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Rigorous coupled-wave theory
for lossy volume grating in Laue geometry

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Abstract

The rigorous coupled-wave analysis of grating diffraction initiated by Moharam and Gaylor [JOSA 71, 811 (1981)] has been extended to lossy media and Fourier gratings for Laue diffraction geometry. In the rigorous approach, the second derivatives of field amplitude in the wave propagation equation together with high-order wave Fourier components are kept and exact boundary conditions are implemented. The problem is reduced to a state differential equation formulated in a matrix form. The restriction to the case of two-waves, the so-called two-wave approximation, and to the first-derivative approximation have been considered and discussed with respect to previous works. The different approaches are numerically compared in the unslanted and slanted cases for a sliced volume multilayer grating dedicated to the x-UV domain.

OCIS codes : (050.7330) Volume gratings, (340.7480) X-ray optics, (300.6560) X-ray spectroscopy
1. Introduction

The recent development of Laue systems for the x-ray domain (Laue lens [1] or Laue grating [2-4]) based on the technology of sliced multilayers or nanostructures requires an accurate evaluation of their performances in terms of efficiency. These systems can be regarded, at least locally, as volume gratings for the calculation of their efficiency as proposed by Maser and Schmähl [5].

The efficiency evaluation of thick gratings started with the classical paper by Kogelnik [6] on the basis of the so-called coupled-wave theory (CWT). This approach that can be viewed as an analytical formulation of the CWT, has been widely implemented for holographic gratings but suffers in its initial formulation of a number of approximations which are not always justified in many applications as mentioned in the paper by Gaylor and Moharam [7]; the latter paper presents the first rigorous coupled-wave theory of thick lossless grating and was of great interest to determine the effects of the different approximations (first-order derivative approximation, two-wave approximation (TWA), ...) done in the previous works. Nevertheless the formulation done in [7] is not directly applicable to lossy grating working in the spectral domain where absorption cannot be neglected (for instance UV, x-UV and x-ray regions).

In the context of sliced multilayers for the x-ray domain, several works [8-10] have been published by Russian scientists assuming generally the TWA; these latters give important practical tools to evaluate the performances and to optimize the performances of sliced multilayer Fresnel lenses or gratings. Always in the context of x-ray optics, Schneider [11] has presented a CWT going beyond the TWA and valid for lossy media but restricted to the first-order derivative approximation; Schneider’s approach is nevertheless largely implemented by the developers of Laue systems [2,3]. Kong has treated the problem by taken into account the second-order derivative but in the framework of the TWA [12]. Let us emphasize that the x-ray diffraction by volume gratings in Bragg or Laue conditions can be also treated by the dynamical theory of x-ray diffraction [13,14] or by modal methods [15]; modal approaches give rise to transcendental equations that are very tricky to solve.

The purpose of this communication is to give a rigorous CWT of volume gratings in Laue geometry valid for lossy systems and going beyond the first-order derivative approximation and the TWA; it can be regarded as an extension of the work by Gaylor and Moharam [7] to lossy media and Fourier gratings (in [7] the grating has a sinusoidal profile) and of the work by Kong [12]. We examine also the first-order derivate approximation and within its framework, we give a generalization of the approach by Levashov and Vinogradov in [9].

The paper is organized as follows: in section 2 we derive the basic set of second-order differential equations of the rigorous CWT for volume gratings; section 3 gives the solution in the framework of the first-order derivative approximation and discuss the TWA for the Laue mode; section 4, using the
state-variable method, presents the complete solution taking into account the second-order derivation and discusses the TWA always in the Laue mode. Numerical examples are shown in section 5. Section 6 is devoted to a discussion of the results together with a global conclusion.

