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Abstract –Langevin's twin experiment does not lead to a contradiction within the frame of special relativity. Though, such is not the case of one of is variants, the forwards and backwards movement, relative to a Galilean space, of a thin elastic spinning cylinder. The crucial difference is that the space between the 'motionless' part and the one 'making the journey' is always bridged by some continuous portion of the cylinder. The study of the cylinder's behaviour and of its changes of shape shows that the experiment can be run with zero intrinsic twist and with intrinsic angular speeds of sections remaining constantly equal to the same value ω. Then, if we calculate the numbers m and n of turns around the axis that two particular points M and N make, we prove $n-m$ is both a zero and a non-zero integer.

1. Presentation of the experiment. – It is run with a 'thin' cylinder, that is, the thickness of which is infinitesimal[1]. The material it is made of is not rigid (in the sense of infinitely rigid): its shape changes when actions are exerted on it[2]. We suppose it is *elastic*: its mechanical properties remain invariable throughout the experiment.

The first part of the experiment goes on in a 'totally uniform' frame. The cylinder (C) is globally motionless relative to the Galilean space (E_0), and freely spinning around its axis. So, at this moment of the experiment, and only at this moment, it has an invariable shape.

(E) is another G.s. (Galilean space), relative to which (C) is in globally uniform translatory movement along its axis (D). So this axis is motionless both in (E) and (E_0), as is the common support of the x and x' axes in the usual presentation of the Lorentz transformation.

(G) is a **generatrix**[3] of (C), 'engraved' on it. (G) intersects a plane (P) motionless in (E) and perpendicular to (D) at a point N which plays the role of the first twin. The second twin's role is played by the point M, which is fixed – 'engraved', also – on (G)[4]. The global translatory speed of (C) relative to (E) is v, and its **intrinsic**[5] angular speed, that is the one observed in (E_0), is ω.

When (C) is in uniform movement relative to (E), one of its generatrix (G), described in (E), is at every instant not a straight line, but a regular helix, because two **sections**[6] (S_1) and (S_2), when considered at the same instant relative to (E), are considered in (E_0) under a temporal gap which, since (C) spins on its axis, generates an angular lag : there is an **extrinsic twist**, that is not linked to any physical change of shape.

[1] Therefore, it is a limit case we examine. The hypothesis of thinness makes the experiment use an arbitrarily small quantity of material, and so legitimates using the 'flat' frame of special relativity.

[2] It has long been known, through various thought experiments, that the rigid body is incompatible with special relativity, as physical actions should be instantaneously propagated through it. No harm is done to special relativity, because rigid bodies just do not exist [1].

[3] We call **generatrix** of (C) a set of points on its surface which, in this early stage of the experiment, when the movement of (C) is totally uniform in all its components, and when (C) is observed from the Galilean space (E_0) in which it is globally motionless, make a straight line parallel to the axis. The generatrix is 'engraved': in a second stage of the experiment, when (C) will be subjected to an action, this cylinder will lose its prime shape, for no rigid body exists, and therefore (G) will also lose its prime shape at the same time (C) does.

[4] One should be wary of confusing N with the orthogonal projection, in the Galilean space (E_0) or in another one, of M onto (P). Instead of the definition which has been given, this erroneous one refers to the simultaneity of that space. Moreover, it cuts out the continuous material link between M and N, which is crucial.

[5] When an object can be considered as motionless (or globally motionless if it is spinning) relative to a given Galilean space, we call its characteristics relative to this space **intrinsic**; relative to another Galilean space, we call them **extrinsic**. Thus the intrinsic angular velocity of (C) is that observed from (E_0).

[6] We call **section** of (C) its cutting by a plane perpendicular to its axis. Sections, like generatrices, are 'engraved' on (C). A **slice** of (C) is the set of the points that are within range made by two sections.

A cylinder (C’) identical to (C), the movement of which being also uniform in all its components, and spinning around (D) at the intrinsic angular speed ω, will strike (C).

The experiment, described relative to (E), begins at the instant i of (E) when the point M crosses the plane (P) forwards and coincides instantaneously with the point N.

