BC}. \end{aligned}$$

■

We can solve (1) as follows

- If $\sin\alpha = 0$, thus $\cos\alpha = \pm 1$, (1) becomes:

$$x_B = x_A \pm \frac{(AB^2 + BC^2 - AC^2)}{2BC}.$$

It is then straightforward to compute the values of x_B and y_B, by substituting the value of x_B to (2).

- If $\sin\alpha \neq 0$, by dividing (1) by $\sin\alpha$, we have:

$$y_B = -\cot\alpha x_B + x_A\cot\alpha + y_A - \frac{AB^2 + BC^2 - AC^2}{2BC\sin\alpha}.$$

Let $b = x_A\cot\alpha + y_A - (AB^2 + BC^2 - AC^2)/(2BC\sin\alpha)$ and $a = -\cot\alpha$. We see that now y_B can be expressed as a function of x_B such that $y_B = ax_B + b$. Substituting y_B to the first equation of (2), we have

$$(x_B - x_A)^2 + (ax_B + b - y_A)^2 = AB^2.$$

Then

$$(1+a^2)x_B^2 - 2x_B(x_A - a(b-y_A)) + x_A^2 + (b-y_A)^2 - AB^2 = 0. \tag{5}$$

The above quadratic equation (5) can be solved easily.

Algorithm 1 shows in detail how to perform 1SM. It outputs two points $B1(x_{B1}, y_{B1})$ and $B2(x_{B2}, y_{B2})$, which are the possible solution of B.

Remark 1 *It is clear that one of the two points, $B1(x_{B1}, y_{B1})$ and $B2(x_{B2}, y_{B2})$, must be the position of the MT (or both of them are, if $B1$ and $B2$ are identical).*

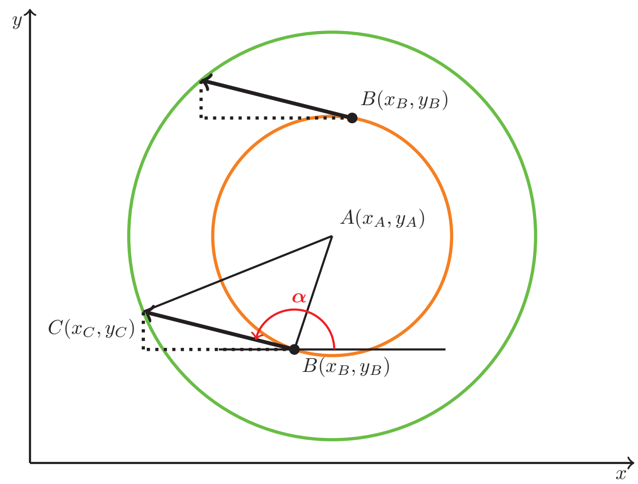


Fig. 2: One-Step Movement (1SM).

B. Two-Step Movement (2SM)

After the first movement, we have two possible locations of the MT given by 1SM using Algorithm 1, but cannot determine which one is the true location. We need to resolve this ambiguity. It is natural to think about performing an additional movement. The basic idea is simple: a Two-Step Movement (2SM) is a combination of two consecutive 1SM's where each move gives two possible positions (in which one of these two positions must be the true position). It is clear that by comparing the results of two 1SM's, we can determine the location of the MT, given that the results of the two 1SM's are not redundant.

Fig. 3 depicts how 2SM works. The MT makes the second movement from C to D in the direction of angle β, which

Algorithm 1 One-Step Movement algorithm

Require: $A(x_A, y_A), AB, AC, BC, \alpha$;

function ONESTEP($A(x_A, y_A)$, AB, AC, BC, α)
  if $\sin\alpha == 0$ **then**
    if $\cos\alpha == 1$ **then**
      $x_B = x_A - (AB^2 + BC^2 - AC^2)/(2BC)$;
    else
      $x_B = x_A + (AB^2 + BC^2 - AC^2)/(2BC)$;
    end if
    $y_{B1} = y_A + \sqrt{AB^2 - (x_B - x_A)^2}$;
    $y_{B2} = y_A - \sqrt{AB^2 - (x_B - x_A)^2}$;
    return $\{B1(x_B, y_{B1}), B2(x_B, y_{B2})\}$;
  else
    ▷ Pre-compute a, b such that $y_B = ax_B + b$;
    $a = -\cot\alpha$;
    $b = x_A \cot\alpha + y_A - (AB^2 + BC^2 - AC^2)/(2BC\sin\alpha)$;
    ▷ Compute x_B, y_B;
    $\Delta = (x_A - a(b - y_A))^2 - (1 + a^2)(x_A^2 + (b - y_A)^2 - AB^2)$;
    $x_{B1} = (x_A - a(b - y_A) + \sqrt{\Delta})/(1 + a^2)$;
    $y_{B1} = ax_{B1} + b$;
    $x_{B2} = (x_A - a(b - y_A) - \sqrt{\Delta})/(1 + a^2)$;
    $y_{B2} = ax_{B2} + b$;
    return $\{B1(x_{B1}, y_{B1}), B2(x_{B2}, y_{B2})\}$;
  end if
end function