2. Basic equations

The scheme of the diffracting grating in Laue mode is displayed in Figure 1. The electric field \( E(x,z) \) in the grating obeys the following wave equation in the s-polarization case:

\[
\frac{\partial^2 E(x,z)}{\partial x^2} + \frac{\partial^2 E(x,z)}{\partial z^2} + k^2 \epsilon(x,z) E(x,z) = 0
\]  

(1)

Figure 1: Scheme of the slanted volume grating in the Laue geometry; the radiation is incoming from the medium 1 at the incident angle \( \theta_0 \) and is diffracted in the outgoing medium 3 at different orders \( \pm m \); the period of the grating is \( d \), its thickness is \( t \) and the slant angle \( \psi \). The reciprocal vector is denoted by \( g \).

The solution of Eq.(1) is written as an expansion of plane waves with spatially varying coefficients:

\[
E(x,z) = \sum_{m=-\infty}^{+\infty} \epsilon_m(x) \exp[i \rho_m \cdot r]
\]

\[
g = \frac{2 \pi}{D} \hat{t}; \quad \rho_m = \rho_0 + m g; \quad r = (x,z)
\]
The dielectric constant can be expanded in Fourier series:

\[ \epsilon(x, z) = \sum_{p=-\infty}^{+\infty} \epsilon_p \exp[i \cdot \mathbf{g} \cdot \mathbf{r}] \] (2)

By inserting (2) in (1), it comes:

\[
\left( \sum_{m=-\infty}^{+\infty} \left[ \ddot{\mathcal{E}}_m(x) + 2i \rho_{mx} \dot{\mathcal{E}}_m(x) + (k^2 \epsilon_0 - \rho_m^2) \mathcal{E}_m(x) \right] \exp[i \cdot \mathbf{g} \cdot \mathbf{r}] + k^2 \sum_{m} \sum_{m'=-\infty}^{+\infty} \epsilon_p \mathcal{E}_m(x) \exp[i \cdot (p + m) \cdot \mathbf{g} \cdot \mathbf{r}] \right) = 0
\] (3)

The dot denotes the derivative with respect to \( x \) and \( p' \) stands for \( p \) different from 0. Multiplying the above expression by \( \exp[-i \cdot n \cdot \mathbf{g} \cdot \mathbf{r}] \) and integrating over a period, one retains the \( n^{th} \) component of \( \mathcal{E}_n(x) \):

\[
\ddot{\mathcal{E}}_n(x) + 2i \rho_{nx} \dot{\mathcal{E}}_n(x) + (k^2 \epsilon_0 - \rho_n^2) \mathcal{E}_n(x) + k^2 \sum_{p'==-\infty}^{+\infty} \epsilon_p \mathcal{E}_{n-p}(x) = 0
\] (4)

Eq.(4) is indeed a set a linearly coupled differential equations forming the basis of the dynamical CWT. Note that this system is rigorous.

For an infinite structure along the \( x \) axis, the coefficients \( \mathcal{E}_m \) do not depend on \( x \) and one recovers the case of 1D-photonic crystal; in this case the derivatives \( \ddot{\mathcal{E}}_n(x) \) and \( \dot{\mathcal{E}}_n(x) \) cancel, and one has:

\[
\mathcal{E}_n = -k^2 \sum_{p'==-\infty}^{+\infty} \frac{\epsilon_p \mathcal{E}_{n-p}}{(k^2 \epsilon_0 - \rho_n^2)}
\] (5)

If the term \( \mathcal{E}_0 \) is dominant one has:

\[
\mathcal{E}_n = k^2 \frac{\epsilon_p \mathcal{E}_0}{(\rho_n^2 - k^2 \epsilon_0)}
\] (6)

It appears that strong coupling between various \( \mathcal{E}_n \) components occurs if

\[
k \sqrt{\epsilon_0} = \pm \rho_n
\] (7)

and the diffraction is then under the Bragg regime.