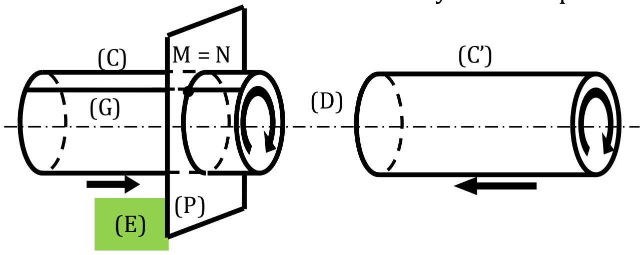

Fig. 1: Beginning of the experiment: M = N.

The second part of the experiment begins when (C’) strikes (C). Their movements then stop being uniform in all their components. A complex process starts, which will make (C) go backwards relative to the G.s. of reference (E). Since actions are propagated at a finite speed, this reversal is not immediate. The shape of (C) changes and its sections no longer go at the same speed[7]. For instance, immediately after the impact, because the head section (S_h) has just been struck by (C’), whose speed relative to (E) is superior to that of (C), the sections close to (S_h) are already going backwards, while those further are still going forwards.

The movement of (C) is no longer uniform; but we shall continue to analyse it relative to a Galilean system of reference, as is necessary. The idea that special relativity must be confined to uniform movements is erroneous: we are only obliged to use Galilean systems of reference. For instance, when Langevin’s traveller twin turns back, he experiences an acceleration[8].

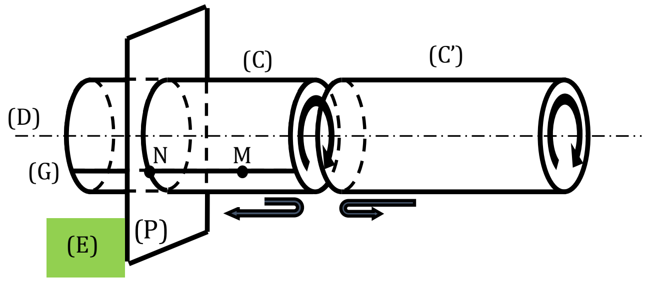

Fig. 2: Beginning of the second part of the experiment: (C’) strikes (C).

M goes through (P) backwards at the (E)-instant j, at which the experiment finishes. This point then coincides again with N. Because (C) does not entirely goes through (P), the point N exists at each instant of the interval $[i, j]$ of the duration of the experiment, at the end of which M has made m turns around (D) and N n turns.

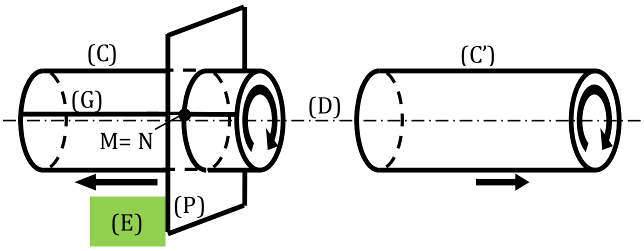

Fig. 3: End of the experiment: M = N.

2. Intrinsic twist and mechanical twist. – During the forwards and backwards experiment, the system maintains its symmetry of revolution. Nevertheless, some **twist** phenomena may or may not happen, which do not alter in any manner this symmetry of revolution.

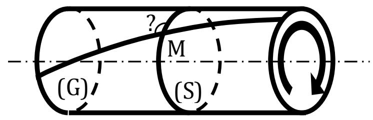

Fig. 4: Twist of (G).

Intrinsic twist. In order to define it, generatrices must have been previously engraved on (C) when it spins freely on its axis, globally motionless in a certain G.s. This cylinder has an **intrinsic twist** at the level of a section (S) if, when observing this tube from the **Galilean space tangent to the movement of translation <u>of one of its points M</u>**[9], we state – for instance with a set square, or by any process equivalent in theory – that the angle between (S) and the generatrix passing through M is not a right angle[10]. The choice of M does not matter, because of the symmetry of revolution. The absence or existence of any intrinsic twist is a local characteristic: the set square is theoretically infinitesimal, and nevertheless of an infinite accuracy.

Let (E_S) be the **G.s. tangent to the movement of translation <u>of (S)</u>**[11]. The movement of (E_M) relative to (E_S) is collinear to the tangent at M to (S), and the Lorentz transformation changes a straight line perpendicular to the direction of the movement into a straight line perpendicular to the direction of the movement, so:

The intrinsic twist at the level of one section (S) is zero if and only if (S) is, in the G.s. tangent to its movement of translation, perpendicular to the generatrices.

