On-line robot dynamic identification based on power model, modulating functions and causal Jacobi estimator

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On-line Robot Dynamic Identification Based on Power Model, Modulating Functions and Causal Jacobi Estimator

Qi GUO[1], Wilfrid PERRUQUETTI[2] and Maxime GAUTIER[3]

***Abstract*— This paper estimates robot dynamic parameters by means of power model associated with modulating functions, which avoids measuring or calculating the joint acceleration. At the same time, an advanced causal Jacobi derivative estimator is applied in order to get on-line robust derivatives from noisy measurements. In the end simulation results on two degrees of freedom planar robot are presented and comparisons with traditional off-line identification method are drawn.**

I. INTRODUCTION

Accurate dynamic models of robots are required in most advanced control schemes formulated in recent literature [1]. The precision, performance, stability and robustness of these schemes depend on, to a large extent, the accuracy of the dynamic parameters. Such parameters include inertia, first moments, masses, friction parameters. In most cases they are estimated off-line, but sometimes it is also necessary to carry out on-line identification, for example when the payload is changing during operation. Thus it is important to estimate robot dynamic parameters on line.

In order to tackle such challenge there exists a huge variety of methods mainly based on least-square techniques which can be divided into three classes according to the required order of derivation of the joint positions:

- Models based on joint force/torque, acceleration, velocity and position [2], [3], [4];
- Models based on joint force/torque, velocity and position [5], [6], [7];
- Models requiring only joint force/torque which is based on a closed loop simulation [8], [9].

The mostly used approaches are the two first methods which require reconstructing some derivatives of the measured signals. Unfortunately the obtained measurements are noisy which makes the derivative estimation problem to be ill-posed in the sense that a small error in measurement can induce a large error in the computed derivatives, specially for high order derivatives. Therefore, various numerical methods have been developed to obtain stable algorithms robust to additive noise. They mainly fall into eight categories:

- finite difference methods [10], [11],
- Savitzky Golay methods [12], [13],
- wavelet differentiation methods [14], [15],
- Fourier transform methods [16], [17],
- mollification methods [18], [19],
- Tikhonov regularization methods [20], [21],
- algebraic methods [22], [23], [24], [25],
- differentiation by integration [26], [27], [28].

This paper makes use of an advanced algebraic derivative estimator, which can offer robust derivatives estimation from noisy measurements by tuning its operator parameters. And its causal property make the on-line identification possible.

For robot on-line dynamic parameter estimation this paper will consider an energy point of view associated with modulating functions. As specified in the literature [5], [6], [7], energy model requires only joint velocity and joint position measurement and gives a scalar equation which is linear with respect to dynamic parameters. The drawback of such model comes from the needs of additional data, which is covered up by the varying modulating functions.

This paper is organized as follows: section 2 introduces the problem formulation, which specifies the energy model, then presents an introducing example of one joint robot and the general case; section 3 gives the precise description of robust algebraic derivative estimator; section 4 presents the obtained results about robot dynamic parameters identification using an energy model and a group of modulating functions; in section 5 simulation is carried out with a two degrees of freedom planar robot model, the simulation result shows that the on-line dynamic parameters estimation has a good precision on inertial parameters; and in last section it comes to a conclusion.

II. PROBLEM FORMULATION

Here robot dynamic parameters identification is based on the energy model in order to eliminate any derivation of velocity.

For the rest part consider that joint position $\mathbf{q}$ and joint torque $\mathbf{\Gamma}$ are measured via sensors. Suppose a rigid robot which is composed of n links, and the power of the system can be described as:

$$\frac{d}{dt}\left(H(\mathbf{q},\dot{\mathbf{q}})\right)+\dot{\mathbf{q}}^T\mathbf{\Gamma_f}=\dot{\mathbf{q}}^T\mathbf{\Gamma_m}, \tag{1}$$

where $H(\mathbf{q},\dot{\mathbf{q}})$ is the total energy of the system, which is the sum of kinetic energy $E(\mathbf{q},\dot{\mathbf{q}})$ and potential energy $U(\mathbf{q})$. $\mathbf{\Gamma_m}$ is the motor torque vector, and $\mathbf{\Gamma_f}$ is the friction torque which is usually modelized at non zero velocity as follows:

$$\mathbf{\Gamma_{fj}}=\mathbf{F_{cj}}\mathrm{sign}(\dot{\mathbf{q}}_\mathbf{j})+\mathbf{F_{vj}}\dot{\mathbf{q}}_\mathbf{j}+\tau_{\mathbf{off_j}}, \tag{2}$$