3. **First-order derivative and two-wave approximation**

In different previous works, the second derivative in Eq.(4) is neglected on the basis of different arguments (near normal incidence, …). In this section
we will consider this situation ; Eq.(4) becomes the linear first-order matrix differential equation :

\[ \dot{\mathbf{E}}(x) = \mathbf{M}\mathbf{E}(x) \]

(8)

where \( \mathbf{E}(x) \) is a the column matrix and \( \mathbf{M} \) the following matrix :

\[
\mathbf{M} = [\mathcal{M}_{ij}] = \begin{bmatrix}
\vdots \\
\mathcal{E}_{-1}(x) \\
\mathcal{E}_0(x) \\
\mathcal{E}_1(x) \\
\vdots 
\end{bmatrix}
\]

\[
= \begin{bmatrix}
\frac{k^2\varepsilon_0 - \rho_l^2}{2\rho_{lx}}\delta_{lj} + \frac{k^2\Delta\varepsilon \gamma \text{sinc}((l - j)\gamma \pi)}{2\rho_{lx}} (1 - \delta_{lj})
\end{bmatrix}
\]

(9)

where \( \delta \) is the Kronecker symbol. We use the fact that in our case

\[
\varepsilon_p = \Delta\varepsilon \text{sinc}(p\gamma \pi) = \varepsilon_0 = \bar{\varepsilon}
\]

(10)

The calculation of the diffraction efficiencies in Laue mode consists in solving Eq.(8) with the boundary conditions \( \mathcal{E}_n(0) = \delta_{n0} \) and a truncation of the system. Since the coefficients of the matrix \( \mathbf{M} \) do not vary with \( x \) (they are constant), Eq.(8) is formally solved as :

\[
\mathbf{E}(x) = \exp[\mathbf{M} \cdot x] \cdot \mathbf{E}_0
\]

(11)

\( \mathbf{E}_0 \) being the value of \( \mathbf{E}(x) \) at \( x = 0 \), that is :

\[
\mathbf{E}_0 = \begin{bmatrix}
\vdots \\
0 \\
\mathcal{E}_0(0) = 1 \\
0 \\
\vdots 
\end{bmatrix} = \delta_{n0}
\]

(12)

Alternatively, the solution of Eq.(8) can be found as an eigenvector expansion, that is :

\[
\mathbf{E}(x) = \sum_k c_k \mathbf{V}_k e^{\lambda_k x}
\]

(13)

where \( \mathbf{V}_k \) is the \( k^{th} \) eigenvector of \( \mathbf{M} \) associated with the eigenvalue \( \lambda_k \).

Let us now consider the TWA. When the conditions for a strong coupling only between the \( m^{th} \) mode and the \( 0^{th} \) mode are fulfilled (i.e. Bragg regime for the \( m^{th} \) mode), it is possible to keep only the two fields \( \mathcal{E}_0(x) \) and \( \mathcal{E}_m(x) \) in Eq.(8). Then, the equation reduces to :

\[
\mathcal{E}_0'(x) = \mathbf{M}_{0m}\mathcal{E}_0(x)
\]

(14)

with
\[
\mathcal{M}_{0m} = \begin{pmatrix}
\frac{i (k^2 \epsilon_0 - \rho_0^2)}{2 \rho_{0x}} & \frac{i k^2 \Delta \epsilon \gamma \text{sinc}(m \gamma \pi)}{2 \rho_{0x}} \\
\frac{i k^2 \Delta \epsilon \gamma \text{sinc}(m \gamma \pi)}{2 \rho_{mx}} & \frac{i (k^2 \epsilon_0 - \rho_m^2)}{2 \rho_{mx}}
\end{pmatrix}
\]

and

\[
\mathcal{E}_{0m}(x) = \begin{pmatrix}
\mathcal{E}_0(x) \\
\mathcal{E}_m(x)
\end{pmatrix}
\]

