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► **To cite this version:**

Philippe Nabonnand. The Poincaré-Mittag-Leffler Relationship. *Mathematical Intelligencer*, Springer Verlag, 1999, 21, pp.58 - 64. 10.1007/BF03024848 . hal-01083380

HAL Id: hal-01083380

<https://hal.archives-ouvertes.fr/hal-01083380>

Submitted on 17 Nov 2014

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The Poincaré - Mittag-Leffler Relationship

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The Swedish mathematician Gösta Mittag-Leffler and the French mathematician Henri Poincaré maintained a regular correspondence between 1881 and 1911.¹ 259 of these letters are kept in the Mittag-Leffler Institute in Djursholm (Sweden) (119 letters or rough drafts written by Mittag-Leffler and 140 by Poincaré). By Mittag-Leffler's standards, this correspondence is not exceptionally big. Indeed, he was almost as prolific with Hermite, Weierstrass, Appell or Painlevé. But, for Poincaré, his correspondence with Mittag-Leffler is by far the most important. It is not clear why Poincaré, who did not co-operate mathematically with Mittag-Leffler, maintained such an intense relation with him, and in what follows I shall try to explain that.

We can distinguish four periods in this correspondence. In the first one (10 letters between April and August 1881), the two mathematicians get to know each other. The creation of *Acta Mathematica* and the Poincaré's papers about Fuchsian functions are the main subjects of the second period (about 50 letters between August 1881 and March 1887). The third period is devoted to the Poincaré's participation in Oscar II's 60th Birthday Competition (about 50 letters between March 1887 and July 1890). In the last period, we can see two eminent mathematicians managing nominations and positions in Universities and Academies and being busy with propositions for Nobel prizes (about 150 letters between May 1891 and September 1911).

1. Making contact

In 1881, Mittag-Leffler wrote to Poincaré about 'lacunary functions', i.e. complex functions which cannot be prolonged to the whole complex plane. Hermite had communicated his very high opinion of Poincaré to Mittag-Leffler, and Mittag-Leffler wanted more information about the results in Poincaré's thesis and his work on differential equations:

"When do you think you will publish your research on differential equations? I am waiting for it impatiently. I cannot see from M. Hermite's account if your results are the same as M. Fuchs has published recently, or if your research is yet more general." [Mittag-Leffler to Poincaré, 11 April 1881 - IML].

Poincaré was not yet well known at that time - he had published only one communication on the qualitative theory of the differential equations and two others about Fuchsian functions. On the other hand, Mittag-Leffler was already a recognized mathematician. He had been one of the more brilliant of Weierstrass' students, and his theorem about the existence of meromorphic functions with prescribed poles and zeros [Mittag-Leffler 1879] is a fine generalization of Weierstrass' result about the existence of holomorphic functions with prescribed zeros [Weierstrass 1876]. Therefore, it is not surprising that, in the opening letters, Mittag-Leffler adopts a patronizing attitude to the 'jeune homme'.² Moreover, in the first version of his paper [Poincaré 1881a], Poincaré did not quote the Weierstrass' results concerning analytic functions, and in particular those about functions with 'lacunary spaces':

¹ The entire Poincaré correspondence will be edited by the Archives Poincaré (Nancy 2 University). The first volume will be the annotated correspondence, with a commentary, of the Poincaré-Mittag-Leffler correspondence and it will be published in the summer of 1998.

² 'M. Poincaré est un jeune homme encore, je suppose.' [Letter from Mittag-Leffler to Hermite, 6th April 1881-AS]

“Permit me to say to you frankly and loyally that I think you must explain the relationship of your work to that of Monsieur Weierstrass, published in the *Berliner Monatsbericht* for August 1880 with the title ‘*Zur Functionenlehre*’”. [Mittag-Leffler to Poincaré, 22 May 1881 — IML]

Mittag-Leffler always defended Weierstrass’ work firmly. Over the years he became worried about Weierstrass’ priority, and more generally the diffusion of his work, because Weierstrass did not publish all his results and a great part of them were only communicated in his lectures. Consequently, many were known only to his students and French mathematicians were unaware of them. Mittag-Leffler wrote to Poincaré on several occasions about questions of priority concerning Weierstrass’ results.

Mittag-Leffler, who was an adept of the Berlin School of rigour, agreed with Weierstrass’ point of view and often criticized Poincaré’s manner of writing mathematics:

What do you make of Poincaré’s second paper “Sur les fonctions fuchsienues”? It is indeed regrettable that he is not a graduate of a German University. As full of new ideas as his papers are, they leave, it seems to me, far too much to be desired in their formal presentation. [Mittag-Leffler to Weierstrass, 11th May 1883 — IML]

Mittag-Leffler’s opinion of Poincaré’s work always remained ambiguous and ambivalent. On the one side, he admired the “genius” of Poincaré, on the other, he was also critical of his lack of rigor:

But he [Poincaré] has however a fault which is very much to be regretted. It cannot be denied that he writes with too little care and his memoirs are full of inexactitudes. That is something, which can only be said between us! One must let the great geniuses follow their own paths and to accept with gratitude what they give us, even if one might hope to receive it in a more digestible form. [Mittag-Leffler to Hermite, 27th October 1887 — IML]

2. The creation of the *Acta mathematica*

Nevertheless, in 1881, impressed by the notes about Fuchsian functions and convinced by Hermite’s arguments, Mittag-Leffler understood that Poincaré was a real mathematical genius:

I congratulate you heartily on the great success you have had in your research, and I find that our dear master M. Hermite was entirely right when he wrote to me that ‘you are a veritable mathematical genius’. I only wish you would publish a great work where you would bring together all your researches, which are so important, into Fuchsian functions’. [Mittag-Leffler to Poincaré, 22nd June 1881 — IML].

