

Maintaining a system subject to uncertain technological evolution

Khanh Nguyen, Bruno Castanier, Thomas G. Yeung

▶ **To cite this version:**

Khanh Nguyen, Bruno Castanier, Thomas G. Yeung. Maintaining a system subject to uncertain technological evolution. Reliability Engineering and System Safety, 2014, 128 (n128), pp.56-65. 10.1016/j.ress.2014.04.004 . hal-01061374

HAL Id: hal-01061374
https://hal.science/hal-01061374

Submitted on 3 Oct 2018

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Open Archive Toulouse Archive Ouverte

OATAO is an open access repository that collects the work of Toulouse researchers and makes it freely available over the web where possible

This is an author's version published in: http://oatao.univ-toulouse.fr/19982

Official URL: http://dx.doi.org/10.1016/j.ress.2014.04.004

To cite this version:

Nguyen, Thi Phuong Khanh and Castanier, Bruno and Yeung, Thomas *Maintaining a system subject to uncertain technological evolution.* (2014) Reliability Engineering and System Safety, 128. 56-65. ISSN 0951-8320

Any correspondence concerning this service should be sent to the repository administrator: tech-oatao@listes-diff.inp-toulouse.fr

Maintaining a system subject to uncertain technological evolution

T.P.K. Nguyen [a,*], Bruno Castanier [b], Thomas G. Yeung [b]

[a] The French Institute of Science and Technology for Transport, Development and Networks, 20 rue Élisée Reclus, F-59666 Villeneuve d'Ascq Cedex, France
[b] Ecole des Mines de Nantes, IRCCyN, 4, rue Alfred Kastler, B.P. 20722, F-44307 Nantes Cedex 3, France

Keywords:
Technology change
Maintenance/replacement investment
Markov decision processes
Dynamic programming
Forecast horizon

A B S T R A C T

Maintenance decisions can be directly affected by the introduction of a new asset on the market, especially when the new asset technology could increase the expected profit. However new technology has a high degree of uncertainty that must be considered such as, e.g., its appearance time on the market, the expected revenue and the purchase cost. In this way, maintenance optimization can be seen as an investment problem where the repair decision is an option for postponing a replacement decision in order to wait for a potential new asset. Technology investment decisions are usually based primarily on strategic parameters such as current probability and expected future benefits while maintenance decisions are based on "functional" parameters such as deterioration levels of the current system and associated maintenance costs. In this paper, we formulate a new combined mathematical optimization framework for taking into account both maintenance and replacement decisions when the new asset is subject to technological improvement. The decision problem is modelled as a non-stationary Markov decision process. Structural properties of the optimal policy and forecast horizon length are then derived in order to guarantee decision optimality and robustness over the infinite horizon. Finally, the performance of our model is highlighted through numerical examples.

1. Introduction

Industry has a large stake in maintenance optimization as it can reduce production costs, extend the useful life of industrial equipment and also alter the strategy for new investments in equipment. This interest can be observed through a steady increase in research in the open literature which has led to the construction of maintenance optimization models which are more advanced and better adapted to modelling of practical industrial concerns. As demonstrated by Mannan [12], there are four reasons for replacing equipment: (1) it has failed, (2) it is about to fail, (3) it has deteriorated and (4) an improved version has become available. One of the most common assumption in maintenance optimization is that maintenance is "perfect" and the system can be reduced to "as good as new". Imperfect maintenance is less commonly considered, but can be found in several models [21]. However, neither of these assumptions considers the optimization replacement according to the reason (4) and leads to a simplified decision based only on the current system characteristics in a given economic environment. In detail, based on the information of the system degradation modes and the associated observation data, we can evaluate the life cycle cost corresponding to the maintenance policies and then choose the optimal policy [4].

Taking into account potential improvements in technology increases the complexity of the maintenance decision problem in several aspects. First, the number of possible choices is increasing in the number of available technologies on the market. Second, technological evolution is a highly stochastic in nature in terms of the appearance time on the market, the purchase price and the expected profitability. Finally, in the context of maintenance of industrial systems, there is a significant difference in the frequency of maintenance decisions (operational level) and the frequency investment in new technology (strategic decision).

In the literature, maintenance decisions with technological change are generally approached in two ways.

The first approach, in close relation to the [12] definition of the replacement model, assumes that new technology is already available on the market. The overall performance of this new technology is known and the question is whether it is worth moving to this new technology given the price of such a change. In this context, the problem is to determine the conditions on the set of characteristics (purchase price, reliability improvements, etc.) which lead to move from one technology to another [3,6,7].

Borgonovo et al. [3] consider a geometric sequence model of technology evolution, represented by the exponential decrease of the failure rate over time. In this model, they take into account several types of maintenance such as minimal repair, imperfect maintenance and replacement. The impact of these actions is modelled directly on the system failure rate. While Borgonovo et al. [3] focus on a single component, Clavareau and Labeau [6,7] examine preventive and

* Corresponding author.
E-mail addresses: khanh.nguyen@ifsttar.fr, nguyenthipk85@gmail.com (T.P.K. Nguyen).

corrective replacement strategies of N identical components with the relevant logistic policy. Nguyen et al. [17] consider a technological innovation sequence with an uncertain appearance time but with deterministic revenue and an estimated purchase price of new technology.

The second approach takes into account the uncertainty in unknown future technology and is mainly found in the management science literature. In this context, the maintenance issue takes even greater importance and may be considered as an economic investment policy as it allows delaying replacement with existing technology in order to await new technology. The uncertainty of technology breakthrough time is captured by Nair [15]. He presents a model in which technological change is stochastic over time with a non-stationary appearing probability. The high level of uncertainty in the cost and the associated revenues of new technology is examined by a number of articles such as Mauer and Ott [13], Bethuyne [2], and Huisman and Kort [11]. Their approaches are mainly based on the modelling of the maintenance process through cost functions such as evolution of the operating cost [19]. These models allow managers to determine the best time for replacement investment of equipment under technological evolution but do not consider the maintenance strategies as well as the impact of technology change on them. Considering maintenance, or more specifically, imperfect maintenance actions, the system to be improved (albeit to a less than perfect state) at a lower cost compared to a complete renewal. Imperfect maintenance thus provides a useful alternative for waiting to invest in existing or future technology. This underscores the benefit of combining operational maintenance decisions with a strategic investment context.