[7] A body whose parts are in the same uniform movement loses its shape if and only if its parts cease to be in that same uniform movement.

[8] We find in manuals of special relativity the formulae giving how acceleration changes when the Galilean system of reference changes. Moreover, *no physical experiment* can be run in a universe where all the movements are uniform.

[9] When a material system can at a certain instant be considered as made of points all going at the same speed, we call **G.s. tangent to its movement of translation** the unique G.s. in which its speed is zero at this instant.

[10] (S) is considered here as a curve and not as a plane. For intrinsic twist can also exist when the changes of shape of (C) make its radius vary, so that this cylinder may have, in the neighbourhood of (S), a form equivalent to that of a portion of a cone. The criterion for (S) and (G) being perpendicular is then the one for two curves, and not the one for a curve and a plane.

[11] We call **G.s. tangent to the movement of translation of <u>a section</u>** the unique G.s. in which its translatory speed is zero at the (E)-instant it is considered. The section is then globally motionless relative to this space, although spinning on its axis.

Mechanical twist. We say that (C) shows at a certain instant a **mechanical twist** at one of its sections (S) if the slices on each part of (S) exert torque[12] on each other.

Relationship between mechanical twist and intrinsic Twist. In order to know whether mechanical twist is present at the level of a section (S) at the instant t of the system of reference, we have a simple test: we can consider a situation *exactly identical, but* in which (S) is replaced by a zero thickness cutting-line, which divides (C) into two consecutive parts (C_1) and (C_2). The operation replaces (S) by two adjacent faces (S_1) and (S_2), that we suppose to be perfectly slippery. Since the strike is compressive, they remain adjacent in the instants immediately after t.

Two possibilities may occur in these instants:

a) (S_1) and (S_2) begin spinning at different angular speeds[13]. Because they turn at the same speed when (C) has not been cut, we conclude the existence of mechanical twist at (S), in a direction given by the sign of the difference of the angular speeds.

b) (S_1) and (S_2) still turn at the same speed. We conclude there is no mechanical twist at (S).

Mechanical twist at (S) will not be present at the instant t if and only if a perfectly slippery cut of (C) at (S) is such that the adjacent faces still spin at the same velocity during the instants immediately after t.

However, we can also know if there is any mechanical twist at (S) by studying how the shape of (C) is changed in the neighbourhood of (S).

By the symmetry of revolution of the situation, the manner the shape is changed is the same at every point of (S). So it is sufficient to study it at any of its points.

The specific shape of (C) in the neighbourhood of a point M of (S), considered in the Galilean space tangent to the movement of this point, is indicative of the existence or non-existence of mechanical twist at (S).

Under the principle of relativity, the local intrinsic properties of the elastic material of which (C) is made do not depend on its translatory speed. Thus, for a neighbourhood of M, being such that an instantaneous slippery cut along (S) may or may not make the two parts immediately glide one on the other, this fact depends only on the intrinsic shape of this neighbourhood, and not on its translatory movement.

So let us consider the case where angular speed is zero. In any possible interaction with a cylinder (C') the angular speed of which is also zero, the initial situation shows a symmetry about the plane containing (D) and the generatrix (G) passing through M. This symmetry will remain for the length of the interaction. So the changes of shape of (G) will in every case be such that (G) remains included in that plane, and thus perpendicular to (S). On the other hand, as the angular speeds of all the sections are zero, the exchanges of angular momentum are constantly zero, and so is the mechanical twist.

Because the local intrinsic shape is an indicator for local mechanical twist[14]:

There is mechanical twist at a section (S) if and only if there is intrinsic twist at (S).

3. Interactions between two thin rings. – *A sufficient condition so that two interacting rings do not exert torque on each other.* Let (A) and (A') be two identical elastic rings, which have the same axis (D), and of which the width and thickness are infinitesimal. Both of them are free from mechanical twist.

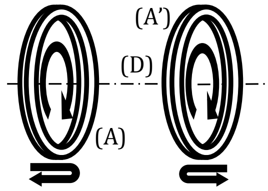

Fig. 5: Interaction between two rings.