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$\dot{\mathbf{q}}_\mathbf{j}$ stands for velocity of joint j and $j \leq n$, sign() denotes each sign of element of vector, $\mathbf{F_{cj}}$, $\mathbf{F_{vj}}$ are the Coulomb and viscous friction coefficients of joint j, $\tau_{\mathbf{off_j}} = \tau_{\mathbf{offFc_j}} + \tau_{\mathbf{off}\tau_\mathbf{j}}$ is an offset parameter which regroups the amplifier offset $\tau_{\mathbf{off}\tau_\mathbf{j}}$ and the asymmetrical Coulomb friction coefficient $\tau_{\mathbf{offFc_j}}$.

Due to linearity with respect to parameters, (1) reads as :

$$\frac{d}{dt}\left(\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})\right)\mathbf{\Theta} + \dot{\mathbf{q}}^T \mathrm{sign}(\dot{\mathbf{q}})\mathbf{F_c} + \dot{\mathbf{q}}^\mathrm{T}\dot{\mathbf{q}}\mathbf{F_v} + \dot{\mathbf{q}}^\mathrm{T}\tau_\mathbf{off} = \dot{\mathbf{q}}^\mathrm{T}\mathbf{\Gamma_m}, \tag{3}$$

where $\mathbf{X} = [\mathbf{\Theta}\ \mathbf{F_c}\ \mathbf{F_v}\ \tau_\mathbf{off}] \in \mathbb{R}^p$ is the vector containing the dynamic parameters to be identified and $\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})$ is a vector function of $\mathbf{q}$, and $\dot{\mathbf{q}}$. In order to avoid in the estimation of $\mathbf{X}$, integration by part is combined with modulating functions.

Let $l \in \mathbb{N}^*$, $T \in \mathbb{R}^*_+$, and g be a function satisfying the following properties: $g \in \mathcal{C}^l([0,T])$, $g^{(i)}(0) = g^{(i)}(T) = 0,\ for\ i = 0,1,...,l-1$, where $\mathcal{C}^l([0,T])$ refers to the set of functions being $l-$times continuously differentiable on $[0,T]$ with $l \in \mathbb{N}^*$. Then g is called l^{th} **order modulating function** on $[0,T]$.

Modulating functions transform a differential expression into a sequence of algebraic equations using noisy data signals [29]. They have low pass filtering property. These features make the modulating functions method interesting in several real processes. In recent years many authors have focused on the choice of different modulating functions types such as Hermite functions [30], Fourier modulating functions [31], Hartley modulating functions [32] and spline-type functions [33].

In continuous time domain, for a given function of time $f(v)$, a modulating function $g(v)$ and a given time interval $K \subset \mathbb{R}$, we will use through out the paper the following notation

$$M_g^K(f) = \int_K g(v)f(v)dv. \tag{4}$$

The following part gives an introducing example and a general form method on how to estimate parameters using modulating functions.

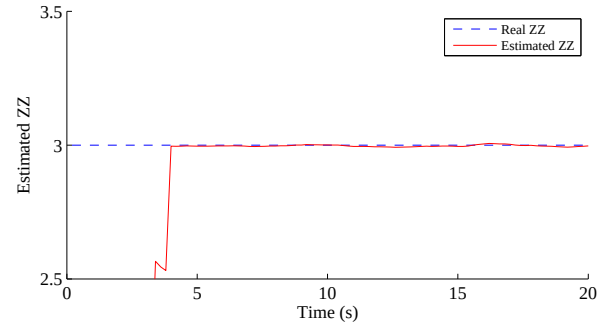
A. An introducing example

Consider a simple one revolute joint described by:

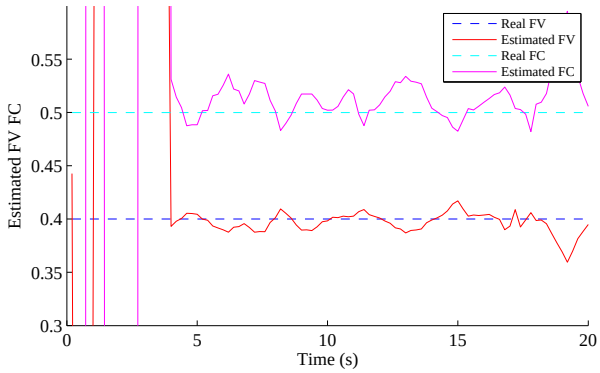
$$ZZ\ddot{q} + F_v\dot{q} + F_c\mathrm{sign}(\dot{\mathrm{q}}) = \tau, \tag{5}$$

where ZZ ($kg \cdot m^2$) is the inertial parameter, F_v (N/(m/s)) and F_c (N) are the viscous and Coulomb friction parameters respectively.