This opportunity of publishing Poincaré’s great papers helped Mittag-Leffler decide to embark on his project of creating a new mathematical journal. During the 1870s, the level of French mathematical journals, such as the *Journal de mathématiques pures et appliquées* and the *Journal de l’Ecole polytechnique* had declined [Gispert 1996]. On the other hand, during the same period, German mathematical journals, such as the *Journal für die reine und angewandte Mathematik* (Crelle’s *Journal*) or the newly-founded *Mathematische Annalen* reflected the great activity and the creativeness of the German mathematical community. Nevertheless, these journals were not very widely read outside the natural domain of the German language, and particularly not in France.³ Mittag-Leffler took it into his head to create a new journal which would be a bridge

³ Furthermore, the nationalist and vengeful ideology in France did not favour an exchange between French and German mathematicians.

between German and French mathematicians. In addition, thanks to his friendship with both Hermite and Weierstrass, he expected the collaboration of the two communities.

But favorable circumstances would not be enough. Mittag-Leffler remembered that the success of the Crelle's *Journal* was grounded on Abel's major contributions on elliptic functions. He thought that Fuchsian functions were the most remarkable functions to have been discovered since elliptic functions and he claimed that the diffusion of this new theory needed a new journal:

I do not need to say you that I am full of admiration for your genius and the beauty of the results that you have achieved. I do not think that I am wrong when I assure you that your discoveries will compete with those of Abel and that your functions are the most remarkable since the discovery of elliptic functions. [...] Now I have a proposition for you. We, the Scandinavian mathematicians, have the project to publish a new mathematical journal according to the model of the Crelle's journal. [...] Now, we, Mr. Gylden and I, have thought that, you, French, will be generous enough to assure the success of our journal. Would you accept to give your memoir "Sur les Groupes Fuchsien" to be published the first in the journal. [...] I ask you not to say anything about our project because the realization of this project depends on you. If you decline, my opinion is that we have to wait two or three years. It is only the huge advantage of publishing your discoveries that can convince me. [Letter from Mittag-Leffler to Poincaré, 29th March 1882 — IML]

[J'ai guère besoin de vous dire que je suis frappé de la plus grande admiration de votre génie et de la beauté des résultats que vous avez obtenus. Je ne crois que je me trompe quand je vous assure que vos découvertes feront la concurrence avec celles d'Abel et que vos fonctions sont les plus remarquables qui ont été trouvées d'après les fonctions elliptiques. [...] Et maintenant j'ai une proposition à vous faire et une prière à vous adresser. Nous, les mathématiciens dans les pays scandinaves, ont le projet de publier un nouvel journal mathématique d'après le modèle du journal de Crelle. [...] Maintenant, nous avons pensé M. Gylden et moi que vous, un français, serez peut-être assez généreux pour vouloir faire le succès de notre journal. Voudriez vous nous donner votre mémoire «Sur les groupes fuchsien» pour être publié le premier mémoire dans le journal. [...] Je vous prie de ne rien dire à personne encore sur notre projet parce que la réalisation de ce projet dépend de vous. Si vous refusez je suis de l'avis que nous devons attendre deux ou trois ans encore. C'est seulement l'avantage énorme de pouvoir publier vos découvertes qui pourrait m'y décider.] [Letter from Mittag-Leffler to Poincaré, 29th March 1882 — IML]

Poincaré agreed to publish his five great papers on Fuchsian functions in the new journal, *Acta Mathematica*. The letters in which Poincaré promised his help to Mittag-Leffler are lost and we can only conjecture why Poincaré decided to publish in *Acta Mathematica*. First, we can presume that Poincaré felt flattered by Mittag-Leffler's proposition. Moreover, Poincaré was convinced that he had to move quickly to establish his priority and his reputation and so that he must make his theory known in Germany, the pre-eminent nation for mathematics. In addition, we must not forget the quarrel between Klein and Poincaré about the priority and the denomination of the Fuchsian functions. In fact, Poincaré took care to make his results known to some prominent mathematicians:

The authors are, in general, hurried of having their “tirages à part” not to widely distribute them to their friends, but to send as immediately as possible an exemplar to ten or dozen great names to which they wish to present their works. [Letter from Poincaré to Eneström, 3rd June 1884 — CHS]