The objective of this paper is to quantitatively analyse of the benefits of combining operational maintenance and strategic investment goals. We formulate an optimization model that also considers the market flexibility in terms of revenue volatility and uncertainty in the new technology's purchase price. Nguyen et al. [18] developed a finite horizon model, however, the chosen time horizon can alter substantially the optimal decision. In practice, it is essential to ensure the consistency of the investment decision regardless of the planning horizon. In the literature, very few articles consider the forecast horizon for maintenance and replacement decisions under technological change. Hopp and Nair [9] proposed a method for identifying the forecast horizon that is specially tailored to the equipment replacement problem with the single new technology assumption. Nair [15,16] extended the approach by considering a new technology sequence. However, this approach cannot be utilized for the models that take into account stochastic characteristics. In fact, they suppose that the revenue in a decision period is known a priori, decreasing over time and independent of the technical system performance. In this paper we propose a method to identify the forecast horizon for a stochastic model of technological change in the maintenance and replacement optimization problem.

The remainder of this paper is structured as follows: In Section 2, we present the classic stationary maintenance and investment problem without technology change. The mathematical formulation model for the maintenance and investment problem under technology evolution is then introduced. In Section 3, its structural properties are derived. The method for identifying the forecast horizon is developed in Section 4. The efficiency of the horizon identification method and the performance of the model are highlighted through numerical examples in Section 5. Finally, conclusions and future works are presented in Section 6.

2. Construction of the mathematical model for optimizing the maintenance decision

We first present the maintenance optimization model under assumption that only one major technology change will occur during the planning horizon. Thus we consider only a breakthrough technology and its associated investment. We assume a coherent market i.e., new technology is always more profitable than current technology. We model this improvement in terms of revenue rate per time period. In addition, we assume an increasing probability of appearance of new technology on the market within a finite time interval. Past this time interval, we assume that if the new technology is not yet on the market, then it will never appear. This assumption may be seen as restrictive, however we justify it by considering the technology breakthrough time interval as an interval of interest after which the impact of the decision becomes negligible. In Section 4 of this paper, we examine the optimal length of such an interval to ensure the robustness of decisions.

We model the optimization problem as a Markov decision process on an infinite time horizon in two steps. First, a model where technological evolution is not considered will be constructed and analysed. This assumption represents both the states of (1) "the new technology will never appear" and (2) "the transition to the new technology has already been made". Second, the global optimization model with potential future technological change will be considered.

2.1. Maintenance/investment problem without technology change

2.1.1. Problem statement

We consider a continuously operating system which generates a continuous revenue stream. We assume that the revenue stream is a function of both the asset deterioration level and the random market in which it performs. Due to the increase in the deterioration over time, the revenue is a non-increasing process in average. At each decision epoch, maintenance or replacement may be performed, if necessary to improve the state of the system. Thus, we define the optimal policy as a function of the current revenue stream generated by the system.

2.1.2. Model formulation

We model the revenue process as geometric Brownian motion (GBM) with drift $\mu < 0$ (characterizing the decreasing technical performance of the machine due to deterioration) and the volatility per unit time σ (characterizing market uncertainty).

Brownian motion is recognized as a very effective method to model revenue/cost flow in the management science literature, especially when considering the problem of investment in new technology [2,11,13]. For example, in Huisman and Kort [11], the authors use GBM to model the profit flow of the firms in a competition game. As the article focuses on the problem of technology adoption from a strictly economic point of view, they consider only the profit flow that is the result of the revenue generated by selling the product/service and the operation/maintenance cost. In this paper, we examine the revenue flow and the maintenance cost individually and also consider the problem in a reliability context.

Let g_n be the revenue rate generated during the decision period n. We assume that the technical performance is decreasing in the deterioration state of the asset. In average, the revenue is decreasing from g_0, the initial revenue rate generated by a new asset, to 0 over time. Let τ be the length of the decision interval, μ_j the drift, σ the volatility and $W_\tau \sim N(0,\tau)$. Given g_n, the revenue rate at the beginning of next period $n+1$ is then

$$g_{n+1} = g_n \exp\left[\left(\mu - \frac{\sigma^2}{2}\right)\tau + \sigma W_\tau\right] \tag{1}$$

The expected cumulative revenue Z_n^0 within a decision period n, based on revenue rate g_n at the beginning of the period, with discount factor per unit time r, is given by the conditional

expectation on g_n. We can deduce

$$E[Z_n|g_n=g]=mg \quad \text{with } m=\frac{1}{\mu-r}(\exp[(\mu-r)\tau]-1) \tag{2}$$

Proof. Given $w=(\sigma^2/4)u$, we have

$$E[Z_t|g_t=g]=\frac{4g}{\sigma^2}E\left[g\int_0^{\tau\sigma^2/4}\exp\left[2\left(\frac{2(\mu-r)}{\sigma^2}-1\right)w+2W_w\right]dw\right]$$
$$=\frac{4g}{\sigma^2}E\left[\int_0^h \exp[2\nu w+2W_w]\,dw\right]$$

with

$$h=\frac{\sigma^2\tau}{4},\quad \nu=\frac{2(\mu-r)}{\sigma^2}-1$$

From Corollary 2 (Section 2.4.2, p. 33) by Yor [23], with $\lambda=2$ and $n=1$, we have

$$E\left[\int_0^h \exp[2\nu w+2W_w]\,dw\right]=\frac{1}{4}[c_0^{\nu/2}+c_1^{\nu/2}\exp[(2+2\nu)h]]$$

$c_j^{\nu/2}$ is defined at the beginning of p. 32 and the function $\varphi(x)$ is given in the equation 4.a, p. 31 [23]:

$$c_0^{\nu/2}=\left[\frac{1}{2}\ \frac{\nu^2}{4}-\frac{(\nu+1)^2}{4}\right)\right]^{-1}=\frac{-2}{\nu+1}$$
$$c_1^{\nu/2}=\left[\frac{1}{2}\ \frac{(\nu+1)^2}{4}-\frac{\nu^2}{4}\right)\right]^{-1}=\frac{2}{\nu+1}$$

Hence

$$E\left[\int_0^h \exp[2\nu w+2W_w]\,dw\right]=\frac{1}{2(\nu+1)}(-1+\exp[2(\nu+1)h])$$

By replacing $h=\sigma^2\tau/4$ and $\nu=2(\mu-r)/\sigma^2-1$, we obtain (2). □

To compute the one-period transition probabilities $P(g'|g)$, we discretize the revenue rate state as follows. g_n is the first value of N_g discrete intervals of length l on $[g_{min},g_{max}]$ with $g_{min}=0$ and $g_{max}=g_0$. More specifically, if the revenue rate at the beginning of current decision period belongs to the intervals $([0,l[,\ldots,[(N_g-1)l,g_0[,[g_0,\infty[)$, we approximate it by $g_n\in\Theta:\{0,\ldots(N_g-1)l,g_0\}$. Note that $P(g'=0|g=0)=1$ and $P(g'\neq 0|\ g=0)=0$.