The purpose is to recover in real time the three dynamic parameters ZZ, F_v, F_c described in this model only by using the measured angular position q and the known applied torque. At time instant t, the estimation make use of the data from time interval $[t-T,t]$, where T is the time window length. Consider a combination of modulating function $g_\ell(v) = \frac{1}{T}e^{-j\ell\omega_0(v-t+T)}(e^{-j\omega_0(v-t+T)}-1)^2$ with resolving frequency $\omega_0 = \frac{2\pi}{T}$, and $g_\ell(v) = (v-t+T)^2(v-T)^2e^{\frac{(t-T)\ell}{0.8T}}$ since $g_\ell(t-T) = g_\ell(t) = \dot{g}_\ell(t-T) = \dot{g}_\ell(t) = 0$, multiplying



(a) Estimated ZZ



(b) Estimated F_v and F_c

Fig. 1. Estimation when q is of SNR=40dB, with $\ell = \frac{i}{50}$, $i = 1,2,...200$ and $T = 4s$

(5) by $g_\ell(v)$ and perform an integration by part on interval $[t-T,t]$, it gives

$$\begin{aligned} &ZZM_{\dot{g}_\ell}^{[t-T,t]}(q) - F_vM_{\dot{g}_\ell}^{[t-T,t]}(q) \\ &+ F_cM_{g_\ell}^{[t-T,t]}(\mathrm{sign}(\dot{\mathrm{q}})) = \mathrm{M}_{\mathrm{g}_\ell}^{[\mathrm{t-T,t}]}(\tau). \end{aligned} \tag{6}$$

Take $\mathbf{X}_p = [ZZ, F_v, F_c]^T$ as unknown and it requires at least 3 equations to solve them. Notice that this relation is a scalar equation. Thus it need additional data to form multi-equations, which can be realized by replacing ℓ by a sequence of N elements of ℓ_i where $\ell_i \in \mathbb{R}$. After setting a sequence of ℓ_i the list of equations are expressed as:

$$\mathbf{A}(t,q,\dot{q})[ZZ, F_v, F_c] = \mathbf{B}(t,\tau), \tag{7}$$

where $\mathbf{A}(t,q,\dot{q})$ is a $N \times 3$ observation matrix and the i-th line of $\mathbf{A}(t,q,\dot{q})$ is given by $\mathbf{A}(t,q,\dot{q}) = [M_{\dot{g}_\ell}^{[t-T,t]}(q), -M_{\dot{g}_\ell}^{[t-T,t]}(q),\ M_{g_\ell}^{[t-T,t]}(\mathrm{sign}(\dot{\mathrm{q}}))]$; and the i-th element of vector $\mathbf{B}(t,\tau)$ is given by $\mathbf{B}(t,\tau) = M_{g_\ell}^{[t-T,t]}(\tau)$.

This forms the general over-determined linear system $\mathbf{AX}_p = \mathbf{B}$, which can be resolved by least square approaches. In the simulation a white Gaussian noise is used with SNR=40dB. The joint velocity is computed numerically by an advanced causal Jacobi estimator which is presented in the next section. Take $\ell = \frac{i}{50}$, $i = 1,2,...200$ and sliding time window length $T = 4s$, a robust estimation result is shown in figure 1. In conclusion, for one joint robot case we utilize the measured joint position from a

certain time window to estimate the dynamic parameters. The estimation gives good result with a time window of 4 seconds and this makes it possible to do on-line identification. With a large sequence of ℓ, the estimator forms an over-determined observation matrix and can be solved by least square techniques. The drawback of this method lies on the fact that the estimation is out of precision before it acquires enough sampling data, as in figure 1 for the first 4 seconds the estimation is meaningless. After this period, the inertial parameter ZZ is well estimated and is robust with respect to noise, but the small parameters F_v and F_c are much disturbed by noise although they can be estimated around the real values.

