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Stability study of the milling process using an exponential force model in frequency domain.

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Abstract: In the last decade the prediction of the stability of different milling processes has experimented a great advance. Part of these advances are related to the successful application of frequency models that provide sufficiently precise analytic solutions for the industrial environment. Most of these models of regenerative chatter are based on linear models of cutting force. Following these models, the cutting forces vary linearly with the chip thickness; therefore it is supposed that the stability of the process does not depend on the feed rate. In practice, it is easy to find cases where chatter grows with low feed rates and it is eliminated increasing the feed. This effect can be partially considered by means of cutting coefficients that depend exponentially on the average chip thickness. The present work considers an exponential force model that takes into account the variation of cutting coefficients with the instantaneous chip thickness and proposes a way to study stability in frequency domain. Obtained results are compared with time domain simulations.

Keywords: milling, chatter, stability lobes.

1. INTRODUCTION

The self excited vibrations keep on being a classic problem that limits productivity of milling processes. The presence of vibrations affects the surface finish quality, causes loss of dimensional accuracy and accelerates the premature failure of cutting tools and other mechanical components. The prediction of optimal cutting conditions using a chatter stability model or measuring the frequency of the self excited vibration have allowed an improvement in the productivity, specially in high speed machining.

The pioneers in the development of these kind of mathematical tools were [Tobias and Fishwick, 1958], and [Tlusty and Polacek, 1965]. They stated that the main reasons for self-excited vibrations are the regeneration of the chip thickness and the mode coupling.
The milling process simulation is more complex than the other machining processes due to the discontinuous nature of cutting process—there are usually several cutting edges machining at the same time—and due to nonlinearities of this type of process. In addition, the cutting forces vary in magnitude and direction affecting the vibrations to the chip thickness. The first milling process modelling attempts were carried out by [Koegnisberger and Tlusty, 1970], [Sridhar et al., 1968], and [Opitz and Bernardi, 1970].

Time domain simulations permit to analyse non-linearities of the process and specially the possibility that the tool may lose contact with the workpiece. In the last two decades many research efforts have been carried out in time domain modelling approach ([Tlusty and Ismail, 1981], [Altintas and Spence, 1991], [Bediaga et al., 2004]. In order to obtain stability areas, time domain techniques require simulations under a great quantity of cutting conditions which is time consuming. However, in these kind of simulations it is easy to consider a nonlinear force model which includes the feed rate effect over the force magnitude. The most common model to describe this effect is the exponential model. The chip thickness is raised to a power lower than the unity, in order to represent the increment of the cutting coefficients when the chip thickness is decreasing.

Searching for an analytical solution, [Minis and Yanushevsky, 1993] used Floquet’s theorem and Fourier series to formulate the stability. [Altintas and Budak, 1995] developed an analytic method. It was based on a two dimensional single frequency analysis and allowed calculating stability diagrams very fast. Later, the model was extended to three-dimensional cases ([Jensen and Shin, 1999], [Altintas, 2001], [Munoa et al, 2005]).

These frequency domain models use linear cutting force equations. The effect of the feed rate on stability is ensured considering that the cutting coefficients vary exponentially with the average chip thickness ([Budak, 1994]). Hence, each feed rate has a different cutting coefficient and the stability lobes diagram will vary with the feed rate value. In this way, [Jensen and Shin, 1999] studied the effect of the feed rate on the stability for a face milling process.

These frequency domain models offer good results with large immersions of the mill near slotting cut, but as the cut becomes more interrupted the results differ from the reality, mainly due to the presence of additional lobes that limit the stability ([Davies et al, 2000], [Zatarain et al, 2004]). The stability of these interrupted cuts has been determined by way of alternative methods ([Davies et al, 2000], [Bayly et al, 2003], [Insperger and Stépán, 2004]).

The same problem can be approached within the frequency domain considering that vibrations comprise multiple harmonics of the tooth passing frequency from a generic
chatter frequency. These multi frequency models ([Budak, 1994], [Merdol and Altintas, 2004], [Zatarain et al, 2004]) also produce additional lobes.