Let $m_1=(\mu-\sigma^2/2)\tau$, $m_2=\sigma\sqrt{\tau}$. $\forall g\in\Theta\backslash\{0\}$, $\forall g'\in\Theta\backslash\{0,g_0\}$, the transition probability is

$$P(g'|g)=\Phi\left(\frac{\ln\left(\frac{g'+l}{g}\right)-m_1}{m_2}\right)-\Phi\left(\frac{\ln\left(\frac{g'}{g}\right)-m_1}{m_2}\right)$$

$$\forall g\in\Theta\backslash\{0\},\quad g'=g_{min}:P(g'|g)=\Phi\left(\frac{\ln\left(\frac{g'+l}{g}\right)-m_1}{m_2}\right)$$

$$\forall g\in\Theta\backslash\{0\},\quad g'=g_{max}:P(g'|g)=1-\Phi\left(\frac{\ln\left(\frac{g'}{g}\right)-m_1}{m_2}\right)$$

At the beginning of any decision epoch n, the possible actions are:

- Do nothing ($\tilde{DN}$), the asset continues to operate until the next decision epoch and generates revenue according to the described process. Given that the value observed at the beginning of epoch is g_n, the expected economic reward in this period is equal to $E[Z_n|g_n]$.
- Maintenance ($\tilde{M}$) restores the asset to a better given state, g^M with $g^M=\varrho g_0$ and $\varrho<1$. ϱ is called hereafter the maintenance efficiency factor. The assumption of a constant deterioration state after maintenance based repair consisting of replacing the critical component. The maintenance cost, $c_M(g)$, is assumed to be an increasing linear function of maintenance effect, as in Yeh et al. [22]:

$$c_M(g)=\nu+h_1(g^M-g) \tag{3}$$

where ν, h_1 are constant. The decision of maintaining can be chosen if and only if $g<g^M$.
- Investment ($\tilde{I}$) replaces the asset by a new one. Note that in this case where new technology is not considered, investment is exactly equivalent to a complete asset replacement. The cost of such a replacement is given by the difference between the purchase price of the new asset c_0 and the salvage value $b_0(g)$ of the current asset. After a replacement, the system generates a revenue rate g_0.

$$b_0(g)=v+h_2g \tag{4}$$

where v is defined as the junk value and v and h_2 are constant.

We assume that the action time is negligible. Let $\tilde{V}(g)$ denote the maximum expected discounted value over infinite horizon and the complete model formulation is given by

$$\tilde{V}(g)=\max\left\{\begin{array}{l}\tilde{DN}(g)=mg+e^{-r\tau}\sum_{\forall g'}P(g'|g)\tilde{V}(g')\\ \tilde{M}(g)1_{\{g<g^M\}}=-c_M(g)+\tilde{DN}(g^M)\\ \tilde{I}(g)=-c_0+b_0(g)+\tilde{V}(g_0^0)\end{array}\right\} \tag{5}$$

2.1.3. Structural properties and the optimal policy

The objective of this subsection is to give the conditions for the existence of the monotone optimal policy. To simplify notations, we assign numerical values to the three possible actions: 1→ Do nothing, 2→ Maintenance, 3→ Investment.

Lemma 1. *Let $q(k|g,a)=\sum_{g'=k}^{g_{max}}P(g'|g,a)$, $q(k|g,a)$ is non-decreasing in g for all k, $\forall g\in\Theta$, $\forall a\in A:\{1,2,3\}$.*

Proof. $q(k|g,1)=\sum_{g'=k}^{g_{max}}P(g'|g)=1-\Phi((\ln(k/g)-m_1)/m_2)$, so $q(k|g,1)$ is an increasing function in g for all k.

For maintenance action, we have $q(k|g,2)=\sum_{g'=k}^{g_{max}}P(g'|g^M)$.
And for replacement action $q(k|g,3)=\sum_{g'=k}^{g_{max}}P(g'|g_0)$.
They are not dependent on g.
Hence, $q(k|g,a)$ is a non-decreasing function in g for all k. □

From Theorem 6.2.10 of Puterman [20], there exists only one optimal policy that is deterministic and stationary.

Theorem 1.

1. *$\tilde{V}(g)$ is non-decreasing in g.*
2. *$\forall a\in A:\{1,2,3\}$, the optimal policy $\pi_{\tilde{V}}(g)$ is non-increasing in g with the two following conditions:*
   - *$\pi_{\tilde{V}}(g^M)=1$.*
   - *$m\geq h_1\geq h_2$ with m be the factor between the current system state and the revenue accumulated in a decision period, h_1 be the factor between the system state and the maintenance cost, and h_2 be the factor between the system state and the residual value.*

Proof.

1. We proceed by induction on the steps of the value iteration algorithm. Let $\tilde{V}_n(g)$ be the maximum expected discounted

value over n decision periods, and $\tilde{V}(g)$ be its asymptotic value when n tends to infinity. Without loss of generality, let $\tilde{V}_0(g)=0,\ \forall g\in\Theta$

$$\tilde{V}_n(g)=\max\begin{Bmatrix} D\tilde{N}_n(g)=mg+e^{-r\tau}\sum_{\forall g'}P(g'|g)\tilde{V}_{n-1}(g') \\ \tilde{M}_n(g)1_{\{g<g^M\}}=-c_M(g)+D\tilde{N}_n(g^M) \\ \tilde{I}_n(g)=-c_0+b_0(g)+D\tilde{N}_n(g_0) \end{Bmatrix}$$

First, for $n=1$, we have:

- $D\tilde{N}_1(g)=m\cdot g$, is non-decreasing in g.
- $\tilde{M}_1(g)$ and $\tilde{I}_1(g)$ are non-decreasing in g as sums of non-decreasing functions.

We deduce $\tilde{V}_1(g)$ is non-decreasing in g.
We assume now that $\tilde{V}_{n-1}(g)$ is non-decreasing in g.
From Lemma 1 of this paper and Lemma 4.7.2 (p. 106) of Puterman [20], we have $\sum_{\forall g'}P(g'|g)\tilde{V}_{n-1}(g')$ is non-decreasing in g.
So $D\tilde{N}_n(g)$ is non-decreasing in g, $\forall n$.
And $\tilde{M}_n(g)$ and $\tilde{I}_n(g)$ are non-decreasing in g.
Hence, the assertion is proved for all n.
Finally, $\tilde{V}_n(g)$ is non-decreasing in g and while $n\to\infty$, $\tilde{V}_n(g)\to\tilde{V}(g)$.