B. General case

The idea of solving this kind of system is to increase the order of observed value by partial integration with modulating functions. For a general system

$$\sum_{i=0}^{N} \alpha_i f_i(\theta, \theta^{(1)}, ..., \theta^{(n)}) = \gamma, \tag{8}$$

where N is the number of terms, n is the largest order of derivative of θ, α_i are constant parameters, $\theta^{(i)}$ is i-th order of derivative of θ and f_i is a general function. Now suppose a family of modulating functions $g_\ell(v)$ satisfying

$$g_\ell^{(i)}(0) = g_\ell^{(i)}(t) = 0, \ i \leq n. \tag{9}$$

For example $g_\ell(v) = \frac{v^{(n+1)}(t-v)^{(n+1)}}{\ell^{(n+1)}}$. Then multiply $g_\ell(v)$ with the general system formulation and do integration on the interval $[0, t]$. When the function $f_i(\theta, \theta^{(1)}, ..., \theta^{(n)})$ is analytically integrable, one can perform integration by part according to partial integration theory using (9)

$$\begin{aligned} &\int_0^t \alpha_i g_\ell f_i(\theta, \theta^{(1)}, ..., \theta^{(n)}) dv \\ &= -\int_0^t \alpha_i g_\ell^{(1)} f_i^{[1]}(\theta, \theta^{(1)}, ..., \theta^{(n-1)}) dv, \end{aligned} \tag{10}$$

where $f_i^{[1]}(\theta, \theta^{(1)}, ..., \theta^{(n-1)})$ is the analytical form of integral function of $f_i(\theta, \theta^{(1)}, ..., \theta^{(n)})$.

In a similar way, if $f_i(\theta, \theta^{(1)}, ..., \theta^{(n)})$ is k-th order integrable, the highest order derivative of θ can be degraded to $(n-k)$-th order, which avoids to use noisy high order derivatives. With this method, estimation of the observed part can be numerically more precise and thus gives better estimation result. Finally in the k-th order integrable case, (10) can be written as a counterpart of

$$\begin{aligned} &\int_0^t \alpha_i g_\ell f_i(\theta, \theta^{(1)}, ..., \theta^{(n)}) dv = \\ &(-1)^k \int_0^t \alpha_i g_\ell^{(k)} f_i^{[k]}(\theta, \theta^{(1)}, ..., \theta^{(n-k)}) dv. \end{aligned} \tag{11}$$

In the rest of the paper, we will use the following family of Fourrier modulating functions: $g_{\ell,n}(t) = \frac{1}{T} e^{-j\ell\omega_0 t}(e^{-j\omega_0 t} - 1)^n$, where l is the pulse index, $\omega_0 = \frac{2\pi}{T}$ is the pulse resolution, n is the order of the system and T is the interval of time for the observation of system.

III. Numerical differentiation

This paper considers a frame of algebraic methods based on Jacobi polynomials. This approach extends the numerical differentiation by integration method proposed by Lanczos in [26] and it is originally introduced by Mboup, Fliess and Join in [22]. This method makes advantage of the truncation of the Taylor expansion and the mismodelling due to the truncation is compensated allowing a small time-delay in the derivative estimation. This Jacobi estimator relies on a group of non negative integer parameters κ, μ and has three version of forms: causal, anti-causal and central forms. By tuning these parameters the errors can be reduced to certain limits. Moreover recently Da-yan Liu, Gibaru and Perruquetti extend the parameters κ, μ used in the estimation from $\mathbb{N}$ to $]-1, +\infty[$, which provides more choices for tuning parameters, an analysis can be found in [24].

Consider a noisy observation $x_m = x + \omega_x$ of real valued smooth signal x on a finite time open interval $I \subset \mathbb{R}^+$, where ω_x is the noise component. The aim is to estimate the nth derivative of x_m. Assume that $x \in C^{n+1}(I)$, for any $t_0 \in I$, denote that $D_{t_0} = t \in \mathbb{R}^+; t_0 + \beta t \in I$, where $\beta = \pm 1$. If and only if noise ω_x is integrable, the Jacobi estimator can be applied. Forget the noise for a while, introduce the continuous time version of nth order derivative Jacobi causal or anti-causal estimators $D^{(n)}_{\kappa,\mu,\beta T,q} x(\beta T\xi + t_0)$:

$$D^{(n)}_{\kappa,\mu,\beta T,q} x(\beta T\xi + t_0) = \frac{1}{(\beta T)^n} \int_0^1 Q_{\kappa,\mu,n,q,\xi}(\tau) x(\beta T\tau + t_0) d\tau, \tag{12}$$

where T is the length of integration time interval, $\mu, \kappa \in]-1, +\infty[$ and $q \in \mathbb{N}$ are tuning parameters impacting the estimation error, ξ is a fixed value on [0,1] which is related to the delay and can be evaluated here as related to the estimation errors [34], and if $\beta = -1$ denotes causal Jacobi estimator and $\beta = 1$ denotes anti-causal Jacobi estimator.