They move towards each other, strike and go backwards. As the situation between (A) and (A') is symmetrical, they do not exchange any angular momentum during the interaction. So the torque one exerts on the other is always zero. This result does not depend upon the speed of one ring relative to the other.

By reason of symmetry, their angular speeds are identical during the interaction, and so the points of them that coincide at the very first instant of the impact still coincide during the whole interaction: the rings do not slip on each other. Their adjacent surfaces may be either perfectly slippery or rough, that makes no difference.

The common intrinsic angular speed of the two rings is ω at the beginning of the impact; but this value might vary during the interaction, because their angular momentum μ remains unchanged, though their temporary changes of shape could make their intrinsic moment of inertia J vary, and thus make their intrinsic angular speed μ/J vary.

We shall prove further the possibility of interaction with invariable moments of inertia, so we shall restrict our analysis to this case. Let us substitute for the ring (A') a thin elastic ring (Z) of the same size, moving like (A') at a uniform translatory speed along (D), and spinning at the angular speed ω'. Like (A), it is made of a homogeneous and isotropic material, of which other characteristics may be different. It has no intrinsic twist before being struck.

Let us study how the situation depends on ω', beginning with the case $\omega' < \omega$.

[12] The torque exerted by one slice on the other is the quantity (actual or virtual) of angular momentum they exchange per unit of time, that is to say its time derivative. It depends on the system of reference gliding along (D) that we chose, for the angular momentum does not depend on this choice (well-known consequence of the principle of relativity) while the time depends on the choice. But its being zero do not depend on it.

[13] In other words, if Δt is a positive duration neighbouring zero, for case a) $\Delta\omega \sim k\Delta t$, with $k \neq 0$; for case b) $\Delta\omega = o(\Delta t)$.

[14] Our reasoning seems to ignore centrifugal forces, but actually it does not, because they take part at their own exact rate to the generation of the local shape of (C).

First, let us suppose that the sides that are adjacent during the interaction are perfectly slippery. The exchange of angular momentum is thus zero throughout the impact.

An observer carried along the movement of (A), staying close to the contact area, watches the situation. If the intrinsic angular speed of (Z) were ω, he would, according to what we have just established, see at the instant when begins the interaction the points of (Z) turning at the same speed as those of (A). But the intrinsic angular speed of (Z) is strictly inferior to ω, so he sees the points of (Z) sliding along (A) in the opposite direction to the rotation of (A). He concludes that if there were friction, (Z) would act against the rotation of (A), that is to say it would exert a torque contrary to this movement.

Similarly, if $\omega' > \omega$ and the friction were not zero, (Z) would exert on (A) a torque acting in the direction of its movement.

As the torque exerted by (Z) on (A) continuously depends on ω', the intermediate value theorem states that it is zero when $\omega' = \omega$ and friction is not zero.

When two thin rings in translation along the same axis and spinning around it at the same intrinsic angular velocity interact in such a manner that their intrinsic momenta of inertia constantly remain invariable, then, throughout their interaction,
- *they exchange no angular momentum,*
- *they do not exert torque on each other,*
- *their intrinsic angular speed is constant.*

Possibility of an interaction with invariable intrinsic moments of inertia. Let us now prove we can make a ring be **neutral**, that is to say, whose moment of inertia remains unchanged during the impact. It suffices to exert on (A) a lateral pressure that cancels out the variations of its moment of inertia. In order to do that, let us bombard (A) during the interaction with a continuous flow of particles that all strike it all at the same angle and at the same speed, respecting its symmetry of revolution. If we want to cancel out an increase of the intrinsic moment of inertia, they will strike the outside cylindrical surface; if a decrease, the inside cylindrical surface. Let us WLOG[15] consider the first case.

According to the way the flow arrives on (A), it can make the momentum relative to (E) increase or decrease, as it can also make the angular momentum increase or decrease.

Let (R) be an infinitesimal rectangle on the outside cylindrical surface of (A), drawn from one edge to the other, and P be the centre of (R).