2. From Lemma 1, $q(k|g,a)$ is non-decreasing in g.
Let $r(g,a)$ be the reward function received in a period when the system state is g and action a is selected, $a\in A$: action set.

$$\forall g<g^M,\ A=\{1,2,3\};\quad \forall g\geq g^M,\ A=\{1,3\}$$

$r(g,a)$ is non-decreasing in g as sums of non-decreasing functions. Then, with assumption that $m\geq h_1\geq h_2$, we can deduce directly that $r(g,a)$ is a sub-additive function as defined in Puterman [20].
Finally, $q(k|g,a)$ is also a sub-additive function by Lemma 1.
From Theorem 6.11.6 in Puterman [20], we deduce directly that the optimal policy $\pi_{\tilde{V}(g)}$ is non-decreasing in g if $\pi_{\tilde{V}(g^M)}=1$. □

From the non-increasing property of the optimal policy (Theorem 1), we can deduce the control limit structure. Let (y^M,y^I) be the respective maintenance and replacement thresholds. Cost assumptions and control limit policy existence state the following decision rules:

- replace as soon as the revenue rate becomes lower than y^I;
- maintain if the revenue rate belongs to (y^M,y^I);
- do nothing if the revenue rate remains greater than y^M.

2.2. The maintenance and investment problem under technological evolution

In case of technological evolution, the decision becomes much more complex with the anticipation and later opportunity to invest in new technology. As mentioned previously, maintenance can be a valuable alternative to replacement while waiting a better investment conditions. We introduce technology evolution directly into the model discussed in the previous section.

2.2.1. Technology evolution modelling

We assume that only one new technology can appear in the future with an increasing probability over time. This assumption is justified under "breakthrough" conditions and commonly used in the literature, see Hopp and Nair [9], Mauer and Ott [13] and Huisman and Kort [11]. In fact, Hopp and Nair [9] consider the appearance probability q_n of new technology at time n. Mauer and Ott [13] assume that technological change follows a Poisson process with constant rate and Huisman and Kort [11] assume that the probability distribution of appearance time of new technology follows the exponential law:

$$p_{n+1}=1-\delta\kappa^n;\quad \delta,\kappa\in(0,1) \tag{6}$$

The δ factor reflects the non-appearance probability of new technology at the next decision epoch. Factor κ characterizes the increasing rate of the appearance probability of new technology over time.

Technological innovation is characterized by a higher initial revenue rate, g_0^1 ($g_0^1>g_0^0$) and lower drift of revenue flow, μ_1 ($\mu_0<\mu_1<0$) .

The salvage value of the asset at decision epoch n depends on the cumulated or expected revenue at time n and on its technological generation ($j=0$ or 1).

We use geometric Brownian motion to model the estimated purchase price of new technology in order to take into account the uncertainty of the new technology's appearance time and the increase in the volatility of the forecast over time. Under the risk neutral measure, the estimated purchase price of the new technology that appears at time t is described as follows:

$$c_{1,t}=c_{1,0}\exp\left[\left(r-\frac{\sigma_c^2}{2}\right)u+\sigma_c W_t\right] \tag{7}$$

where $c_{1,0}$ is the given initial purchase price of new technology. To ensure both the profit of the manufacturer and the attractiveness of a new technology, we assume that the fluctuation range of the new technology purchase price belongs to $[c_{1\ min},c_{1\ max}]$ within $c_{1\ min}=c_0$. Geometric Brownian motion of $c_{1,t}$ is discretized similar to the revenue process. Moreover, as mentioned previously, the new technology is assumed to be more attractive than the old one (A1). This assumption that management is no longer interested in the old technology and will invest directly in new technology after its appearance on the market is also used in many references [9,14,6,7].

Assumption (A1). $-c_0+\tilde{V}(g_0^0)<-c_{1\ max}+\tilde{V}^1(c_{1\ max},g_0^1)$.

Note that $\tilde{V}^1(c_1,g)$ is the optimal value function over the infinite horizon for the new technology with initial price c_1. After investment in new technology, the problem becomes equivalent to the problem presented in Section 2.1, thus $\tilde{V}^1(c_1,g)$ has the same structural properties as $\tilde{V}(g)$ (Theorem 1). It also has the following property:

Lemma 2. *$\tilde{V}^1(c_1,g)$ is non-increasing in c_1, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$.*

Proof. We can prove Lemma 2 directly by induction. □

2.2.2. Model formulation under technological evolution

Let $V_n^N(g)$ denote the maximum expected discounted value from decision epoch n to the last epoch N given that new technology has not yet appeared. $\hat{V}(c_1,g)$ represents the maximum expected discounted value given that the new technology has appeared with the purchase price c_1. Finally, the optimization problem can be formulated as follows:

$$V_n^N(g)=\max\{DN_n^N(g),M_n^N(g)1_{\{g<g^M\}},I_n^N(g)\},\quad \forall n<N \tag{8}$$

with

$$DN_n^N(g)=mg+e^{-r\tau}[(1-p_{n+1})\sum_{\forall g'}P(g'|g)V_{n+1}^N(g'). \\ +p_{n+1}\sum_{\forall c_1}p_{n+1,c_1}\sum_{\forall g'}P(g'|g)\hat{V}(c_1,g')] \tag{9}$$

$$M_n^N(g)=-c_M(g)+DN_n^N(g^M)$$
$$I_n^N(g)=-c_0+b_0(g)+DN_n^N(g_0^0)$$

and

$$\hat{V}(c_1,g)=\max\left\{\begin{array}{l}\widehat{DN}(c_1,g)=mg+e^{-r\tau}\sum_{\forall g'}P(g'|g)\hat{V}(c_1,g')\\ \hat{M}(c_1,g)1_{\{g\le g^M\}}=-c_M(g)+\widehat{DN}(c_1,g^M)\\ \hat{I}(c_1,g)=-c_1+b_0(g)+\tilde{V}^{1}(c_1,g_0^1)\quad{}^{(1)}\end{array}\right\}\qquad(10)$$

where

- $P(g'|g)$: probability that the old technology system generates a revenue rate g' at the beginning of next period given that the revenue rate generated at the beginning of current period is g.
- p_{n+1}: appearance probability of the new technology during period $n+1$ if it is not yet available at n.
- $p_{n+1,c_1'}$: probability that the purchase price of new technology is c_1' during the next period, given at n, new technology has not yet appeared. It is calculated similarly as $P(g'|g)$ in Section 2.1.