is measured from the positive x-axis counter-clockwise. The distance CD and β are known by the MT, whereas the distance AD from the MT to the RP is measured from the received signal strength by standard techniques. The underlying idea is that, we now consider the movement of 2SM case similarly as that of 1SM case in which the starting point is now B and the ending point is D. We can compute the distance BD and the angle γ analytically (see Algorithm 2: line 5–10) and then use the method of Algorithm 1 to determine B. Algorithm 2 details how 2SM works. By comparing the results from the two 1SM's computation, we determine the location of the MT.

Remark 2 *Note that the directions of the two movements should not be in parallel, i.e.,* $\beta \neq \alpha$ *and* $\beta \neq \alpha \pm \pi$*, otherwise the ambiguity cannot be resolved since the system of equations generated by the second movement would be equivalent to that of the first one.*

In practice with estimation error or system imperfection, say the existence of noise, we may not have a common solution from the two 1SM's computation. In this case, the first movement may give us two possible solutions denoted by $B1(x_{B1}, y_{B1})$ and $B2(x_{B2}, y_{B2})$, but the second movement may give us another two possible solutions denoted by $B3(x_{B3}, y_{B3})$ and $B4(x_{B4}, y_{B4})$. However $\{B1, B2\}$ and $\{B3, B4\}$ may not have a common point as shown in Fig. 4. To solve this problem, we can choose the pair of points that

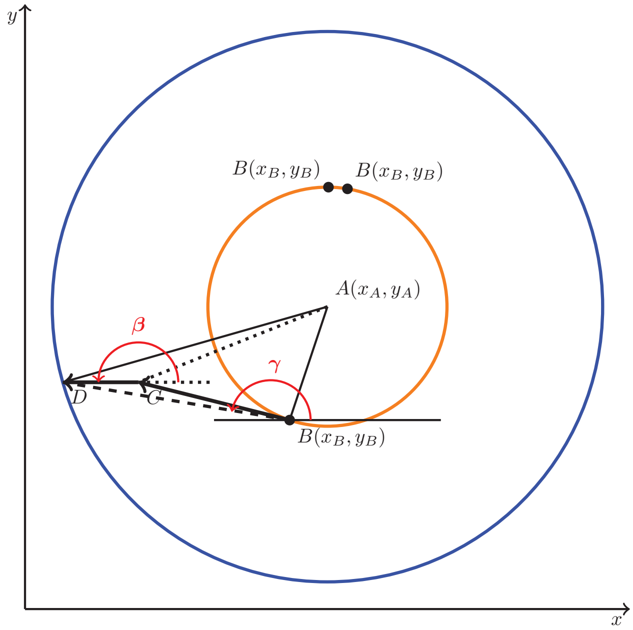


Fig. 3: Two-Step Movement (2SM).

have the smallest distance, i.e., solving $\min\{d(P1, P2) | P1 \neq P2\}$, for $P1, P2 \in \{B1, B2, B3, B4\}$ where $d(P1, P2)$ denotes the Euclidean distance of points $P1$ and $P2$. After that, we take their mean (e.g., the mid-point of $B1$ and $B3$ in Fig. 4) as the estimate of the MT's position for minimizing the error. In general, one can formulate it as an optimization problem and find the optimal result.

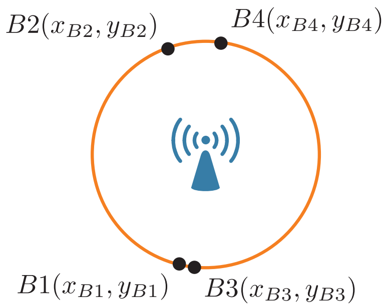


Fig. 4: Ambiguity elimination in case of noise.

IV. SIMULATION

Simulation is performed to investigate the performance of the proposed scheme (2SM) under noisy environment. The RP is placed at the center of a room, i.e., $A = (0, 0)$. We are going to determine the MT's location, denoted by $B(x_B, y_B)$, which is randomly distributed in the room. In the following analysis, we consider three distances (1, 5, and 10 meters) between the MT (denoted by B, in Fig. 3) and the RP. Also, we assume that the direction from B to A is uniformly distributed in $(0, 2\pi]$.