The present work is focused on the study of the stability problem of the steel face milling process. Face milling of steel is carried out using big mills, high number of inserts and high radial immersion [Munoa et al, 2004]. Therefore, the milling process is far to be interrupted. Considering these aspects, a single frequency model is adapted ([Budak, 1994], [Altintas and Budak, 1995]) to include a novel solving method of exponential cutting force equations.

2. FACE MILLING PROCESS STABILITY.

1.1. Force Models:
In the present work, three different cutting models have been considered: the linear-edge force model and two different expressions of the exponential force coefficient model. The used notation is described in Figure 1.

- **Linear-edge force model:**
The linear model difference two type of forces: cutting forces proportional to the chip area \((h \cdot b)\) and edge forces related to the ploughing effect and proportional to the length of edge \((S)\) ([Budak, 1994]). If the cutting coefficients are considered constant, they do not describe the effect of the feed rate \((f_t)\).

![Figure 1: Geometry of the face milling.](image)
The frequency domain analysis of the exponential and the average force models were carried out by Budak and Altintas ([Budak, 1994]; [Altintas and Budak, 1995]; [Budak and Altintas, 1998]). The present work will be focused on the stability analysis of the exponential instantaneous force model. This model is used to obtain frequency domain stability lobes.

1.2. Dynamic Chip Thickness:
The chip thickness consists of static part \( h_s \), and a dynamic component \( h_d \). Considering the influence of the lead angle \( \kappa \), the chip load can be expressed by,

\[
\begin{align*}
    h_s(\phi) &= h_s + h_d \cdot g(\phi), \\
    h_t(\phi) &= f_s \cdot \sin(\phi) \cdot \sin \kappa, \\
    h_d(\phi) &= [u]^T \cdot [\Delta r] = \{\sin(\phi) \cdot \sin \kappa \cdot \cos(\phi) \cdot \sin \kappa - \cos \kappa\} \cdot [\Delta x \ \Delta y \ \Delta z]^T
\end{align*}
\]

Considering:
\[\Delta x = x(t) - x(t-T), \ \Delta y = y(t) - y(t-T), \ \Delta z = z(t) - z(t-T)\]
\(\kappa\): Lead Angle
In order to linearize the expression of the chip thickness, a Taylor/McLaurin series expansion is applied around the null dynamic chip thickness. Considering the first two terms the total chip thickness equation becomes a linear expression.

\[
 h_j(\phi) = (h_{s0} + h_{d0})^{1-X} = f(h_d) \approx f(0) + f'(0) \cdot h_d = (h_r)^{1-X} + (1 - x) \cdot (h_r)^{-X} \cdot h_d
\]  

(5)

### 1.3. Dynamic Milling Forces:

Considering the equation (4) and neglecting the static chip thickness, the tangential (\(F_t\)), radial (\(F_r\)) and axial (\(F_a\)) forces can be expressed as,

\[
\begin{align*}
\{F_t\} &= K_T \cdot b \cdot \left[ (1 - m_t) \cdot (h_r)^{-\omega} \right] \{u\} \cdot \{\Delta r\} \\
\{F_r\} &= K_T \cdot [D] \cdot \{k\} \cdot \{u\}^T \cdot \{\Delta r\} \\
\{F_a\} &= K_T \cdot [D] \cdot \{k\} \cdot \{A(t)\} \cdot \{\Delta r\}
\end{align*}
\]

(6)

The elemental forces are projected to cartesian axis using the transformation matrix:

\[
\begin{align*}
\{F_x\} &= \left[ \begin{array}{ccc} -\cos(\phi_j) & -\sin(\phi_j) & 0 \\
\sin(\phi_j) & \cos(\phi_j) & 0 \\
0 & 0 & 1 \end{array} \right] \{F_t\} \\
\{F_y\} &= \left[ \begin{array}{ccc} -\cos(\phi_j) & -\sin(\phi_j) & 0 \\
\sin(\phi_j) & \cos(\phi_j) & 0 \\
0 & 0 & 1 \end{array} \right] \{F_r\} \\
\{F_z\} &= \left[ \begin{array}{ccc} -\cos(\phi_j) & -\sin(\phi_j) & 0 \\
\sin(\phi_j) & \cos(\phi_j) & 0 \\
0 & 0 & 1 \end{array} \right] \{F_a\}
\end{align*}
\]

(7)

The matrix \([D]\) is the unique element that depends on the feed rate. If a linear model \((m_t=m_s=m_k=0)\) is considered, the matrix \([D]\) becomes just the identity matrix and the stability is independent of feed rate. At this point the stability of the process can be obtained in the same way described by Budak and Altintas. The originality of the analysis lies in the dependency of the time-dependent cutting coefficient matrix on the magnitude of the feed rate. Therefore the stability analysis will be explained roughly and more details can be obtained in the bibliography ([Altintas y Budak, 1995]; [Budak y Altintas, 1998]).