(1) From Assumption (A1), we deduce the following corollary:

Corollary (C1). $-c_0+\hat{V}(g_0^0)<-c_{1\ max}+\tilde{V}^{1}(c_{1\ max},g_0^1)$. □

An investment is equivalent to replacement with the best available technology.

On the other hand, we define a time horizon of the interest for a new technology appearance for which, beyond this, we no longer consider the probability of occurrence. Hence, we consider $V_N^N(g)=\tilde{V}(g)$.

3. Structural properties of the maintenance and investment problem under technological evolution

In this section, structural properties of the optimal policy are examined and the associated necessary conditions are derived. We first present Lemma 3 that is the foundation for studying monotone properties of the optimal policies.

Lemma 3.

- $\hat{V}(c_1,g)$ *is non-decreasing in* g, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$.
- $\hat{V}(c_1,g)$ *is non-increasing in* c_1, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$.
- $V_n^N(g)$ *is non-decreasing in* g, $\forall g\in\Theta$.

Proof.

1. We can prove directly by induction (similar to Theorem 1).
2. We proceed by induction on the steps of the value iteration algorithm. We consider $\hat{V}_n(c_1,g)$, when $n\to\infty$, $\hat{V}_n(c_1,g)\to\hat{V}(c_1,g)$. Let

$$\hat{V}_n(c_1,g)=\left\{\begin{array}{l}\widehat{DN}_n(c_1,g)=mg+e^{-r\tau}\sum_{\forall g'}P(g'|g)\hat{V}_{n-1}(c_1,g')\\ \hat{M}_n(c_1,g)1_{\{g\le g^M\}}=-c_M(g)+\widehat{DN}_n(c_1,g^M)\\ \hat{I}_n(c_1,g)=-c_0+b_0(g)+\tilde{V}^{1}(c_1,g_0^1)\end{array}\right\}$$

   Without loss of generality, let $\hat{V}_0(c_1,g)=0$, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$. $\hat{V}_0(c_1,g)$ is non-increasing in c_1, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$.
   Now assume that $\hat{V}_n(c_1,g)$ is non-increasing in c_1, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$, we have:
   - $\widehat{DN}_{n+1}(c_1,g)$ is non-increasing in c_1 because $\sum_{\forall g'}P(g'|g)\hat{V}_n(c_1,g')$ is non-increasing in c_1.
   - $\hat{M}_{n+1}(c_1,g)$ is non-increasing in c_1 because of the $\widehat{DN}_{n+1}(c_1,g^M)$ monotonic property.
   - $\hat{I}_{n+1}(c_1,g)$ is non-increasing in c_1 by Lemma 2.

   Hence $\hat{V}_{n+1}(c_1,g)$ is non-increasing in c_1. This result holds in the limit.
3. We have $V_N^N(g)=\tilde{V}(g)$, so $V_N^N(g)$ is non-decreasing in g, $\forall g\in\Theta$, $\forall c_1\in[c_{1\ min},c_{1\ max}]$ (Theorem 1).
   Similar to the proof of Theorem 1, we can prove that $V_n^N(g)$ is a non-decreasing function in g by induction. □

Recall that g^M is the asset state after the maintenance action, we define the respective differences in action costs for the non-obsolescence and obsolescence cases.

$$\forall g<g^M,\quad g\in\Theta\left\{\begin{array}{l}d12_n^N(g)=M_n^N(g)-DN_n^N(g)\\ d23_n^N(g)=I_n^N(g)-M_n^N(g)\\ \Delta12(c_1,g)=\hat{M}(c_1,g)-\widehat{DN}(c_1,g)\\ \Delta23(c_1,g)=\hat{I}(c_1,g)-\hat{M}(c_1,g)\end{array}\right.$$

$$\forall g\in\Theta\left\{\begin{array}{l}d13_n^N(g)=I_n^N(g)-DN_n^N(g)\\ \Delta13(c_1,g)=\hat{I}(c_1,g)-\widehat{DN}(c_1,g)\end{array}\right.$$

Theorem 2. *If the proportional factors between the system state and the revenue accumulated in a decision period (m), or the maintenance cost (h_1), or the residual value (h_2) follow the non-increasing order ($m\ge h_1\ge h_2$), then:*

- $\forall n=1,2,3\ldots N$ *with the action set is* $A=\{1\to DN_n^N,2\to M_n^N,3\to I_n^N\}$, *the optimal policy* $\pi_{V_n^N}(g)$ *is non-increasing in* g *if* $\pi_{V_n^N}(g^M)=1$.
- *With the action set is* $A=\{1\to\widehat{DN},2\to\hat{M},3\to\hat{I}\}$, *the optimal policy* $\pi_{\hat{V}}(c_1,g)$ *is non-increasing in* g *if* $\pi_{\hat{V}}(c_1,g^M)=1$.

Proof.

- (a) At $n=N$, $V_N^N(g)=\tilde{V}(g)$ so $\pi_{V_N^N}(g)$ is non-increasing in g (Theorem 1).
  When $n<N$, $\forall g^-<g^+<g^M$:

$$\begin{aligned}&d12_n^N(g^+)-d12_n^N(g^-)=(h_1-m)(g^+-g^-)\\&+e^{-r\tau}\left[(1-p_{n+1})\left(\sum_{\forall g'}P(g'|g^-)V_{n+1}^N(g')-\sum_{\forall g'}P(g'|g^+)V_{n+1}^N(g')\right)\right.\\&\left.+p_{n+1}\sum_{\forall c_1}p_{n+1,c_1}\left(\sum_{\forall g'}P(g'|g^-)\hat{V}(c_1,g')-\sum_{\forall g'}P(g'|g^+)\hat{V}(c_1,g')\right)\right]\end{aligned}$$

  From Lemma 3 of this paper and Lemma 4.7.2 of [20], we have

$$\sum_{\forall g'}P(g'|g^-)V_{n+1}^N(g')\le\sum_{\forall g'}P(g'|g^+)V_{n+1}^N(g')$$

$$\sum_{\forall g'}P(g'|g^-)\hat{V}(c_1,g')\le\sum_{\forall g'}P(g'|g^+)\hat{V}(c_1,g')$$

  (b) It follows $\forall g<g^M,g\in\Theta: d12_n^N(g^+)\le d12_n^N(g^-)$ if $h_1\le m$.
  Similarly, we can deduce

$$\forall g<g^M,\quad g\in\Theta: d23_n^N(g^+)\le d23_n^N(g^-)\quad\text{if } h_2\le h_1.\qquad(c)$$

$$\forall g\in\Theta: d13_n^N(g^+)\le d13_n^N(g^-)\quad\text{if } h_2\le m.\qquad(d)$$