[Si les auteurs sont généralement pressés d’avoir leurs tirages à part, ce n’est pas pour faire une ample distribution à tous leurs amis, mais pour envoyer aussitôt que possible un exemplaire à une dizaine de grands noms à qui ils désirent faire connaître leurs travaux.] [Letter from Poincaré to Eneström, 3th June 1884 — CHS]

So, Poincaré needed to diffuse his work in Germany; he had published a survey about the Fuchsian functions [Poincaré 1882] in *Mathematische Annalen*, the Klein’s Journal, but, on account of the Germanophobia in France after the 1870 war, we can think that he could not publish his main papers in a German journal⁴ and Mittag-Leffler’s offer allowed him to be read in Germany without offending the nationalist ideology of his compatriots. In any way, by publishing in *Acta Mathematica*, he could reach both French and German audience, good for his reputation in both countries. If Mittag-Leffler built the success of *Acta Mathematica* on Poincaré’s participation, Poincaré built his international fame on his many publications in that journal.

Poincaré published 10 papers in the first ten issues of *Acta Mathematica* (676 of the 1594 pages written by the French authors).⁵ Not all were on Fuchsian functions, for Poincaré was especially creative and prolific during this period. For example, in 1883 he showed that a meromorphic functions of two variables is the quotient of two holomorphic functions:⁶

I have tried for a long time to find if a meromorphic Function $F(x, y)$ can be always written as the form $G(x, y)/G_1(x, y)$ but without reaching a satisfying result. I know that Mr. Weierstrass consider this problem as one of the most essential and one of the most difficult in the Analysis. And you announce me that you are on the way of the solution. I hope that you will not be long to communicate to me the result that you obtain when you will have a final result. [Letter from Mittag-Leffler to Poincaré, 5th December 1882 — IML]

[J’ai cherché depuis bien longtemps de trouver si une fonction méromorphe $F(x, y)$ peut toujours se mettre sous la forme $G(x, y)/G_1(x, y)$ mais sans parvenir à un résultat satisfaisant. Je sais aussi que M. Weierstrass regarde ce problème comme un des plus essentiels et des plus difficiles dans l’analyse. Et vous m’annoncez maintenant que vous êtes sur la voie de trouver la solution. J’espère que vous ne tardez pas de me communiquer le résultat que vous obtenez quand vous aurez un résultat définitif.] [Letter from Mittag-Leffler to Poincaré, 5th December 1882 — IML]

Other important papers are ‘Sur l’équilibre d’une masse fluide’ (1885a), and ‘Sur les intégrales irrégulières des équations linéaires’ (1885b); this last paper was the occasion of a polemic with Thomé. These mathematical disputes are almost the only circumstances in which Poincaré broke through his reserve, although, as is well-known, the young Poincaré defended his views about the priority and the name of Fuchsian functions in a very sharp controversy with Klein without making any concessions. Mittag-Leffler referred to this polemic on several occasions, as on the 18th July 1882, when he related his visit to Schwarz and Schwarz’s fury:

⁴ More, Poincaré was born in Nancy, a town of Lorraine and half of this region has been annexed by Germans.

⁵ From a quantitative point of view, Poincaré’s papers stands for 10% of the 10 first issues of *Acta Mathematica*

⁶ Poincaré makes use of the Dirichlet’s Principle to show this theorem. It is worth noting that he did not know this principle a few months before: Je ne connaissais pas ce principe à cette époque, [...]. [Poincaré 1901, p. 46]

It was not the same thing with Mr. Schwarz. I have found him full of indignation with you. He thinks that he is the first who has given an example of those groups that you call Fuchsian, which it cannot be found in theory of elliptic functions. [...] From your point of view, you have to find in this squabble between German mathematicians about your new names a proof of the importance of your discoveries. Mr. Schwarz did not hide that especially the Fuchsian Function were the cause of his fury. [IML]

[Ce n'était pas la même chose avec Monsieur Schwarz. Je l'ai trouvé plein d'indignation contre vous. Il trouve que c'est lui qui le premier a donné un exemple des groupes que vous appelez Fuchsiennes et qui ne se rencontrent pas dans la théorie des fonctions elliptiques. [...] Pour votre part, vous devez trouver une preuve de l'importance de vos découvertes dans toutes ces querelles entre les géomètres allemands à cause de vos nouvelles dénominations. Monsieur Schwarz ne cachait nullement que c'étaient surtout les fonctions fuchsiennes qui lui faisaient suffoquer de fureur]. [IML]

Before answering, Poincaré quickly studied Schwarz's papers and concluded that he saw no reason to change his mind:

I do not hope to mollify Mr. Schwarz. What are the reasons of his fury? First, he is in a rage because he has had in his hands an important result and he has not known how to take advantage with it. For that, I cannot do anything.

Next, he is dissatisfied with the name Fuchsian. He would prefer Schwarzian. I have said the reasons why for that also, I cannot do anything. [Letter from Poincaré to Mittag-Leffler, 27th July 1882 — IML]

[Je n'espère pas ainsi calmer M. Schwarz. Quelles sont en effet les causes de sa fureur ? D'abord il est furieux d'avoir tenu entre les mains un résultat important et de n'en avoir pas su tirer profit. A cela, je ne puis rien.