For a given AB, the movement from B to C or from C to D is equal to 0.1, 0.2, and 0.5 times of AB. The directions of the movement, i.e., α and β, are uniformly distributed in

Algorithm 2 Two-Step Movement algorithm

Require: $A(x_A, y_A)$;

function TWOSTEP($A(x_A, y_A)$)

  MT makes the first movement from B to C; measure AB, AC, BC, α;

  $\{B1(x_{B1}, y_{B1}), B2(x_{B2}, y_{B2})\}$ = ONESTEP($A(x_A, y_A), AB, AC, BC, \alpha$); ▷ Obtain $B1$ and $B2$

  MT makes the second movement from C to D; measure CD, AD, β; make sure that $\beta \neq \alpha$ and $\beta \neq \alpha \pm \pi$;

  $X = BC \cos \alpha + CD \cos \beta$; ▷ The change in x-coordinate after the second move

  $Y = BC \sin \alpha + CD \sin \beta$; ▷ The change in y-coordinate after the second move

  $BD = \sqrt{X^2 + Y^2}$;

  $\cos \gamma = X/BD$;

  $\sin \gamma = Y/BD$;

  Compute $\gamma \in (0; 2\pi]$ from $\cos \gamma$ and $\sin \gamma$;

  $\{B3(x_{B3}, y_{B3}), B4(x_{B4}, y_{B4})\}$ = ONESTEP($A(x_A, y_A), AB, AD, BD, \gamma$); ▷ Obtain $B3$ and $B4$

  $B(x_B, y_B) = \{B1(x_{B1}, y_{B1}), B2(x_{B2}, y_{B2})\} \cap \{B3(x_{B3}, y_{B3}), B4(x_{B4}, y_{B4})\}$; ▷ Determine MT location $B(x_B, y_B)$ from the set of $B1$, $B2$, $B3$ and $B4$;

  return $B(x_B, y_B)$;

end function

$(0, 2\pi]$. Estimation error to the measurement of distances AB, AC, AD, and BC, is considered to be bounded in $[-1\%, 1\%]$, $[-2\%, 2\%]$, and $[-5\%, 5\%]$, for comparison. We use e_d to denote the above bound such that $e_d = 1\%$, 2%, and 5%, respectively. Estimation error to the measurement of angles α and β is considered to be bounded in $[-1^\circ, 1^\circ]$, $[-2^\circ, 2^\circ]$, and $[-5^\circ, 5^\circ]$. The bound on the angle measurement error is denoted by e_a such that $e_a = 1$, 2, and 5 degrees. For each (e_d, e_a) setup, the errors are randomly generated to corrupt the proposed algorithm in determining $B(x_B, y_B)$. Results are shown in Figs. 5, 6, and 7 for $BC = CD$ is equal to $0.1 \times AB$, $0.2 \times AB$ and $0.5 \times AB$, respectively. Note that each curve in the figures is obtained by 10,000 runs. During these runs, we observe that about 10% of the time the system fails to find the MT position (i.e., the quadratic equation (5) has no solution since Δ in Algorithm 1 is negative) due to the noise (which would be accumulated to Δ). We find that when $\Delta < 0$, the system is indeed heavily corrupted. In this case, the movement is considered as bad and is not used to locate the MT. Note that it would be interesting to derive the position of the MT even when $\Delta < 0$ and see how to extract useful information to optimize results. This is left as future work.

As shown in Figs. 5-7, the estimation error in determining the position of the MT increases as e_d increases. Note that the estimation error is defined by the distance between the real position of the MT and the result given by Algorithm 2. Clearly, $e_d = 1\%$ (curves in "red") results in smaller estimation error than that $e_d = 3\%$ or $e_d = 5\%$ (curves in "blue" and "black", respectively) makes, given that e_a is the same.

As expected, the estimation error in determining the position of the MT also increases as e_a increases. However, when e_d is relatively large (5%), the impact of the considered e_a is relatively less significant. This can be clearly shown by Figs. 6 and 7. Roughly speaking, e_d is more dominating.

In comparing Figs. 5-7, we observe that when increasing BC and CD from $0.1 \times AB$ to $0.5 \times AB$, the error in estimating the position of the MT decreases quite substantially. In Fig. 6 and 7, the curves shift to the left. The distance of the movement is a significant factor. We can improve the system performance by requiring a longer movement distance. However, a longer distance may be less favorable in some scenarios. Furthermore, from the obtained simulation results which are not shown in this paper, we see that the improvement is indeed decreasing and starts to get flat at $0.5 \times AB$.