  Note that $\forall g\ge g^M$, $g\in\Theta: A=\{1,3\}$, since (d) the optimal policy $\pi_{V_n^N}(g)$ is non-increasing in g.
  (e) In addition, if $\pi_{V_n^N}(g^M)=1$, so $\pi_{V_n^N}(g)=1$, $\forall g\ge g^M$, $g\in\Theta$.
  (f) $\forall g<g^M$, $g\in\Theta$, from (b)+(c)+(d) we can conclude that the optimal policy $\pi_{V_n^N}(g)$ is non-increasing in g.
  Since (a)+(e)+(f), Theorem 2.1 is proved.
- Similar to the previous proof, by considering the action difference functions: $\Delta12(c_1,g)$, $\Delta13(c_1,g)$, $\Delta23(c_1,g)$, Theorem 2.2 is deduced easily. □

Similar to Theorem 1, the non-increasing property of the optimal policy presented in Theorem 2 allows us to deduce the control limit structure for maintenance and replacement activities when considering technological change.

Now, we present Lemma 4 that is a key result to consider the sensitivity of the replacement threshold after the new technology occurrence.

Lemma 4. $\forall g \in \Theta$, $\forall c_1^-, c_1^+ \in [c_{1\ min}, c_{1\ max}]$ *and* $c_1^- < c_1^+$: $\hat{V}(c_1^-, g) - \hat{V}(c_1^+, g) \le \tilde{V}^1(c_1^-, g_0^1) - \tilde{V}^1(c_1^+, g_0^1) + c_1^+ - c_1^-$.

Proof. We consider $\hat{V}_n(c_1, g)$. When $n \to \infty$: $\hat{V}_n(c_1, g) \to \hat{V}(c_1, g)$.

Let B be $\{\tilde{V}^1(c_1^-, g_0^1) - \tilde{V}^1(c_1^+, g_0^1) + c_1^+ - c_1^-\}$.

Without loss of generality, let $\hat{V}_0(c_1, g) = 0, \quad \forall g \in \Theta$, $\forall c_1 \in [c_{1\ min}, c_{1\ max}]$.

From Lemma 2.1, we have

$$\hat{V}_0(c_1^-, g) - \hat{V}_0(c_1^+, g) = 0 < B$$

If $\hat{V}_{n-1}(c_1^-, g) - \hat{V}_{n-1}(c_1^+ g) \le B$, $\forall g \in \Theta$, then

$$\widehat{DN}_n(c_1^-, g) - \widehat{DN}_n(c_1^+, g) = e^{-r\tau} \sum_{\forall g'} P(g'|g)(\hat{V}_{n-1}(c_1^-, g) - \hat{V}_{n-1}(c_1^+, g))$$

$$\le e^{-r\tau} \sum_{\forall g'} P(g'|g) B$$

Hence, we have

$$\widehat{DN}_n(c_1^-, g) - \widehat{DN}_n(c_1^+, g) \le B$$
$$\hat{M}_n(c_1^-, g) - \hat{M}_n(c_1^+, g) \le B$$
$$\hat{I}_n(c_1^-, g) - \hat{I}_n(c_1^+, g) = B$$

Now we demonstrate $\hat{V}_n(c_1^-, g) - \hat{V}_n(c_1^+, g) \le B$.

In fact, if $\hat{V}_n(c_1^-, g) = \widehat{DN}_n(c_1^-, g)$ then

$$\hat{V}_n(c_1^-, g) - \hat{V}_n(c_1^+, g) \le \widehat{DN}_n(c_1^-, g) - \widehat{DN}_n(c_1^+, g) \le B$$

Similarly, if $\hat{V}_n(c_1^-, g) = \hat{M}_n(c_1^-, g)$ or $\hat{V}_n(c_1^-, g) = \hat{I}_n(c_1^-, g)$

Hence, $\hat{V}_n(c_1^-, g) - \hat{V}_n(c_1^+, g) \le B$, $\forall g \in \Theta$.

When $n \to \infty$, $\hat{V}_n(c_1, g) \to \hat{V}(c_1, g)$, Lemma 4 is proved. □

Theorem 3. *After the new technology appears, the replacement threshold is non-increasing in* c_1.

Proof. We define the replacement threshold $y^I_{c_1}$ such as

$$y^I_{c_1} = \max\{g : \Delta 23(c_1, g) \ge 0 \text{ and } \Delta 13(c_1, g) \ge 0\}$$

Hence we invest in the new technology, $\forall g \le y^I_{c_1} : \pi_{c_1}(g) = 3$

$\forall c_1^-, c_1^+ \in [c_{1\ min}, c_{1\ max}]$ and $c_1^- < c_1^+$:

$y^I_{c_1^{-(+)}} = \max\{g : \Delta 23(c_1^{-(+)}, g) \ge 0 \text{ and } \Delta 13(c_1^{-(+)}, g) \ge 0\}$.

From Lemma 4, we have

$$\Delta 13(c_1^-, g) - \Delta 13(c_1^+, g)$$
$$= \tilde{V}_1(c_1^-, g_0^1) - \tilde{V}_1(c_1^+, g_0^1) + c_1^+ - c_1^- - e^{-r\tau}(\hat{V}(c_1^-, g) - \hat{V}(c_1^+, g))$$
$$\ge (1 - e^{-r\tau})(\tilde{V}_1(c_1^-, g_0^1) - \tilde{V}_1(c_1^+, g_0^1) + c_1^+ - c_1^-)$$

Then $\Delta 13(c_1^-, g) \ge \Delta 13(c_1^+, g)$.

Similarly, we deduce $\Delta 23(c_1^-, g) \ge \Delta 23(c_1^+, g)$.

On the other hand, we have $\Delta 13(c_1^-, g)$, $\Delta 23(c_1^-, g)$ is non-increasing in g, $\forall c_1 \in [c_{1\ min}, c_{1\ max}]$ (Theorem 2.2), so we can conclude that $y^I_{c_1^-} \ge y^I_{c_1^+}$. □

Next, we deduce Lemma 5 that is an important result to identify the forecast horizon in the next subsection.

Lemma 5. $\forall g \in \Theta$, $n = 1, 2, 3 \ldots N$: $\tilde{V}(g) \le V_n^N(g) \le \hat{V}(c_1, g)$.

Proof. We prove this lemma by 2 steps.