$$Q_{\kappa,\mu,n,q,\xi}(\tau) = w_{\mu,\kappa}(\tau) \sum_{i=0}^{q} C_{\kappa,\mu,n,i} P_i^{(\mu+n,\kappa+n)}(\xi) P_{n+i}^{(\mu,\kappa)}(\tau),$$

where $Q_{\kappa,\mu,n,q,\xi}(\tau)$ is described by Jacobi polynomial $P_n^{(\mu,\kappa)}(\tau)$, weight function $w_{\mu,\kappa}(\tau)$ and $C_{\kappa,\mu,n,i}$ which are given below:

$$C_{\kappa,\mu,n,i} = \frac{(\mu+\kappa+2n+2i+1)\Gamma(\kappa+\mu+2n+i+1)\Gamma(n+i+1)}{\Gamma(\kappa+n+i+1)\Gamma(\mu+n+i+1)},$$

$$P_n^{(\mu,\kappa)}(\tau) = \sum_{j=0}^{n} \binom{n+\mu}{j} \binom{n+\kappa}{n-j} (\tau - 1)^{n-j} \tau^j,$$

$w_{\mu,\kappa}(\tau) = (1-\tau)^\mu \tau^\kappa$, and Γ is the gamma function.

The causal n^{th} Jacobi estimator has a time delay of $\tau = T\xi$ which indicates $D^{(n)}_{\kappa,\mu,T,q} x(T\xi + t_0) \approx x^{(n)}(t_0 - \tau)$. Similarly it has for anti-cause Jacobi estimator variant, $D^{(n)}_{\kappa,\mu,-T,q} x(-T\xi + t_0) \approx x^{(n)}(t_0 + \tau)$.

When taking noise into consideration, the Jacobi estimators $D^{(n)}_{\kappa,\mu,\beta T,q} x(\beta T\xi + t_0)$ can deal with a large class of noises for which the mean and covariance are polynomial in time, with degree smaller than the order of derivative to be estimated. The noise contribution is investigated in [34] and they can be bounded according to the type of noise.

In summary the Jacobi estimators are corrupted by three sources of errors [24]:

- the noise error contribution $e^\beta_\omega(t_0; n, \kappa, \mu, T, \xi, q)$,

- the bias term error e_a^β due to the truncation, which produces an amplitude error in estimation,
- the drift error e_d^β.

Table I shows the influence of causal Jacobi estimator parameters for each error contribution. By well tuning these parameters the estimation results can be optimised. The notations $a \uparrow, b \nearrow$ and $c \searrow$ mean that if increase the value for parameter a then the error b increases and the error c decreases. To verify the precision of causal Jacobi estimators,

Parameters	e_a^β	e_d^β	e_ω^β
$\kappa \uparrow$	$\nearrow$	$\nearrow$	$\nearrow$
$\mu \uparrow$	$\searrow$	$\searrow$	$\nearrow$
$q \uparrow$	$\searrow$	$\searrow$	$\nearrow$
$T \uparrow$	$\nearrow$	$\nearrow$	$\searrow$

TABLE I

INFLUENCE ON ERRORS OF CAUSAL JACOBI ESTIMATORS PARAMETERS

add perturbation of SNR=30 white Gaussian noise to a joint position trajectory then apply the causal Jacobi estimators. By well tuning the parameters a good estimation of first order derivative is obtained with a delay of 0.1 second and the error bound is less than 0.05. The following figure 2 shows the trajectories of reference joint velocity and estimated joint velocity.

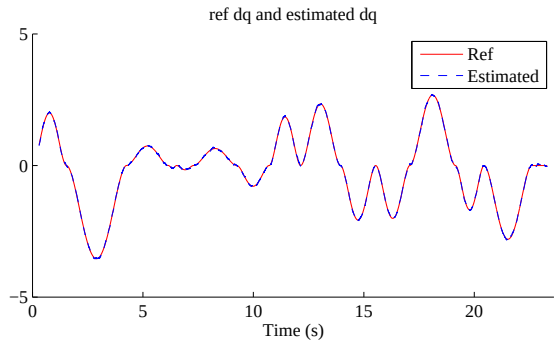


Fig. 2. Reference joint velocity and shifted estimated joint velocity by causal Jacobi estimators

IV. DYNAMIC PARAMETERS ESTIMATION

Consider a family of modulating functions $g_\ell(v)$ such that

$$g_\ell(v-t+T) = g_\ell(v-t) = 0, \tag{13}$$

where t is the instant time, and T is the sliding time window.