Table II shows the average error in determining the position of the MT under different AB (at 1, 5, and 10 meters) and various BC, CD, and noise levels (e_d, e_a). For $AB = 5$, the results are plotted in Figs. 5-7. For $AB = 1$ and 10, the results have characteristics very similar to those in Figs. 5-7 so that they are not shown in this paper. Comparing the results at $AB = 1$, 5, and 10, we see that the magnitude of the error increases roughly proportional to AB. It is clear that the estimation error is minimized when (e_d, e_a) are small and the movement distance is relatively large. Roughly speaking, if $BC = CD = 0.5 \times AB$, the performance is quite desirable for $e_d \leq 2\%$ and $e_a \leq 5\%$. When the movement distance is at the level of $0.2 \times AB$, the same performance can be achieved for a smaller $e_d \leq 1\%$. The average error can be limited to within about 10% of AB. In the best case, the average error can be less than 5% of AB.

V. CONCLUSION

In this paper, we have proposed a new method called Two-Step Movement (2SM) to estimate the position of MT. It requires only one reference point (RP) by exploiting useful information given by the position change of the MT or user movement. One can therefore reduce the number of RPs required and also the system cost. Analytical result shows that the user position can be derived and given in simple closed-form expression with low complexity. Simulation is conducted to study its performance under noisy environment. It is possible to achieve average error within about 10% of the distance between the RP and MT, or even less. Note

	$e_d = 1\%$ $e_a = 1°$	$e_d = 1\%$ $e_a = 2°$	$e_d = 1\%$ $e_a = 5°$	$e_d = 2\%$ $e_a = 1°$	$e_d = 2\%$ $e_a = 2°$	$e_d = 2\%$ $e_a = 5°$	$e_d = 5\%$ $e_a = 1°$	$e_d = 5\%$ $e_a = 2°$	$e_d = 5\%$ $e_a = 5°$
$AB = 1$ (BC=CD= 0.1AB)	0.1412	0.1434	0.1581	0.2583	0.2640	0.2708	0.5530	0.5608	0.5691
$AB = 1$ (BC=CD= 0.2AB)	*0.0808*	*0.0859*	*0.1036*	0.1463	0.1508	0.1631	0.3202	0.3340	0.3417
$AB = 1$ (BC=CD= 0.5AB)	***0.0484***	*0.0566*	*0.0896*	*0.0753*	*0.0804*	*0.1086*	0.1701	0.1759	0.1868
$AB = 5$ (BC=CD= 0.1AB)	0.7194	0.7279	0.8027	1.2797	1.3012	1.3668	2.7913	2.9031	2.9226
$AB = 5$ (BC=CD= 0.2AB)	*0.3957*	*0.4235*	*0.5513*	0.7145	0.7480	0.8246	1.6222	1.6372	1.6587
$AB = 5$ (BC=CD= 0.5AB)	***0.2193***	*0.2831*	*0.4481*	*0.4136*	*0.4412*	*0.5448*	0.8738	0.8829	0.9134
$AB = 10$ (BC=CD= 0.1AB)	1.4165	1.4348	1.6130	2.4798	2.6257	2.7011	5.7899	5.8059	5.8929
$AB = 10$ (BC=CD= 0.2AB)	*0.8006*	*0.8602*	*1.0845*	1.4779	1.1508	1.5112	3.2304	3.3131	3.3873
$AB = 10$ (BC=CD= 0.5AB)	***0.4987***	*0.5601*	*0.9362*	*0.8180*	*0.8798*	*0.1058*	1.7551	1.7652	1.8750

TABLE II: Average error (in meter) under various AB, BC, CD, and noise levels (e_d, e_a).

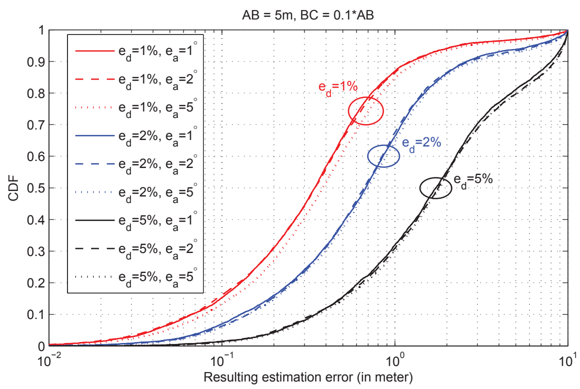


Fig. 5: Resulting error when $AB = 5$ meters, $BC = CD = 0.1 \times AB$ under various (e_d, e_a).