Ensuite, il est mécontent du nom de Fuchsiennes auquel il préférerait Schwarzianes. A cela non plus je ne puis rien pour les remarques que j'ai dites.] [Letter from Poincaré to Mittag-Leffler, 27th July 1882 — IML]

Poincaré replied to Weierstrass' questions about his note "Sur l'intégration des équations différentielles" in the same way. In this paper, Poincaré had shown that there is always a change of variables so that solutions of a differential equation are representable by a convergent series for all values of the new variable. Weierstrass' objection concerned collisions in the three-body problem. Poincaré answered that in case of collision, his new variable tends to infinity and so, after the collision, "the formulas do not give anything [...] and that is the best they have to do". Poincaré's answer to the old Master is a bit airy...

Later, after his success in Oscar II's Prize competition,⁷ Poincaré came to Mittag-Leffler's help in a polemic with the Swedish astronomer Gylden. In his prize-winning paper, Poincaré discussed the convergence of the new expansions (without secular terms) used by the astronomers.

The present day geometer have endeavored to replace these developments by some new ones, which contain only trigonometric terms. Recently, they have succeeded and Mr. Gylden's series or Mr. Lindstedt's ones contain only terms of the form

A sin at or B cos at.

[...]. Nevertheless, that is not the end. We may ask if these series are convergent and as the presence of "small divisors" result in making some terms very big, this convergence is dubious. This work will show that these

⁷ For more information about Oscar II's Prize competition, see Barrow-Green's book, *Poincaré and the Three Body Problem* [1997].

doubts are grounded; all of these series are divergent. However, I have to exclude those proposed by Mr. Gyldén in his last memoir. I do not have any way to know if they are convergent or divergent. [Poincaré 1888, p. 6]

[Aussi les géomètres contemporains se sont ils efforcés de remplacer ces développements par d'autres séries ne contenant que des termes trigonométriques. Ils y sont enfin parvenus dans ces derniers temps et les séries de M. Gyldén comme celles de M. Lindstedt ne contiennent que des termes en

$$A \sin at \quad \text{ou} \quad B \cos a \quad [\dots].$$

Tout n'est pas fini cependant. On peut se demander si les séries ainsi obtenues sont convergentes et comme la présence de «petits diviseurs» a pour effet de rendre certains termes très grands, on peut avoir des doutes sérieux au sujet de cette convergence. Le présent travail montrera que ces doutes sont fondés ; toutes ces séries divergent ; je dois réserver toutefois les séries proposées par M. Gyldén dans son dernier mémoire ; en ce qui les concerne je n'ai aucun moyen de reconnaître si elles sont convergentes ou divergentes. [Poincaré 1888, p. 6]

Without having read Poincaré's paper, Gyldén maintained the convergence of his own expansions and claimed priority over Poincaré about some results concerning asymptotic solutions. King Oscar required a response from Mittag-Leffler, so Mittag-Leffler called for Poincaré's help. In his answer, first, Poincaré said that reading Gyldén's papers [1887] and understanding his results was very hard. He explained also his method of working that is an ideal standard:

Shall I confess to you that I think Mr. Gyldén's style a bit tedious and it is very difficult for me to read it? I am used, when I read a memoir, to glance over first quickly so as to have a general impression and then I come back on the points which seem to me obscure. I feel more convenient to do again a proof than to examine thoroughly those of the author. My proof is generally by far less proper but they have for me the advantage that they are mine. [Letter from Poincaré to Mittag-Leffler, 5th February 1889 — IML]

[Vous avouerai-je que je trouve le style de M. Gyldén un peu rebutant et qu'il me donne beaucoup de mal à lire. J'ai l'habitude, quand je lis un mémoire, de le parcourir d'abord rapidement de façon à me donner une idée de l'ensemble et de revenir ensuite sur les points qui me semblent obscurs. Je trouve plus commode de refaire des démonstrations que d'approfondir celles de l'auteur. Mes démonstrations peuvent être généralement beaucoup moins bonnes mais elles ont pour moi l'avantage d'être miennes.] [Letter from Poincaré to Mittag-Leffler, 5th February 1889 — IML]

As Gyldén's algorithm is not precisely defined, deciding its convergence is hard. Poincaré showed that Gyldén's argumentation is not rigorous and his conclusion is pitiless:

That [Gyldén's proof] amounts to accept the following principle:

Every entire series with a lower than one variable is convergent unless we have very serious reason to be doubtful of this convergence. [Letter from Poincaré to Mittag-Leffler, 1st March 1889 — IML]

[Ce qui revient à admettre le principe suivant :

Toute série procédant suivant les puissances croissantes d'une variable plus petite que 1 est convergente à moins qu'on ait des raisons sérieuses de douter de cette convergence.] [Letter from Poincaré to Mittag-Leffler, 1st March 1889 — IML]

Later in the same letter, Poincaré moderated his judgement and admitted that Gyldén's expansions are sufficient for calculations in astronomy:

Mr. Gyldén says that he has proved the existence of asymptotic solutions and we say that he has not proved it. Where does that come from? The reason is that the words proof and convergence have not the same signification for him and for us. Mr. Gyldén thinks that he has proved the convergence of a serie if he has shown that the starting terms are decreasing and that it is improbable one of the 99 starting terms could be large. This is very sufficient for the astronomic applications but it not enough for the geometer. [Ibid.]