Let us observe the situation from the G.s. (E_P) tangent to its movement. The action exerted on (R) can be represented one-to-one by a vector the terminal point of which is P, parallel to the flow, and the length of which is proportional to its intensity. The original points of these vectors can be chosen everywhere in the half-space above (R). The flows making the momentum of (A) relative to (E) increase are characterised WLOG by a vector pushing (R) rightwards; and those making the momentum decrease, by a vector pushing (R) leftwards. This discrimination splits the original points of vectors into two regions separated by a revolution surface (S_A) transversal to the axis of the ring. In a similar way, the flows that make the angular momentum increase and those that made it decrease are on opposite sides of a surface (S_L) set along the axis. The two surfaces intersect on a curve (Γ), which characterizes the flows that modify neither the momentum of (A) relative to (E) nor its angular momentum.

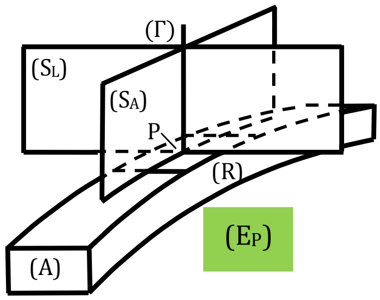

Fig. 6: Discrimination of the actions possibly exerted on (R).

Among these flows, some have an intensity I too weak to cancel out the increase of the moment of inertia: for instance, the flow zero.

Let us decide from now on to run the experiment at '*very reasonable*' speeds of approach and rotation of the rings, that is to say in such a way that changes of shape and relativistic effects be 'very tiny', and even 'negligible". Without intending to neglect anything, on the contrary we want to reason *with the absolute accuracy of geometry*, which infinitely exceeds that of our actual experiments[16]. But, since these phenomena are 'negligible', we can, by acting on elastic material, overpower them. A sufficiently high intensity I of the flow will have the effect of surpassing this 'negligible' and will make the momentum of inertia decrease. As it is continuously dependant on I, at every instant of the interaction, there exists an intermediate value of I that *exactly* cancels out its variation.

We can exert on (A) throughout the interaction a time-varying homogeneous pressure which, without transmitting to it momentum nor angular momentum, is such that its moment of inertia remains unchanged.

4. Possibility of running a forwards and backwards experiment at constant intrinsic angular speed and with zero intrinsic twist. – Let us resume our analysis of the interaction of the two cylinders. The situation is studied from the G.s. (E).

[15] Without loss of generality.

[16] Special relativity is a geometrical theory, meaning that it is ruled by axioms the accuracy of which is not supposed to be approximate but perfect. So one can, as with geometry, produce reasoning whose precision goes infinitely further than our actual ability to observe and verify; and in particular concerning bodies the velocity of which can be as low as we want, much slower than that of a snail. Mathematically speaking, relativistic effects are actual as soon as speeds are not zero.

Let us assume the mathematical induction hypothesis that at a certain (E)-instant t, we know the whole set of positions and speeds of the points of the cylinders, and that

– each section turns at the angular speed ω;

– there is no intrinsic twist anywhere;

– the angular momentum of each infinitesimal slice is the same as it was the first time of the experiment, when the movement of (C) was uniform in all its components.

As we know the whole set of positions and speeds of the points of the system, and because this data, added to the complete knowledge of the mechanical characteristics, determines its evolution at this instant t, we are able, at least in theory, to calculate this evolution, and so to obtain the knowledge of the whole set of positions and speeds at the instant $t + dt$. But there is no reason why the induction hypothesis would still be valid at this new instant, because the changes of shape make the moments of inertia of infinitesimal slices vary. As a consequence, intrinsic angular speeds do not remain unchanged, thus phenomena of exchange of angular momentum occur, and thus so do phenomena of mechanical twist, that is to say, of intrinsic twist. In order to counteract this inconvenience, we have to interfere a little: exert on each infinitesimal slice of (C) and (C') the action we have just studied, which exactly cancels out the variations of its intrinsic moment of inertia.

The successive infinitesimal slices making (C) are joined together, but that makes no change to the fact they then do not exert torque on each other. Indeed, let us suppose (C), at the instant t when we study its behaviour at the level of any section (S), is suddenly cut in two parts along (S). The two adjacent cylinders thus obtained (that compression phenomena prevent from separating), considered at this instant in a neighbourhood of the cutting-line,

– are turning at the same intrinsic angular speed everywhere,

– have no intrinsic twist anywhere.