In this case, Eq.(16) can be solved analytically and we found:

\[
\mathcal{E}_m(x) = \frac{i}{2} k^2 \exp(\chi z) \Delta \epsilon \gamma \text{sinc}(m \gamma \pi) \frac{\sinh(\beta x)}{\beta \rho_{mx}}
\]

with \(\beta\)

\[
\sqrt{4 \frac{k^4 \Delta \epsilon^2 \gamma^2 \text{sinc}^2(m \gamma \pi) \rho_{0x} \rho_{mx}}{\rho_{0x} \rho_m}} + \left(\rho_0^2 \rho_{mx} - \rho_{0x} \rho_m^2 + k^2 \epsilon_0 (\rho_{0x} - \rho_{mx})\right)^2
\]

\[
= \frac{i}{4 \rho_{0x} \rho_{mx}} \left(\frac{k^2 \epsilon_0 (\rho_{0x} + \rho_{mx}) - (\rho_0^2 \rho_{mx} + \rho_{0x} \rho_m^2)}{\rho_{0x} \rho_{mx}}\right)
\]

One can show that:

\[
|\beta^2 - \chi^2| = \left|\frac{(k^2 \epsilon_0 - \rho_0^2)(k^2 \epsilon_0 - \rho_m^2) - k^4 \Delta \epsilon^2 \gamma^2 \text{sinc}^2(m \gamma \pi)}{4 \rho_{0x} \rho_{mx}}\right|
\]

Formula (17-18) generalize the formula (22-23) by Levashov and Vinogradov in [9]; Eq.(17) is formally identical to Eq.(22) of the previous reference provided one makes the substitution: \(i \chi \rightarrow -i \frac{B}{2} \); \(\beta \rightarrow i \frac{\Delta}{2} \); \(\frac{k}{\rho_{mx}} \rightarrow \frac{1}{\kappa_m}\) with the notations of reference [9]. Thus,

\[
|\beta^2 - \chi^2| = \left|\frac{k^2}{4} (\Delta^2 - B^2)\right|
\]

Since from Eq.(23) of reference [9], one has:

\[
|\Delta^2 - B^2| = \left|\frac{\epsilon_m^2}{\kappa_0 \kappa_m}\right| \equiv \left|\frac{\Delta \epsilon^2 k^2 \gamma^2 \text{sinc}^2(m \gamma \pi)}{\rho_{0x} \rho_{mx}}\right|
\]

Our Eq.(19) is satisfied if \((k^2 \epsilon_0 - \rho_m^2) = 0\), if the \(m^{th}\) mode is in the Bragg regime (see Eq.(7)); this result is in agreement with the assumption of [9] which stipulates that their equations (22-23) are valid close to the Bragg resonance.

From Eq.(17), the efficiency of the \(m^{th}\) transmitted order is given by:
\[\varepsilon_m = |\varepsilon_m(x)|^2 = \frac{1}{4} k^4 \exp(-\text{Im}[\chi]d) d^2 |\Delta \varepsilon \gamma \text{sinc}(m \gamma \pi)|^2 \left| \frac{\text{sinhc}(\beta d)}{\rho_{mx}} \right|^2\]  

where \(d\) is the thickness of the Laue grating and \(\text{sinhc}\) stands for the cardinal hyperbolic sine: \(\text{sinh}(y)/y\). The term \(|\text{sinhc}(\beta d)|^2\) can be splitted into two terms:

\[|\text{sinc}(\text{Re[\beta d]})|^2 + |\text{sinhc}(\text{Im[\beta d]})|^2\]  

The first term corresponds to the Pendellösung oscillations while the second one is for anomalous transmission, i.e. the Borrmann effect. By the way, let us mention that formula (22) makes it possible to find the optimal parameters of a volume grating in Laue geometry in a way similar to the one performed in [9].