[M. Gyldén dit avoir démontré l'existence de solutions asymptotiques et nous, nous prétendons qu'il ne l'a pas fait. D'où vient cela ! de ce que les mots démonstration et convergence n'ont pas le même sens pour lui et pour nous. M. Gyldén croit avoir démontré la convergence d'une série lorsqu'il a fait voir que les premiers termes vont en décroissant et qu'il est invraisemblable qu'un des 99 premiers termes par exemple ait une valeur très grande.

Cela peut être très suffisant pour les applications astronomiques mais cela ne saurait contenter le géomètre. [Ibid.]

In any case, these mathematical polemics are the few occasions where Poincaré is somewhat expansive. In his university correspondence he was very reserved and showed no personal feelings. He rarely referred to political and social events. Furthermore, although interested in academic and university life, unlike Mittag-Leffler he did not participate in the usual gossip and intrigue of the scientific community.

3. The Oscar II 's Prize

Thanks to the book of J. Barrow-Green [1997], the story of the Oscar II's Competition is now well known. For the 60th birthday of Oscar II, the King of Sweden, who was a friend of the Sciences, Mittag-Leffler organized a mathematical competition and a commission whose members were Hermite, Weierstrass and Mittag-Leffler. They asked four questions "which from different points of view equally engage the attention of analysts, and the solution of which would be of the greatest interest for the progress of science" [Announcement of the Oscar Competition]. The first one, posed by Weierstrass, concerned the n -Body Problem and in particular, the stability of the orbits. Poincaré decided to compete:

I have not forgotten the King Oscar's prize and I will even say that this prize is exclusively holding my attention for one or two months. [Letter from Poincaré to Mittag-Leffler, 16th July 1887 — IML]

[Je n'ai pas oublié le prix du roi Oscar et je vous dirai même que ce prix me préoccupe exclusivement depuis un ou deux mois.] [Letter from Poincaré to Mittag-Leffler, 16th July 1887 — IML]

Poincaré tried to answer to the first question. The main result of his work was a proof of the stability in the case of the restricted 3-Body Problem. Poincaré obtained this result quickly and hoped to obtain a more general one:

Nevertheless, I have obtained some interesting results and I want to quote one of them. It concerns the special case where the first and the second bodies have a finite mass and the third a null one. The first and the second bodies trace circles centered in their mutual center of gravity and the third move in the plane of these circumferences. In this special case, I have found a rigorous proof of the stability and a way to determine precisely bounds of some parameters of the third body.

[...] *I hope now to be able to attack the general case and the 1st June not to have completely solved the question (I do not hope that) but to have found some results which will be sufficiently complete to be sent for the competition.* [Ibid.]

J'ai toutefois obtenu quelques résultats qui ne sont pas sans intérêt et dont je ne veux vous citer qu'un seul. Il s'agit du cas particulier où des trois corps, le 1er et le 2d ont une masse finie et le 3e une masse nulle. Le 1er et le 2d décrivent une circonférence autour de leur centre de gravité commun et le 3e se meut dans le plan de ces circonférences. Dans ce cas particulier, j'ai trouvé une démonstration rigoureuse de la stabilité et un moyen de déterminer des limites précises pour des éléments du 3e corps.

[...] J'espère maintenant que je pourrai aborder le cas général et que d'ici au 1er Juin j'aurai, sinon résolu complètement la question (cela, je ne l'espère pas) mais trouvé des résultats assez complets pour pouvoir être envoyés au concours.] [Ibid.]

Poincaré's hopes were frustrated because he arranged his memoir around this result. He expressed his disappointment in the introduction of his original publication:

The present memoir has been undertaken to answer to the first of the four questions of the competition; but the results I have obtained are so uncompleted that I should hesitate to publish them if I do not know that the importance and the difficulty of this problem give some interest with all which is concerning it and that we may expect a definitive solution only after a long succession of attempts. [...] I had to restrict myself with a special case. I have only handled the equations of the dynamic in the case [...] of two degrees of freedom.

[Poincaré 1888, p. 5-8]

[Le présent mémoire a été entrepris pour répondre à la première des quatre questions du concours ; mais les résultats que j'ai obtenus sont tellement incomplets que j'aurais hésité à les publier si je ne savais que l'importance et la difficulté du problème donne de l'intérêt à tout ce qui s'y rapporte et qu'on ne peut attendre une solution définitive que d'une longue série d'efforts successifs. [...] j'ai dû me restreindre à un cas particulier. J'ai traité seulement des équations de la dynamique quand il n'y a (pour employer une expression usitée en Angleterre) que deux degrés de liberté (degrees of freedom).] [Poincaré 1888, p. 5-8]

During the summer of 1888, Mittag-Leffler and Weierstrass studied Poincaré's memoir. Mittag-Leffler was very enthusiastic, but thought that Poincaré's work was very difficult and that some proofs were incomplete. Poincaré answered his questions with 100 pages of supplementary notes.⁸ On the 21 January 1889, King Oscar awarded the Prize to Poincaré.