So they do not exert torque on each other at this instant.

The flow cancelling the variations of the moments of inertia also does not exert torque, thus no mechanical twist will appear in the instants immediately after t, and thus no intrinsic twist.

As the intrinsic moments of inertia and the angular momenta remain unchanged, these neighbourhoods will in the instants immediately after t continue turning everywhere at the unchanged intrinsic angular speed ω and will not exert torque on each other. The adjacent sides can be rough or perfectly slippery, it does not matter. Two adjacent points on each side of the cutting-line will continue coinciding as they did when there was no cut, and that shows that torque is not exerted at (S) when there is no cut.

As this is true wherever the cut is made, the induction hypothesis is true again at the instant $t + dt$: the inductive step is performed[17]. Because the basis is obviously true, in the first part of the experiment, we can state:

It is possible to construct a forwards and backwards experiment throughout which the intrinsic angular speed of all sections remains what it was before the impact, and the intrinsic twist remains zero everywhere.

From now on, we shall consider the experiment is done thus.

5. Angular speed of N. – Let (S) be the section of (C) crossing (P) at the instant t of (E) when we intend to determine this angular speed, (E_S) be the G.s. tangent to the movement of translation of (S), and v the speed of (E_S) relative to (E).

WLOG, we can chose two new systems of reference RG_1 for (E) and RG'_1 for (E_S), placed according to the standard manner: relative to RG_1, the equation of (D) is $y = z = 0$, relative to RG'_1, it is $y' = z' = 0$; the coordinates of the event 'the centre of (S) crosses (P)' are (0, 0, 0, 0) relative to each system of reference ; the coordinates of N in RG'_1 at the instant 0 are (0, R, 0, 0).

Let us first suppose the radius of (C) does not vary. Since (G) has no intrinsic twist, it is intrinsically perpendicular to (S), thus the spatiotemporal coordinates of a generic point P of (G) are, in (E_S), at the instant 0: $\left(x', R + o(x'),\ o(x'), 0\right)$.

The points, being infinitely little, have no physical reality. Only their infinitesimal neighbourhoods do 'exist', whose acknowledgement as pertinent elementary physical entities having a certain speed presupposes the continuity of the speeds. When $x' = t' = 0$ the speed of P relative to RG'_1 is $(0, 0, R\omega)$, thus the spatiotemporal coordinates of a generic point of (G) relative to RG'_1 are:

$$(x' + o(x',\ t'),\ R + o(x',\ t'),\ R\omega t' + \ o(x',\ t'),\ t')$$

that is to say, replacing WLOG $x' + o(x',\ t')$ by x',

$$(x',\ R + o(x',\ t'),\ R\omega t' + \ o(x',\ t'),\ t').$$

An 'event' of coordinates relative to RG_1 $(x,\ y,\ z,\ t)$ is on the generatrix if and only if there exist x' and t' such that its image under the Lorentz transformation from RG_1 to RG'_1 has the form shown above. As the $o(x', t')$ are some of $o(x, t)$, this is equivalent to

$$\begin{cases} y = R \ + \ o(x,t) \\ z = R\omega\,\gamma\left(t - \dfrac{v\,x}{C^2}\right) + o(x,t) \end{cases}$$

If we make $x = 0$, we obtain $\begin{cases} y = R \ + \ o(\ t) \\ z = R\omega\gamma t + o(t) \end{cases}$

The angular speed of N in (E) is thus $\omega\gamma$. As it does not depend upon the radius R, this formula is still valid for a thick cylindrical tube, replacing (G) by an axial surface without intrinsic twist, and still valid again for all the thin revolution surfaces we can cut out of that thick tube. So the

[17] Our reasoning, using mathematical induction concerning an arithmetic progression with infinitesimal increments dt, is legitimate in non-standard analysis – the one we implicitly use when reasoning 'like physicists", in particular employing infinitesimals [2].

formula remains valid when the changes of shape of (C) make R vary.