Applying the following operator $\int g_\ell(\tau)$ to robot energy model equation (3) and doing integration in time interval $[t-T, t]$ leads to the following equation:

$$\int_{t-T}^{t} g_\ell(\tau)\frac{d}{dt}\mathbf{h}^T(\mathbf{q},\dot{\mathbf{q}})d\tau\Theta + \int_{t-T}^{t} g_\ell(\tau)\dot{\mathbf{q}}^T\mathrm{sign}(\dot{\mathbf{q}})\mathrm{d}\tau\mathbf{F_c} + \int_{t-T}^{t} g_\ell(\tau)\dot{\mathbf{q}}^T\dot{\mathbf{q}}d\tau\mathbf{F_v} + \int_{t-T}^{t} g_\ell(\tau)\dot{\mathbf{q}}^T d\tau\tau_{\mathbf{off}} = \int_{t-T}^{t} g_\ell(\tau)\dot{\mathbf{q}}^T\mathbf{\Gamma_m}d\tau. \tag{14}$$

Use the following notations:

$$M_{g_{\ell-1}}^{[t-T,t]}(\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})) = \int_{t-T}^{t} g_{\ell-1}(\tau)\mathbf{h}^T(\mathbf{q},\dot{\mathbf{q}})d\tau \tag{15}$$

$$M_{g_\ell}^{[t-T,t]}(\mathbf{x},\mathbf{y}) = \int_{t-T}^{t} g_\ell(\tau)\mathbf{x}^T\mathbf{y}d\tau, \tag{16}$$

which transfer (14) into the following equation:

$$M_{g_\ell}^{[t-T,t]}(\dot{\mathbf{q}},\mathbf{\Gamma_m}) = M_{g_{\ell-1}}^{[t-T,t]}(\mathbf{h}(\mathbf{q},\dot{\mathbf{q}}))\mathbf{\Theta} + M_{g_{\ell-1}}^{[t-T,t]}(\dot{\mathbf{q}})\tau_{\mathbf{off}} + M_{g_\ell}^{[t-T,t]}(\dot{\mathbf{q}},\mathrm{sign}(\dot{\mathbf{q}}))\mathbf{F_c} + \mathrm{M}_{\mathrm{g}_\ell}^{[\mathrm{t-T,t}]}(\dot{\mathbf{q}},\dot{\mathbf{q}}))\mathbf{F_v}. \tag{17}$$

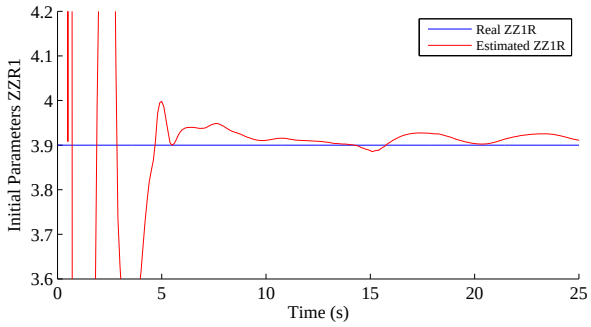
This gives one scalar equation which varies with respect to ℓ and depends on $\mathbf{q}$, $\dot{\mathbf{q}}$. In order to identify the dynamic parameters $\mathbf{X} = [\mathbf{\Theta}\ \mathbf{F_c}\ \mathbf{F_v}\ \tau_{\mathbf{off}}]$, joint velocity $\dot{\mathbf{q}}$ must be estimated from measured joint position data $\mathbf{q}$, which can be realized by using the derivatives estimators presented in section 3; meanwhile select a sequence of $\ell \in \mathbb{R}$ and these scalar equations can give an over-determined system which is linear with respect to unknown parameters, in the form of $\mathbf{AX} = \mathbf{B}$, where $\mathbf{A}_i = [M_{g_{\ell_i-1}}^{[t-T,t]}(\mathbf{h}(\mathbf{q},\dot{\mathbf{q}})), M_{g_{\ell_i}}^{[t-T,t]}(\dot{\mathbf{q}},\mathrm{sign}(\dot{\mathbf{q}})), \mathrm{M}_{\mathrm{g}_{\ell_i}}^{[\mathrm{t-T,t}]}(\dot{\mathbf{q}},\dot{\mathbf{q}})), M_{g_{\ell_i-1}}^{[t-T,t]}(\dot{\mathbf{q}})]$, $\mathbf{B}_i = [M_{g_{\ell_i}}^{[t-T,t]}(\dot{\mathbf{q}},\mathbf{\Gamma_m})]$ are known matrix.