Both, milling forces and \([A(t)]\) matrix are periodic at tooth passing frequency \(T=2\pi/\Omega\). Thus \([A(t)]\) can be expanded into a discrete Fourier series,

\[
[A(t)] = \sum_{m=-\infty}^{\infty} [A_m] \cdot e^{im\Omega t} \quad \text{where} \quad [A_m] = \frac{2}{T} \int_0^T [A(t)] \cdot e^{-im\Omega t} \, dt
\]

(10)

The number of harmonics \((m)\) to be considered to an accurate solution depends on relationship between the immersion and the number of cutting edges. Two kind of
solutions can be obtained Single Frequency Solution and Multi Frequency Solution, ([Budak and Altintas, 1998]).

1.4. Single-Frequency Solution
The milling dynamic force is approximated to the average component of the Fourier series expansion \[ A_o \]. It can be demonstrated that the matrix \[ A_o \] is time-invariant, but depends on radial immersion, feed rate, and cutting coefficient exponent.

\[
[A_o] = \frac{1}{T} \int_0^T [A(t)] \cdot dt = \frac{N}{2\pi} [\alpha(\phi)]
\]

(11)

Thus, the interesting point is that now the average directional cutting coefficient matrix \[ [\alpha] \] is feed rate-dependent. The single frequency chatter model supposes that the regenerative vibrations are dominated by a single chatter frequency \( \omega_c \). Therefore \( \{\Delta r\} \) can be represented as:

\[
\{\Delta r(t)\} = \{r(t)\} - \{r(t-T)\} = (1 - e^{-i\omega_c T}) \cdot \{r(t)\}
\]

(12)

The vibration vector \( \{\Delta(t)\} \) and the cutting forces \( \{F(t)\} \) are related by the frequency response functions matrix \( [\Phi(i\omega_c)] \), which represents the dynamics of the system. By using this matrix, the characteristic equation of the closed loop dynamic cutting forces is reduced to

\[
\{F(t)\} = \frac{N}{2\pi} \cdot b \cdot K_T \cdot \left[1 - e^{-i\omega_c T}\right] \cdot [\alpha(\phi)] \cdot [\Phi(i\omega_c)] \cdot \{F(t)\}
\]

(13)

The stability analysis drives to an eigenvalue problem where the eigenvalues \( \Lambda = \Lambda_R + \Lambda_I i \) can be related with the maximum stable depth of cut and different cutting speeds.

\[
\det \{ [I] + \Lambda [\alpha] [\Phi] \} = 0
\]

(14)

\[
b_{lim} = -\frac{2\pi\Lambda_R}{N \cdot K_t} \left(1 + \left(\frac{\Lambda_I}{\Lambda_R}\right)\right) \quad \Rightarrow \quad a_{lim} = -\frac{2\pi\Lambda_R}{N \cdot K_t} \left(1 + \left(\frac{\Lambda_I}{\Lambda_R}\right)\right) \cdot \sin \kappa
\]

(15)

\[
n = \frac{60}{N \cdot T} = \frac{60 \cdot \omega_c}{N \cdot \pi - 2 \cdot \arctan \left(\frac{\Lambda_I}{\Lambda_R}\right) + 2\pi \cdot k} \quad k = 0, 1, 2, ...
\]

(16)

Different values for the integer value \( k \) yield different spindle speed. Therefore taking into account the relationship between the limiting depth of cut and spindle speeds, the procedure for computing the stability lobes diagram can be fulfilled.
3. OBTEENTION OF DIFFERENT CUTTING COEFFICIENTS

A set of 30 cutting tests has been carried out to obtain different cutting test. The cutting test have been performed using 5 different feed rates, 3 different cutting speeds and a constant depth of cut (a) of 1.5mm. Slotting conditions have been chosen using SANDVIK R245-080Q27-12M mill with SANDVIK R245-12 T3 M-PM-4030 inserts cutting F1140 (C45). This tool has a 45º lead angle.