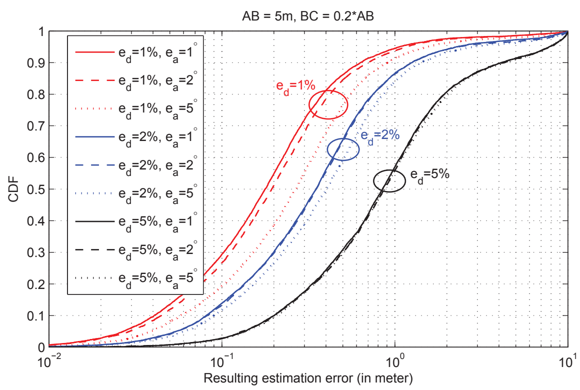


Fig. 6: Resulting error when $AB = 5$ meters, $BC = CD = 0.2 \times AB$ under various (e_d, e_a).

that further analysis of noise impact and issues related to reflection and refraction of signals are important to improve the proposed method. Our method, thanks to the reliance on a single reference point, makes a lot of sense in the context of Internet of Things (IoT) such as home or business office area. It should be also noted that our method can be easily extended to localization in 3D coordinates and to device-to-device (D2D) applications in which both devices could be mobile. The practical implementation is left as future work.

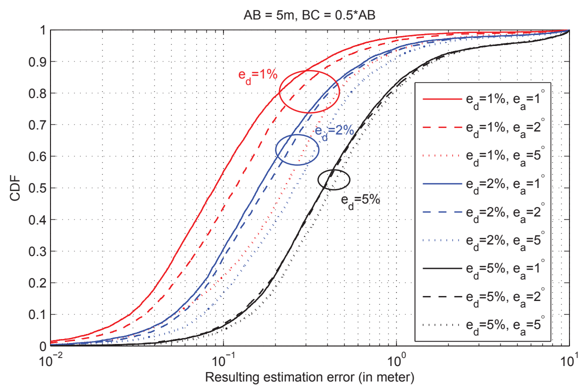


Fig. 7: Resulting error when $AB = 5$ meters, $BC = CD = 0.5 \times AB$ under various (e_d, e_a).

ACKNOWLEDGMENT

The work presented in this paper has been carried out at LINCS (www.lincs.fr). The authors would like to thank Fabien Mathieu and Philippe Jacquet for their discussion and valuable comments.

REFERENCES

[1] G. Gartner and K. Rehrl, *Location Based Services and TeleCartography II: From Sensor Fusion to Context Models*. Lecture Notes in Geoinformation and Cartography, Springer, 2009.

[2] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of wireless indoor positioning techniques and systems," *IEEE Trans. Sys. Man Cyber - Part C*, vol. 37, no. 6, pp. 1067–1080, Nov. 2007.

[3] G. Mao, B. Fidan, and B. D. O. Anderson, "Wireless sensor network localization techniques," *Comput. Netw.*, vol. 51, no. 10, Jul. 2007.

[4] A. Kushki, K. N. Plataniotis, and A. N. Venetsanopoulos, *WLAN Positioning Systems: Principles and Applications in Location-Based Services*. Cambridge University Press, 2012.

[5] A. Cavallini, *iBeacons Bible*, http://meetingofideas.files.wordpress.com/2013/12/ibeacons-bible-1-0.pdf.

[6] M. Yasir, S.-W. Ho, and B. Vellambi, "Indoor positioning system using visible light and accelerometer," *Journal of Lightwave Technology*, vol. 32, pp. 3306–3316, Oct 2014.

[7] Y. Gu, A. Lo, and I. Niemegeers, "A survey of indoor positioning systems for wireless personal networks," *IEEE Commun. Surveys Tuts.*, vol. 11, no. 1, pp. 13–32, Jan. 2009.

[8] R. Harle, "A survey of indoor inertial positioning systems for pedestrians," *IEEE Commun. Surveys Tuts.*, vol. 15, no. 3, 2013.

[9] P. Mirowski, D. Milioris, P. Whiting, and T. Kam Ho, "Probabilistic radio-frequency fingerprinting and localization on the run," *Bell Labs Technical Journal*, vol. 18, no. 4, pp. 111–133, Feb. 2014.

[10] R. Peng and M. Sichitiu, "Angle of arrival localization for wireless sensor networks," in *IEEE SECON*, Sep 2006, pp. 374–382.