4. Beyond the first-order derivative

If one keeps the second-order derivative in the system (4), then in order to apply the matrix formalism, it is valuable to express the differential equations in forms that involve only the first-order differential derivatives, using the so-called state-variable representation as proposed by Moharam and Gaylar[7]. One introduces the state-variable column matrices:

\[\mathbf{E}^{(0)}(x) \equiv \mathbf{E}(x), \mathbf{E}^{(1)}(x) \equiv \mathbf{E}^{(0)}(x), \mathbf{E}^{(2)}(x) \equiv \mathbf{E}^{(1)}(x) = \mathbf{E}^{(0)}(x)\]  

By means of these state-variables, the system (4) can be written:

\[\mathbf{\dot{F}}(x) = \mathbf{S} \mathbf{F}(x)\]  

where \(\mathbf{S}\) is a supermatrix given by:

\[\mathbf{S} = \begin{pmatrix} 2 \rho_{lx} & [0] \\ i \rho_{lx} & \mathcal{M}_{ij} \end{pmatrix} \begin{pmatrix} 1d \\ 2 \rho_{lx} \delta_{ij} \end{pmatrix}\]  

and

\[\mathbf{F}(x) = \begin{pmatrix} \mathbf{E}^{(0)}(x) \\ \mathbf{E}^{(1)}(x) \end{pmatrix}, \mathbf{\dot{F}}(x) = \begin{pmatrix} \mathbf{E}^{(1)}(x) \\ \mathbf{E}^{(2)}(x) \end{pmatrix} = \frac{d}{dx} \mathbf{F}(x)\]  

The problem reduces in solving the differential matrix equation (25) with the four boundary conditions given below. In the incoming medium (1) the electric field is:

\[E_1(x,z) = \exp[i(\zeta_{10x} x + \zeta_{10z} z)] + \sum_m R_m \exp[-i(\zeta_{1mx} x - \zeta_{1mz} z)]\]  

In the outgoing medium (3), the electric field is:
\[ E_3(x, z) = \sum_m T_m \exp[i (\zeta_{3mx} (x - d) + \zeta_{3mz} z)] \] (29)

The terms \( \zeta_{jms} \) are the \( s \) (= \( x \) or \( z \)) component of the wavevector in the \( j \) (= 1, 3) medium for the \( m \)th diffraction order. We apply the different boundary conditions:

1. Continuity of the tangential component of the electric field \( \mathbf{E} \) at \( x = 0 \):
\[ \mathbf{E}(0)(0) = [R_m + \delta_{om}] \] (30)

2. Continuity of the tangential component of the electric field \( \mathbf{E} \) at \( x = d \):
\[ \mathbf{E}(0)(d) = [T_m] \] (31)

3. Continuity of the tangential component of the magnetic field \( \mathbf{H} \) at \( x = 0 \), that is of the first derivative \( \mathbf{E}^{(1)}(x = 0) \):
\[ \mathbf{E}^{(1)}(0) = [-i \zeta_{1mx}(R_m - \delta m_0)] \] (32)

4. Continuity of the tangential component of the magnetic field \( \mathbf{H} \) at \( x = d \), that is of the first derivative \( \mathbf{E}^{(1)}(x = d) \):
\[ \mathbf{E}^{(1)}(d) = [+ i \zeta_{3mx} T_m] \] (33)

Practically \( \mathbf{F}(x) \) can be obtained as an eigenvector expansion
\[ \mathbf{F}(x) = \sum_k c_k \mathbf{W}_k e^{\lambda_k x} \] (34)

where \( \mathbf{W}_k \) is the \( k \)th eigenvector of \( \mathbf{S} \) associated with the eigenvalue \( \lambda_k \). One has:
\[ \mathbf{F}_0 = \sum_k c_k \mathbf{W}_k \] (35)

and
\[ \mathbf{F}_d = \mathbf{F}(d) = \sum_k c_k e^{\lambda_k d} \mathbf{W}_k \] (36)

where \( c_k \) are the coefficients determined by the boundary conditions of the problem. Of course the computation requires a truncation in the expansion, so that practically only \( n \) eigenvectors are retained in the calculation; the choice of the value of \( n \) will be discussed hereafter. If one considers the modal matrix \( \mathbf{M} \) defined from the \( n \) eigenvectors \( \mathbf{W}_k \) by:
\[ \mathbf{M} = (\mathbf{W}_1 \ldots \mathbf{W}_k \ldots \mathbf{W}_n) \] (37)