But, at the beginning of July, Mittag-Leffler transmitted to Poincaré some questions he had been asked by Phragmén who was his assistant for *Acta Mathematica*. They concerned the convergence of expansions of the asymptotic solutions introduced by Poincaré. Poincaré's answer formed the last supplementary note "Sur les solutions asymptotiques" [1888, p. 251-256]. It seems that these questions spurred Poincaré to investigate the whole proof of the convergence of asymptotic solutions more precisely and unfortunately to find an irreparable error. On the 1 December, Poincaré announced in despair to Mittag-Leffler that the major part of his memoir was flawed.

⁸ The whole memoir with its supplementary notes then comprised then 258 pages.

I will not conceal from you the distress this discovery has caused me. In the first place, I do not know if you still think that the results which remain, [...] deserve the great reward you have given them. [Letter from Poincaré to Mittag-Leffler, 1st December 1889 — IML]

[Je ne vous dissimulerai pas le chagrin que me cause cette découverte. Je ne sais d'abord si vous jugerez encore que les résultats qui subsistent [...] méritent la haute récompense que vous avez bien voulu m'accorder.] [Letter from Poincaré to Mittag-Leffler, 1st December 1889 — IML]

Poincaré's proof of stability was based on the fact that asymptotic surfaces, i.e. surfaces generated by asymptotic solutions⁹, are closed and so other solutions are shut in. This last result followed from a more general lemma, but this lemma was wrong.¹⁰

We may imagine that for a while Poincaré hoped to save his stability result by proving the closure of the asymptotic surfaces directly from the convergence of the asymptotic solutions. Indeed, on the 10th December, Hermite wrote to Mittag-Leffler that the error was not as important as Poincaré had believed and that it is only a question of reshaping his "admirable work". But Poincaré discovered that these expansions were not convergent but asymptotic.

Unfortunately, these series are not convergent. [...] But although divergent, cannot we make use of them? [...] We can say that the series we have obtained in this paragraph stand for the asymptotic solutions with small value of μ as the same manner than the Stirling formula stands for the eulerian functions. [Poincaré 1890, p. 384-386]

[Malheureusement, les séries ainsi obtenues ne sont pas convergentes. [...] Mais quoiqu'elles soient divergentes, ne peut-on en tirer quelque parti ? [...] On peut dire que les séries que nous avons obtenues dans le présent paragraphe représentent les solutions asymptotiques pour les petites valeurs de m de la même manière que la série de Stirling représente les fonctions eulériennes.] [Poincaré 1890, p. 384-386]

Although many results survived, in particular the divergence of Lindstedt's series and the recurrence theorem, it was nevertheless a disaster. Mittag-Leffler kept cool and answered that he did not regret having given the prize to Poincaré. He busied himself with his usual efficiency. By then, Poincaré's memoir had been printed and partially delivered to the editors of *Acta Mathematica*, so recovering them without giving away the motives was the most immediate problem. When that was done Mittag-Leffler proposed that Poincaré writes and publishes a new memoir.

And now here are what I propose you to do and what will be, from my point of view, the most honourable for you as for me. You write a new memoir in which you insert all is remaining from your original memoir and the developments which are in the notes and whatever you think right to add. You write for this new memoir an introduction in which you say that it is a modifying of the awarded memoir and that you have added some developments which were only pointed out and corrected an error which has crept into your early research. [...] I think that this story has to be kept between us until the publication of your memoir. [Letter from Mittag-Leffler to Poincaré, 5th December 1889 — IML]

[Et voici maintenant ce que je vous propose à faire et ce qui sera d'après mon opinion le plus honorable pour vous comme pour nous. Vous écrivez un nouveau mémoire dans lequel vous introduisez tout ce qui reste de

⁹ Nowadays, asymptotic surfaces are called "stable manifolds".

¹⁰ See Barrow-Green [1994, 1997].

vosre mémoire original ainsi que les développements qui se trouvent dans les notes ainsi que tous les autres développements que vous jugez bon d'introduire. Vous écrivez à ce nouveau mémoire une introduction dans laquelle vous dites qu'il est un remaniement du mémoire couronné dans lequel des développements qui se trouvaient seulement indiqués dans le mémoire originaire sont donnés et dans lequel une erreur que vous indiquez et qui s'était glissée dans vos premières recherches a été corrigée. [...] Je trouve que toute cette histoire doit rester entre nous jusqu'à la publication de votre mémoire.] [Letter from Mittag-Leffler to Poincaré, 5th December 1889 — IML]