<table>
<thead>
<tr>
<th>Feed rate (mm/rev)</th>
<th>Average Cutting Force</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fx (N)</td>
</tr>
<tr>
<td>0,05</td>
<td>-369,6</td>
</tr>
<tr>
<td>0,1</td>
<td>-507,1</td>
</tr>
<tr>
<td>0,15</td>
<td>-603,0</td>
</tr>
<tr>
<td>0,2</td>
<td>-684,5</td>
</tr>
<tr>
<td>0,3</td>
<td>-816,5</td>
</tr>
</tbody>
</table>

*Figure 2: Cutting coefficients*

In experimental tests the cutting forces have been measured using a dynamometric table. Average cutting forces have been obtained, and their values have been compared with the theoretical expressions of the average cutting force obtained using different force models (see equations 1-3) ([Budak, 1994], [Altintas, 2000]).

4. STABILITY LOBES.

The development done so far has been applied to a theoretical example of a system with a single degree of freedom. The results were obtained under three different theoretical approaches for the cutting force model: Linear model (classical), Exponential average model (Budak & Altintas) and Exponential instantaneous. The results were compared with those obtained by a time domain model with exponential cutting force approach, which should be the reference model.

The characteristics of the system were: \( m = 376 \text{ kg} \), \( c = 3779.95 \text{ Ns/m} \), \( k = 23.75 \text{ N/μm} \). The corresponding relative damping is 2% and the natural frequency of the system is 40 Hz. Mode direction is Y, perpendicular to feed direction. These dynamic parameters are usually related to machine tool structural modes and can be considered as typical. The tool assumed was 8 fluted with SANDVIK R245-12 T3 M-PM-4030 inserts, with 125 mm diameter. Radial immersion is 100 mm, and feed direction corresponds to down milling. Lead angle is 45º.
Figure 3 shows the lobes diagrams obtained by the three methods. The results of exponential force models have been obtained for two different feed rates: 0.05 mm/tooth and 0.3 mm/tooth.

From the figure, it can be concluded that all the stability charts define the same optimum spindle speeds. However, some differences can be noted in the different stability lobes. The stability obtained using the exponential model depends on the feed and in these cases simulations with low feed rates differ substantially from the predictions fixed with the linear model. The two exponential models establish very similar boundaries in this case. Anyway, a deeper study must be carried out taking into account different radial immersions and dynamic properties.

5. TIME DOMAIN SIMULATIONS.

The predictions of the instantaneous exponential force models (see equation 4) obtained in frequency domain have been compared with time domain simulation performed using the exponential model. The spectrums of the time domain simulations have been synthesised in the next waterfall plots.
The results confirm that the explained methods provide an accurate prediction of the stability of the exponential force model. For instance, if a spindle speed of 250r/min is selected, the time domain simulations predict chatter vibrations around 50Hz. These vibrations grow from a depth of cut 2.5mm with a feed rate of 0.05 mm/flute and around 4mm when a feed rate of 0.3mm/flute is considered. In the case of 400 r/min, chatter vibrations start at 0.3mm and 0.5 when a pair of feed rates of 0.05mm and 0.3mm is selected, respectively. The proposed frequency domain method predicts the same depth of cut in all cases.

6. CONCLUSIONS

A new method to consider the influence of the feed rate on the stability of face milling is proposed. This method is based in an exponential force model where the variation of the cutting coefficients depends on the chip thickness. The chip thickness is linearized using a Taylor expansion of the dynamic chip thickness and in this way the directional
factors become feed rate dependent. Finally feed-rate dependent stability lobes can be obtained considering an extended version of Budak and Altintas single frequency or multi frequency solution. The predicted lobes have been compared with time domain solutions and good correlation has been achieved in the studied cases. In a further work the predictions of the different methods must be compared in different dynamic conditions and experimental validation will be carried out.

7. REFERENCES