1. First, we must prove that $\tilde{V}(g) \le \hat{V}(c_{1\ max}, g)$.
   This assertion is correct at the first rank:
   $$\tilde{V}_0(g) = 0 \le \hat{V}(c_{1\ max}, g), \quad \forall g \in \Theta$$
   By induction, we can deduce $\tilde{V}_n(g) \le \tilde{V}(g)$.
   In fact, assume that $\tilde{V}_{n-1}(g) \le \hat{V}(c_{1\ max}, g)$, $\forall g \in \Theta$:
   $$\sum_{\forall g'} P(g'|g^-) \tilde{V}_{n-1}(g') \le \sum_{\forall g'} P(g'|g^-) \hat{V}(c_{1\ max}, g')$$
   (a) We now have $\tilde{DN}_n(g) \le \widehat{DN}(c_{1\ max}, g)$, $\forall g \in \Theta$.
   From $\tilde{DN}_n(g^M) \le \widehat{DN}(c_{1\ max}, g^M)$, we deduce
   $$\tilde{M}_n(g) \le \hat{M}(c_{1\ max}, g), \quad \forall g \in \Theta. \tag{b}$$
   From Assumption (A1), we have $\tilde{I}(g) \le \hat{I}(c_{1\ max}, g)$.
   (c) Hence $\tilde{I}_n(g) \le \tilde{I}(g) \le \hat{I}(c_{1\ max}, g)$.
   (d) From (a)–(c) we can conclude $\tilde{V}(g) \le \hat{V}(c_{1\ max}, g) \le \hat{V}(c_{1\ min}, g)$.
2. Next, we must prove
   $$\tilde{V}(g) \le V_n^N(g) \le \hat{V}(c_{1\ min}, g), \quad \forall g \in \Theta, \ n \le N.$$
   In fact $V_n^N(g) = \tilde{V}(g) \le \hat{V}(c_{1\ min}, g)$.
   Now assume that $\tilde{V}(g) \le V_{n+1}^N(g) \le \hat{V}(c_{1\ min}, g)$, $\forall g \in \Theta$.
   We can deduce
   $$\begin{cases} DN_n^N(g) \le \widehat{DN}(c_{1\ min}, g); \\ M_n^N(g) \le \hat{M}(c_{1\ min}, g); \\ I_n^N(g) \le \hat{I}(c_{1\ min}, g) \end{cases}$$
   Hence $V_n^N(g) \le \hat{V}(c_{1\ min}, g), \quad \forall g \in \Theta, \ \forall n \in N.$ (e)
   On the other hand, we find that
   $$(1 - p_{n+1}) \sum_{\forall g'} P(g'|g) V_{n+1}^N(g') \ge (1 - p_{n+1}) \sum_{\forall g'} P(g'|g) \tilde{V}(g')$$
   From Lemma 3.2 and (d), we deduce
   $$p_{n+1} \sum_{\forall c_1} p_{n+1,c_1} \sum_{\forall g'} P(g'|g) \hat{V}(c_1, g') \ge p_{n+1} \sum_{\forall g'} P(g'|g) \tilde{V}(g')$$
   Hence $DN_n^N(g) \ge \tilde{DN}(g)$, $\forall g \in \Theta$. Then, we deduce easily
   $$V_n^N(g) \ge \tilde{V}(g), \quad \forall g \in \Theta, \ \forall n \in N \tag{f}$$
   Propositions (d)–(f) imply that $\forall g \in \Theta$, $n = 1, 2, 3 \ldots N$:
   $\tilde{V}(g) \le V_n^N(g) \le \hat{V}(c_{1\ min}, g)$. □

4. An algorithm for identification of the optimal forecast horizon

The term "forecast horizon" is employed here for the time interval denoted N where it is possible for the new technology to appear. From a decision-maker point of view, it is essential to estimate the conditional probability of appearance at each time in this interval. The interval length is a function of the robustness of the optimal decision given N, thus N becomes a decision variable.

We base the algorithm for the forecast horizon on the bounds $d12_n^N(g)$, $d13_n^N(g)$, $d23_n^N(g)$. This method is inspired by Hopp and Nair [9], however, they only consider the replacement investment problem with a deterministic purchase price of new technology. In our problem, we integrate both the maintenance option and the

uncertainty in the new technology purchase price. Let us define

$$V_n^N(g)^+ = \hat{V}(c_{1\ min}, g); \quad V_n^N(g)^- = \tilde{V}(g)$$

$$V_n^N(g)^+ = \max\{DN_n^N(g)^+, M_n^N(g)^+ 1_{\{g<g^M\}}, I_n^N(g)^+\}$$

$$V_n^N(g)^- = \max\{DN_n^N(g)^-, M_n^N(g)^- 1_{\{g<g^M\}}, I_n^N(g)^-\}$$

with $DN_n^N(g)^+$, $M_n^N(g)^+$, $I_n^N(g)^+$, $DN_n^N(g)^-$, $M_n^N(g)^-$, $I_n^N(g)^-$ describe similarly as Eq. (9).

Then we derive the required bounds for $d12_n^N(g)$, $d13_n^N(g)$, $d23_n^N(g)$:

$$\forall g < g^M, \quad g \in \Theta \begin{cases} d12_n^N(g)^+ = M_n^N(g)^+ - DN_n^N(g)^- \\ d12_n^N(g)^- = M_n^N(g)^- - DN_n^N(g)^+ \\ d23_n^N(g)^+ = I_n^N(g)^+ - M_n^N(g)^- \\ d23_n^N(g)^- = I_n^N(g)^- - M_n^N(g)^+ \end{cases}$$

$$\forall g \in \Theta \begin{cases} d13_n^N(g)^+ = I_n^N(g)^+ - DN_n^N(g)^- \\ d13_n^N(g)^- = I_n^N(g)^- - DN_n^N(g)^+ \end{cases}$$

Lemma 6. *$\forall g \in \Theta$, $n = 1, 2, 3\ldots N$, for any $y \in \{1,2\}$; $z \in \{2,3\}$ and $y < z$:*

1. *$dyz_n^N(g)^+$ and $dyz_n^N(g)^-$ are upper and lower bounds for $dyz_n^N(g)$.*
2. *$dyz_n^N(g)^-$ and $dyz_n^N(g)^+$ are non-decreasing and non-increasing in N.*

Proof.