Although Weierstrass complained that he had only learned the news of Poincaré's error through gossip, Mittag-Leffler succeeded rather well in keeping the whole story secret. On 20 December, after Poincaré had accepted Mittag-Leffler's deal¹¹, Mittag-Leffler considered that the crisis was settled from the institutional point of view. Secure at home, he encouraged Hermite to praise Poincaré's work at the *Académie des Sciences*. *At the solemn meeting of the Institute, you must absolutely speak about his memoir. Otherwise, everyone will be astonished and will begin to wonder about the reason of your silence. Moreover, you can really praise the memoir without being afraid of exaggerating, because in any case, it will remain as one of the best masterpiece of our days.* [Letter from Mittag-Leffler to Hermite, 20th December 1889 — AS]

[Il faut absolument que vous parlez de son mémoire à la séance solennelle de l'Institut. Sinon, tout le monde s'étonnera et on commencera de se demander la cause de votre silence. Et vous pouvez vraiment louer le mémoire sans craindre d'exagérer car il restera dans tous les cas un des premiers chefs-d'œuvre de notre temps.] [Letter from Mittag-Leffler to Hermite, 20th December 1889 — AS]

He added:

First, the advice will be very useful to Mr. Poincaré who maybe, will leave in the future his regrettable habit of stating results whose he knows only imperfectly the proof. [...] Of course, I share your opinion about Poincaré and his genius but he has compelled too much to every body and it is not profitable that someone who does not condescend to be more rigorous, holds a so prominent position he has had up to now. He is still young, he will improve himself and mathematical Sciences will gain. [Ibid.]

[D'abord la leçon sera très utile à M. Poincaré qui laissera peut-être à l'avenir son habitude fâcheuse d'énoncer des résultats dont il ne connaît la démonstration que d'une manière imparfaite. [...] Certainement que je partage votre opinion sur Poincaré et sur son génie immense mais il a imposé trop sur tout le monde et ce n'est pas utile qu'un homme qui ne daigne pas être plus exact que lui occupe une position si grande qu'il a eue jusqu'ici. Il est jeune encore, il se reformera et les sciences mathématiques y gagneront.] [Ibid.]

4. The Nobel Prizes in Physics

One of the most interesting points of the Poincaré-Mittag-Leffler relationship is Mittag-Leffler's attempt to obtain the Nobel Prize in Physics for Poincaré. His ostensible goal was to have the Nobel Prize in Physics awarded to theoreticians. As usual, in so doing he combined an undeniable love of Mathematics with more practical pre-occupations. Indeed, he now found a new occasion for fighting his opponents at the Swedish University, all supporters of "the scientific methods of the Oswald-Arrhenius, who does not like

¹¹ In particular, Poincaré accepted to pay the expenses of the new printing (3585 Swedish crowns). In comparison, the value of the price was 2500 crowns and Mittag-Leffler's annual wage was 7000 crowns [Domar 1982].

mathematics”.¹² In her book *The Beginnings of the Nobel Institution* [1984], Crawford shows that behind these trivial cares, nothing less than the definition of the field of the Physics was at stake in the opposition between experimentalists and theoreticians over the Nobel Prize in Physics.

On 18 July 1902, Mittag-Leffler explained his strategy to Painlevé:

Now, I am doing my best to make Lorentz winner of the Nobel Prize. It is necessary to set Lorentz before Poincaré. First, Lorentz is more directly physicist and for having a report signed by a sufficiently competent authority. Poincaré is writing the report. If I succeed, [...] I should win to open the door to the theory, which was surely in the ideas of Nobel, and then first Poincaré, and you will come. [Letter from Mittag-Leffler to Painlevé, 18th July 1902 — IML]

[Je fais mon possible pour l’instant pour faire donner le prix Nobel à Lorentz. C’était nécessaire de mettre Lorentz avant Poincaré. D’abord parce que Lorentz est plus directement physicien et puis pour avoir un rapport par une autorité suffisamment compétente. C’est Poincaré qui a écrit le rapport. Si je réussis, [...] j’aurai gagné d’avoir ouvert la voie pour la théorie ce qui était aussi sûrement dans les idées de Nobel et alors d’abord Poincaré et puis vous-mêmes viendront à la suite.] [Letter from Mittag-Leffler to Painlevé, 18th July 1902 — IML]

Poincaré’s “authority” was meant to convince the Nobel Commission to award the Prize to the theoretician Lorentz. This was the first step, and only then could the commission give the Prize to Poincaré for his works in mathematical Physics, but Mittag-Leffler had to agree to compromise and to accept the division of the Prize between Lorentz and Zeeman, who was more of experimentalist.