This kind of problem can be solved by minimizing the Euclidian length of the residual vector $\min_{\mathbf{X}}||\mathbf{AX}-\mathbf{B}||$, which gives a unique optimal $\hat{\mathbf{X}}$ as solution. In order to decrease the sensitivity of the least square solution to errors in $\mathbf{A}$ and $\mathbf{B}$, the condition number of the observation matrix $\mathbf{A}$, $Cond(\mathbf{A})$, must be close to one before computing $\hat{\mathbf{X}}$. This can be done by running exciting trajectories which offer a good select of noisy samples q, $\dot{q}$. Exciting trajectories can be obtained by non linear optimization of a criterion function of the condition number of observation matrix [35].

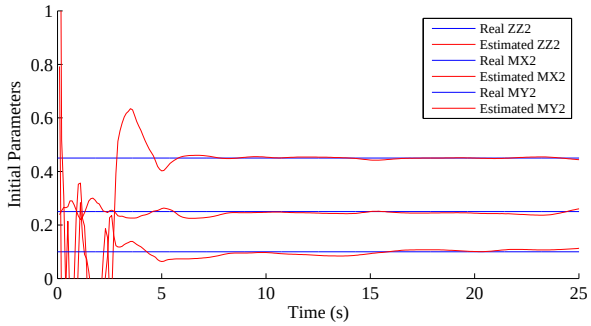
V. SIMULATION RESULTS

The simulation part utilizes a two revolute joints planar robot model which moves in a horizontal plane and has no gravity effect. According to [3], [36], the energy model depends on eight minimal dynamic parameters, considering four friction parameters, where $\tau_{\mathbf{off}}$ is not considered: $\mathbf{X} = [ZZ_{1R}\ ZZ_2\ MX_2\ MY_2\ F_{V1}\ F_{C1}\ F_{V2}\ F_{C2}]$, with the regrouped parameter $ZZ_{1R} = ZZ_1 + M_2L^2$, where L is the length of first link, ZZ_1 and ZZ_2 are drive side moment of inertial of link 1 and 2 respectively, MX_2, MY_2 are first moment of link 2, F_{Vj}, F_{Cj}, are the viscous and Coulomb friction coefficients of joint j. The simulation tests are running with value $\mathbf{X}$ which is all in SI Units: $\mathbf{X} = [3.9\ 0.25\ 0.45\ 0.1\ 0.3\ 0.4\ 0.15\ 0.25]$.

To carry out the estimation, suppose a modulating function $g_\ell(v) = \frac{1}{T}e^{-j\ell\omega_0(v-t+T)}(e^{-j\omega_0(v-t+T)} - 1)$ with resolving frequency $\omega_0 = \frac{2\pi}{T}$, which is null at both ends of sliding time interval $[t-T, t]$. Choose a sequence of $\ell = \frac{i}{50}$, $i = 0, 1, ..., 250$. Here we use QR factorization method to solve the least square problem. In the following part simulation runs a random trajectory in order to test the generality of this method.