and the shifted modal matrix \( \mathbf{Z} \) defined from the \( n \) eigenvectors \( \mathbf{W}_k \) and the corresponding eigenvalues \( \lambda_k \) by:
\[ Z = (e^{\lambda_1 d} W_1 \cdots e^{\lambda_k d} W_k \cdots e^{\lambda_n d} W_n) \]  

one has from Eqs.(35-36):
\[ \mathcal{F}_0 = \mathbf{M} \cdot (c_k) \]  
and
\[ \mathcal{F}_d = Z \cdot (c_k) \]  
where \((c_k)\) is the column matrix formed with the coefficients \(c_k\). Therefore,
\[ (c_k) = \mathbf{M}^{-1} \cdot \mathcal{F}_0 \]  
and
\[ (c_k) = Z^{-1} \cdot \mathcal{F}_d \]  
that is,
\[ \mathbf{M}^{-1} \cdot \mathcal{F}_0 - Z^{-1} \cdot \mathcal{F}_d = 0 \]  

Since \(\mathcal{F}_0\) and \(\mathcal{F}_d\) are linear combinations of \(R_m\) and \(T_m\) respectively (see boundary conditions Eqs(30-34), solving the system of linear equations (43) provides the values of the quantities \(R_m\) and \(T_m\) and finally of the transmission and reflection efficiencies.

As done previously for the first-order derivative approximation, we consider the TWA with the \(0^{th}\) and the \(m^{th}\) orders where the second-order derivative is kept. The matrix \(\mathcal{S}\) reads:
\[ \mathcal{S} = \begin{pmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
-(k^2 \varepsilon_0 - \rho_0^2) & -k^2 \Delta \varepsilon \gamma \text{sinc}(m \gamma \pi) & -2i \rho_{0x} & 0 \\
-k^2 \Delta \varepsilon \gamma \text{sinc}(m \gamma \pi) & -(k^2 \varepsilon_0 - \rho_m^2) & 0 & -2i \rho_{mx}
\end{pmatrix} \]  

If \(\lambda_s (s = 1, \ldots, 4)\) stands for the eigenvalues of \(\mathcal{S}\), the modal matrix \(\mathbf{M}\) is given by:
\[ \mathbf{M} = (W_1 W_2 W_3 W_4) \]  
where \(W_s\) is the \(s^{th}\) eigenvector of \(\mathbf{M}\) which can be represented by the column matrix:
\[ W_s = \begin{pmatrix}
\tau_s \\
1 \\
\lambda_s \\
\lambda_s \tau_s \\
1
\end{pmatrix} \]
with
\[
\tau_s = \frac{\lambda_s^2 + 2i \rho_m \lambda_s - \frac{i (k^2 \varepsilon_0 - \rho_m^2)}{2 \rho_m}}{i k^2 \delta \varepsilon \gamma \text{sinc}(m \gamma \pi) \lambda_s}
\]  
(47)

The shifted modal matrix \(Z\) is given by:
\[
Z = \left( e^{\lambda_1} d W_1 e^{\lambda_2} d W_2 e^{\lambda_3} d W_3 e^{\lambda_4} d W_4 \right)
\]  
(48)

Using Eq.(43) and the boundary conditions, after some algebra, it comes that \(R_0\), \(R_m\), \(T_0\) and \(T_m\) satisfy the Kramer system of the form :
\[
Q \begin{pmatrix} R_0 \\ R_m \\ T_0 \\ T_m \end{pmatrix} = C
\]  
(49)