Between 1904 and 1909, Poincaré was nominated several times for the Nobel Prize in Physics on Darboux’s initiative. In 1910, Mittag-Leffler thought that the circumstances were propitious for awarding the Prize to Poincaré. He wrote to Appell in order to co-ordinate the French and Swedish attempts:

The time is come when we can hope to make Poincaré winner of the Nobel Prize. I send enclosed with the next mail a project written by Fredholm that he subjects to your estimation and those of Mr. Darboux. He has made a large use of the proposition made by Darboux this year. The most important is first to argue about the prominent part played by the pure theory in Physics and then to conclude with the proposition to give the Prize for discoveries related to a sufficiently simple formula. After some discussions, we have found this formula with Poincaré’s discoveries concerning the differential equations of the mathematical Physics. I think that we will gain the victory with this program. [Letter from Mittag-Leffler to Appell, 28 November 1909 – IML]

[Le temps est venu où nous pouvons espérer de faire donner le prix Nobel à Poincaré. J’envoie avec le courrier prochain un projet fait par Fredholm qu’il soumet à votre jugement et à celui de M. Darboux. Il a largement puisé de la proposition de Darboux pour cette année. L’essentiel est d’abord de bien développer le rôle que joue la théorie pure dans la physique et puis de conclure avec la proposition de donner le prix pour des découvertes définies par une formule suffisamment simple. Après quelques discussions nous avons trouvé cette formule dans les découvertes de Poincaré sur les équations différentielles de la physique mathématique. Je crois que nous remporterons la victoire sur ce programme.]

¹² Crawford [1984] describes the constituting shows.

Mittag-Leffler added that the nominators had to avoid “mathematics” and refer to “pure theory” because “like those who are only experimentalists, members of the Nobel committee for Physics are crazily frightened by Mathematics”.

Appell, Darboux and Fredholm signed a report “sur les travaux d’ordre physique de Poincaré”. In the introduction, they claimed that it was impossible to ignore the very important part played by the progress of theory in the advancement of the Sciences. They emphasized that physicists had to propose theories and to build mathematical tools:

In the same time when Physics wants to understand the hidden and inner mechanism of the things, the part of the hypothesis gains, and alike, the need for the physicist of more and more perfect analytical tools. [Report about “les travaux d’ordre physique de M. Poincaré – CHS]

[A mesure que la physique veut pénétrer le mécanisme intérieur et caché des choses, le rôle de l’hypothèse gagne en importance, et dans la même mesure croît le besoin du physicien d’un instrument analytique d’une perfection de plus en plus grande. [Rapport sur les travaux d’ordre physique de M. Poincaré — CHS]

Poincaré’s nomination was justified by his contributions to the general and correct solution of problems of mathematical Physics. Then Mittag-Leffler sent to the whole physics community a circular asking them to nominate Poincaré for the Nobel Prize. 34 eminent Physicists or Mathematicians supported the proposition in favor of Poincaré.¹³ Unfortunately, the experimentalists and Arrhenius’ net were not convinced and the Nobel commission awarded the Prize to an experimentalist, Van der Waals.

For the Nobel Prize, we have been again beaten this time. This crowd of naturalists who do not understand anything about the fundamentals of the things has voted against us. They fear Mathematics because they never have the smallest possibility to understand anything. [Letter from Mittag-Leffler to Poincaré, 6th December 1910 — IML]

[Nous avons été vaincus encore cette fois-ci pour le prix Nobel. Toute cette foule de naturalistes qui ne comprend rien sur le fond des choses a voté contre nous. Ils ont peur des mathématiques parce qu’ils n’ont pas la moindre chance d’y comprendre jamais quelque chose.] [Letter from Mittag-Leffler to Poincaré, 6th December 1910 — IML]

Archives

AS — Archives of the Académie des Sciences — Paris (France).

CHS — Center for the History of Sciences — The royal Swedish Academy of Sciences — Stockholm (Sweden).

IML — Mittag-Leffler Institute — Djursholm (Sweden).

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¹³ Between 1901 and 1930, it is the greater number of supports to a candidature for the Nobel Prize in Physics [See Crawford 1987]. Nevertheless, some physicists like Rutherford did not agree with Poincaré’s nomination because they thought that Poincaré was not a real physicist.

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[1876] Zur Theorie der eindeutigen analytischen Functionen, *Abhandlungen der Königl. Akademie der Wissenschaften vom Jahre 1876*, 11-60 ; *Werke*, 2, 77-124.

«M. Poincaré est un jeune homme encore, je suppose.» [Letter from Mittag-Leffler to Hermite, 6th April 1881–AS]

More, Poincaré was born in Nancy, a town of Lorraine and half of this region has been annexed by Germans.

From a quantitative point of view, Poincaré's papers stands for 10% of the 10 first issues of *Acta Mathematica* Mathematics .

Poincaré makes use of the Dirichlet's Principle to show this theorem. It is worth noting that few months before, he does not know this principle:

Je ne connaissais pas ce principe à cette époque, [...]. [Poincaré 1901, p. 46]

For more precision about the Oscar II's Prize competition, see the Barrow-Green's book, Poincaré and the Three Body Problem [1997].

The whole memoir with the supplementary notes comprised then 258 pages.

Nowadays, asymptotic surfaces are called "stable manifolds".

See Barrow-Green [1994, 1997].

In particular, Poincaré accepted to pay the expenses of the new printing (3585 Swedish crowns). In comparison, the value of the price was 2500 crowns and Mittag-Leffler's annual wage was 7000 crowns [Domar 1982].

Crawford [1984] describes the constituting shows