The angular speed of N relative to (E) is $\omega_N(t) = \omega\,\gamma_N(t)$*, in which formula* $\gamma_N(t)$ *is the Lorentz factor associated with the translatory speed of the section crossing (P) at the (E)-instant t.*

6. Angular speed of M – The section M belongs to is a clock turning at the intrinsic angular speed ω, thus, according to a well-known result:

The angular speed of M relative to (E) is $\omega_M(t) = \frac{\omega}{\gamma_M(t)}$ *in which formula* $\gamma_M(t)$ *is the Lorentz factor – in general different from* $\gamma_N(t)$ *– associated with the translatory speed of the section to which M belongs.*

7. An integer which is both zero and non-zero. – Because $\gamma_N(t)$ and $\gamma_M(t)$, except for isolated instants, are strictly higher than 1, $\omega_N(t) > \omega_M(t)$. As these functions are continuous, $\int_i^j \omega_N(t)\,dt > \int_i^j \omega_M(t)\,dt$. So $n > m$.

Moreover, M and N coincide at the instants i and j. The difference between the numbers of their turns is thus an integer.

$n - m$ is a positive integer.

All other things remaining unchanged, let us now vary a parameter: the position of (P), which is still motionless in (E) throughout the experiment, and still perpendicular to (D). This position is now characterized by the abscise x of the point at which (P) intersects (D). The set of numbers x such that (C) – whose movement is strictly unchanged – crosses (P) but not entirely, and so generates a forwards and backwards experiment, is an $\mathbb{R}$-interval $[x_1, x_2]$.

The longest possible experiment, in which the almost entire cylinder temporarily crosses (P), except for its last section, is run when $x = x_1$. The shortest one, in which the point M reaches (P) only at a single instant, at the furthest point of its movement, is run when $x = x_2$.

For a given x in $[x_1, x_2]$, the experiment characterized by x gives the two numbers of turns $m(x)$ and $n(x)$. An infinitesimal variation of x can only induce an infinitesimal variation of $m(x)$ and $n(x)$, thus $n(x) - m(x)$ *continuously* depends on x belonging to the *interval* $[x_1, x_2]$. As moreover $n(x) - m(x)$ is an *integer*, it is constant on that interval. This constant is the value obtained above, the positive integer $n - m$. It is also the value obtained when $x = x_2$. Since the experiment has then a duration of zero, $m(x_2) = n(x_2) = 0$. Thus:

$n - m = 0$

8. Temporary conclusion. – So the experiment, when analysed in accordance with special relativity, leads to contradiction. Concerning mathematics, it is a catastrophe. The theory, because asserting the existence of an integer which is both zero and non-zero, allows, by multiplying it by a an arbitrary number, to state that all the real numbers are zero, and thus are equal – which does not prevent us from stating, at the same time, that they are different from zero: when a theory is contradictory, one can prove both anything and its opposite.

This situation looks similar to that which Greek mathematics seems to have known when their prime belief that all numbers be rational suddenly collapsed with the discovery that the square root of 2 is irrational. Because if we suppose that $\sqrt{2} = {}^{p}/_{q}$, with p and q as mutually prime integers, we can show that q is both even and odd, and thus that $0 = 1$. As a result, the theory collapses, which is unfair as well as incomprehensible considering the great many proofs it has already shown of its value. It is precisely due to this great number of proofs that the theory could not be scrapped. It was too efficient, too powerful to be ruined by a simple question of parity, completely outside the range of the experiment because rational numbers are dense among real numbers. There was necessarily a solution, which the Greeks finally discovered: to admit that not all numbers are rational. Once they had admitted this unconceivable solution, everything returned to normal. The old theorems recovered their prime solidity, and geometry, which had been ruined for a time, was reinforced by that temporary disaster. Moreover, mathematics were from now on enriched by an essential and fruitful knowledge, the existence of irrational numbers.

Like Euclidian geometry at that time, special relativity has given enough evidence of its adequacy to reality to survive the catastrophe, even if it has to be modified in at least one point [4]. As the modification is necessarily minor, we can consider it as being only a ‘detail’. But, since this ‘detail’ is necessarily related to some erroneous conception, as was the naïve belief that all real numbers are rational, it is more than a simple ‘detail’ and must teach us something of interest

REFERENCES. – As this thought experiment is original, it is not possible to provide a specific bibliography, except for some papers written by the same author. Nevertheless, only general knowledge is required to read and understand the given demonstration.

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