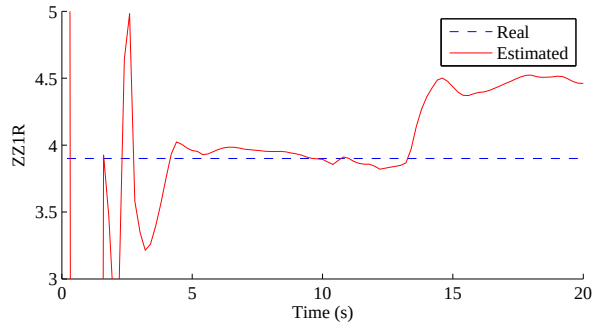


(a) Estimated $ZZ1R$

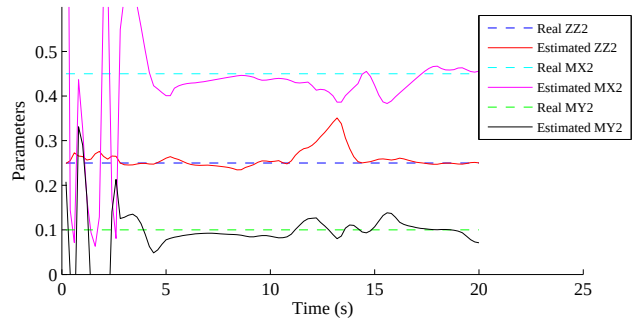


(b) Estimated $ZZ2$, $MX2$, $MY2$

Fig. 3. Estimation with $\ell = 0 : 0.05 : 10$ and time window 12s



(a) Estimated $ZZ1R$



(b) Estimated $ZZ2$, $MX2$, $MY2$

Fig. 4. Estimation with time window $7s$ and ZZ_{1R} changes from 3.9 to 4.5 at instant $t = 9.4s$

A. On-line estimation using modulating functions

In noise free case the estimated values are identical with reference inertial parameters values when sliding time window length reaches 2 second. When considering noise component identification becomes difficult as the dynamic parameters are sensitive to noise. Add white Gaussian noise of SNR=30 to the joint position and torques trajectories. The joint velocities are computed numerically using the causal Jacobi estimator. From experiments it can be found that sliding time window should be much larger in order to decrease the influence of derivative error in joint velocity.

An estimation result is given in figure 3 with sampling time window length $T = 12s$. As time moves on, the inertial parameters update according to the sampling data from the previous 12 seconds. In the beginning the estimation is pointless because the observation matrix is rank deficient and ill-conditioned. When sampling data are enough the inertial parameter estimation result is robust. But the estimated friction parameters are disturbed and can only be estimated approximately. This is because the friction parameters are not dominant parameters and make relatively weak contribution in the model, the noise component will have great influence on these weak parameters.

B. Non stationary inertial parameter

This part simulates the abrupt change of inertial parameter. The initial dynamic parameters are set the same as those in previous section. At instant $t = 9.4s$, ZZ_{1R} changes from 3.9 to 4.5. Apply the on-line identification using modulating functions. The results are shown in figure 4. Notice that from $t = 9.4s$ to $t = 15s$ there exists a delay about 5 seconds before getting the correction estimation. This delay is necessary because it needs enough data to re-estimate the changed parameters. During this transition period all the estimated inertial parameters are varying smoothly to the correct value. But the delay is too big for control and more study should be done to reduce the delay.

C. Comparison with off-line identification using explicit dynamic model

The traditional off-line approach uses robot explicit dynamic model and requires extra joint acceleration data which are difficult to measure and are usually obtained by numerical differentiation. Each sampling point can give an equation. After collecting all sampling points of the trajectory, an optimal solution is given by least square method. This method also need the observation matrix has rank efficiency and small condition number.

To compare, both identification methods run the same trajectory with noise component of SNR=30 white Gaussian noise and use causal Jacobi estimator to get derivatives. The inertial parameters to be estimated are $ZZ1R = 3.9$, $ZZ2 = 0.25$, $MX2 = 0.45$, and $MY2 = 0.1$. Simulation results are given in table II with sliding time window length $T = 20s$. Results show that under the same noise level, on-line identification method is competitive with off-line method and the estimation results have the same precision.

Parameters	Real Value	off-line	on-line
$ZZ1R$	3.9	3.8959	3.8855
$ZZ2$	0.25	0.2528	0.2499
$MX2$	0.45	0.4441	0.4524
$MY2$	0.1	0.0967	0.0961
$Fv1$	0.3	0.2840	0.2917
$Fc1$	0.4	0.4436	0.4382
$Fv2$	0.15	0.1484	0.1472
$Fc2$	0.25	0.2645	0.2623

TABLE II

COMPARISON BETWEEN OFF-LINE AND ON-LINE IDENTIFICATION METHOD WHEN TIME WINDOW LENGTH $T = 20s$

VI. CONCLUSION

This paper discusses within an energy point of view, the possibility that robot dynamic parameters can be estimated on-line associated with modulating functions. Joint acceleration is useless in this approach so that it can avoid the second order derivative computation of the joint position. Meanwhile the algebraic way of causal Jacobi estimator offers an on-line and robust estimation of the derivatives of joint position. Once the joint velocities are well estimated using a causality based way, this energy-based method can give a good estimation, in large sense, of the inertial parameters, while the friction parameters are difficult to estimate because they are sensitive to noise. This can be applied in real application to update on-line the inertial parameters during robot motion. In the end on-line identification method is compared to off-line one and simulation results show that both methods provide a good estimation of dynamic parameters.

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