The coefficients of the 4 x 4 matrix \(Q\) and the column vector \(C\) can be written in a compact form but too long to be given in the paper. Indeed for practical application, it is enough to determine them numerically from (43) and then to inverse the matrix \(Q\) to get the amplitudes \(R_0\), \(R_m\), \(T_0\), \(T_m\)
\[
\begin{pmatrix} R_0 \\ R_m \\ T_0 \\ T_m \end{pmatrix} = Q^{-1} C
\]  
(51)

At this stage, let us outline that although the grating is supposed to work in the Laue mode, the equation (43) or (51) allowing to get not only the transmission amplitudes (\(T_m\) terms) but also the reflection ones (\(R_m\) terms). It follows that in principle, it is possible to calculate the reflection efficiency of a reflection grating by using (43) or (51) and assuming a thick grating, that is taking a large value of the thickness \(t\).

5. Numerical examples

We analyse the case of a Laue volume grating designed for the X-UV domain. We consider a sliced MoSi\(_2\)/Si multilayer with a period \(d = 100\) nm, a ratio of the MoSi\(_2\) layer thickness to the period \(d\) equals to 0.3. This system diffracts the radiation of wavelength \(\lambda = 15\) nm in the geometry shown in Fig.1; the incoming angle \(\theta_0\) equals 4.3°which corresponds to the Bragg angle for the first diffraction order. The optical indices are taken from [16]; let us recall that in the X-UV spectral domain, absorption is important, that is the materials cannot be considered as lossless, and the optical indices have an imaginary part of the order of \(10^{-3}\).
Figure 2 displays the grating efficiency $\varepsilon$ for the orders 0 and ±1 versus the thickness $t$ of the grating calculated from different theories, Schneider’s model [11], Levashov-Vinogradov theory [9] and our rigorous theory presented in Section 4, for the slant angle $\psi$ of 0°. In our approach and in Schneider’s one, 15 orders are retained. Figure 3 gives the same results but taking into account the slant angle $\psi$ of 10°. We observe that the shape of the curves $\varepsilon(t)$ differs from a model to another. One notes also that in our model, periodic oscillations with fringes separated by around $\lambda/2$ appears in the curve $\varepsilon(t)$, see the inset of Fig.2(b). They are given neither by Schneider’s model nor by the Levashov-Vinogradov approach. They can be regarded as the results of thin-film interference taking place between the two faces of the grating. In a general manner, Schneider’s model gives efficiency values slightly higher than our rigorous model while Levashov-Vinogradov theory overestimates the efficiency.
Figure 2: Efficiency versus grating thickness at the slant angle $\psi = 0^\circ$ for the orders $-1$ (a), $0$ (b) and $+1$ (c). Our model: red solid line; Schneider’s model: blue dotted line; Levashov-Vinogradov model: green dashed line.
Figure 3: Efficiency versus grating thickness at the slant angle $\psi = 10^\circ$ for the orders -1 (a), 0 (b) and +1 (c). Our model: red solid line; Schneider’s model: blue dotted line; Levashov-Vinogradov model: green dashed line.

6. Conclusion
Using the state-variable method, the coupled-wave analysis of diffraction by volume grating in Laue geometry has been extended to lossy and Fourier gratings; the previous rigorous coupled-wave approach by Moharam and Gaylor was restricted to lossless and sinusoidal gratings. Application to the case of sliced multilayer grating in the x-UV regime shows that it is necessary to go beyond the TWA and the first derivative approximation and that rigorous approach is needed to calculate accurately the diffraction efficiency. Nevertheless a fairly satisfactory value of the optimal thickness can be deduced from the Levashov-Vinogradov or Schneider approaches. To accurately determine the efficiency it is valuable to implement the rigorous method.
To be able to conclude in a more general manner by drawing curves similar to the ones in [7], it would be useful to introduce auxiliary or reduced variables as done by Moharam and Gaylord; unfortunately taking into account the absorption does not allow to easily perform this operation. The p-polarization case can be treated in a similar way by its numerical implementation but gives rise to difficulties in particular in the vicinity of the Brewster angle. This work is underway.

References