1. By definition, $V_N^N(g)^- \le V_N^N(g) \le V_N^N(g)^+$.
   From Lemma 5, we can easily prove by induction:
   $\forall g \in \Theta, \quad \forall n \in N, \quad V_n^N(g)^- \le V_n^N(g) \le V_n^N(g)^+$
   Finally, Lemma 6.1 is directly given with the bounds definition.
2. We consider two forecast horizons N and $N+1$, $\forall g \in \Theta$.
   From Lemma 5 and the definition of $V_N^N(g)^-$, we have
   $V_N^{N+1}(g)^- \ge \tilde{V}(g) = V_N^N(g)^-$
   We can prove easily by induction $\forall g \in \Theta$, $\forall n \in N$:
   $V_n^N(g)^-$ is non−decreasing in N

   (a) The non-decreasing properties for $DN_n^N(g)^-$, $M_n^N(g)^-$ and $I_n^N(g)^-$ are deduced directly the non-decreasing property for $V_n^N(g)^-$.
   (b) We can do exactly the same for proving the non-increasing properties for $V_n^N(g)^+$ and $DN_n^N(g)^+$, $M_n^N(g)^+$, $I_n^N(g)^+$.
   Lemma 6.2 follows directly from (a)+(b). □

Theorem 4. *For all $T > N$ where T, N are time horizons.*

1. *If* $\max\{d12_n^N(g)^+,\ d13_n^N(g)^+\} \le 0$, $\pi_{V_n^T}(g) = 1$.
2. *If* $\min\{d13_n^N(g)^-,\ d23_n^N(g)^-\} \ge 0$, $\pi_{V_n^T}(g) = 3$.
3. *If* $\{d12_n^N(g)^- \ge 0,\ d23_n^N(g)^+ \le 0\}$, $\pi_{V_n^T}(g) = 2$.

Proof. $\pi_{V_n}^N(g) = \arg\max_{\{a \in A\}}\{DN_n^N(g), M_n^N(g) 1_{\{g \le g^M\}}, I_n^N(g)\}$.
From Lemma 6, we can deduce directly Theorem 4. □

With Theorem 4 and the control limit structure of the optimal policy, we present the following algorithm:

Algorithm for identifying the forecast horizon

Step 0: $N = 0$

Step 1: $N = N + 1$

Step 2: For all decision period n on forecast horizon N,

Step 2.1: Calculate the upper and lower bounds $d12_n^N(g)^{-(+)}$, $d13_n^N(g)^{-(+)}$, $d23_n^N(g)^{-(+)}$ for all g.

Step 2.2: Determine the optimal action among $A = \{1,2,3\}$ at the revenue rate state g, $\pi_{V_n^N}(g)$, if one of three conditions in Theorem 4 is satisfied.

Step 3: Determine y^I, the replacement threshold such that: $\pi_{V_n^N}(g) \ne 3$, $\forall g > y^I$ and $\pi_{V_n^N}(y^I) = 3$.

Step 4: If the optimal action for all g: $y^I \le g < g^M$ are determined,
STOP. N is the forecast horizon for the optimal decision at period n. If not, go to step one.

5. Numerical examples

In this section, we present numerical examples to illustrate the performance of the forecast horizon algorithm and discuss the sensitivity of some of the parameters. For the following examples we use a day as the time unit, a decision period of one month and a discount factor, $r = 3 \times 10^{-4}$. Note that the selection of these parameters is arbitrary and made without loss of generality.

5.1. Identification of the forecast horizon

The additional input parameters for Example 1 are given in Table 1, for a purchase price of new technology c_1 belongs to [300, 1837].

Table 2 shows a sequence of finite horizon solutions for the decision at the first period in Example 1 under the forecast given N. Notice that these decisions and the forecast horizon depend on the system states g. We find that:

- For any N lower than 6, we cannot determine the optimal choice for all system states because none of the conditions in Theorem 4 are satisfied.
- When the forecast horizon is $N = 8$, the optimal decision is to invest in a new system for $g \in [0, 0.31]$, to do nothing for $g \in [8, 10]$ while, for any $g \in [0.32, 7.99]$, we do not have enough information for a decision.
- The optimal policy for all system states is not determined until N reaches 15 and does not change where $N > 15$. It prescribes that we invest in a new system for states $g \in [0, 5.06]$, maintain for states $g \in [5.07, 7.31]$ and do nothing for states $g \in [7.32, 10]$.

Therefore, the non-decreasing property in g of the optimal policy is also illustrated.

A comparison of the decision thresholds in the case without new technology highlights the importance of the maintenance option. It allows postponing the investment decision for the opportunity to benefit from a potential better technology in the near future.

Table 1
The input parameters for Example 1.

Appearance prob.	Maintenance	Salvage value	Purchase price	Revenue process
$\delta = 0.9$	$\varrho = 0.8$	$\nu = 5$	$\sigma_c = 1.9 \times 10^{-3}$	$\sigma = 6.3 \times 10^{-3}$
$\kappa = 0.9$	$\nu = 20$	Old tech:	Old tech:	Old tech:
	$h_1 = 20$	$h_{20} = 10$	$c_0 = 300$	$g_0^0 = 10$
				$\mu_0 = -3 \times 10^{-3}$
		New tech:	New tech:	New tech:
		$h_{21} = 12$	$c_{1,0} = 315$	$g_0^1 = 10.5$
				$\mu_1 = -2.7 \times 10^{-3}$

Table 2
The optimal decision at first decision period for Example 1.

Optimal action				
N	Unknown	Invest	Maintain	Do nothing
6	$\forall g \in \Theta$			
8	$g \in [0.32, 7.99]$	$g \in [0, 0.31]$		$g \in [8, 10]$
12	$g \in [5.01, 5.11]$ $g \in [7.3, 7.34]$	$g \in [0, 5]$	$g \in [5.12, 7.29]$	$g \in [7.35, 10]$
15		$g \in [0, 5.06]$	$g \in [5.07, 7.31]$	$g \in [7.32, 10]$
24		$g \in [0, 5.06]$	$g \in [5.07, 7.31]$	$g \in [7.32, 10]$

Table 3
The optimal policy for obsolescence case in Example 1.

c_1	Invest	Maintain	Do nothing
300	$g \in [0, 7.84]$		$g \in [7.85, 10]$
400	$g \in [0, 7.57]$		$g \in [7.58, 10]$
600	$g \in [0, 7.5]$		$g \in [7.51, 10]$
1000	$g \in [0, 7.37]$		$g \in [7.38, 10]$
1700	$g \in [0, 7.07]$	$g \in [7.08, 7.19]$	$g \in [7.2, 10]$
1837	$g \in [0, 6.15]$	$g \in [6.16, 7.33]$	$g \in [7.34, 10]$

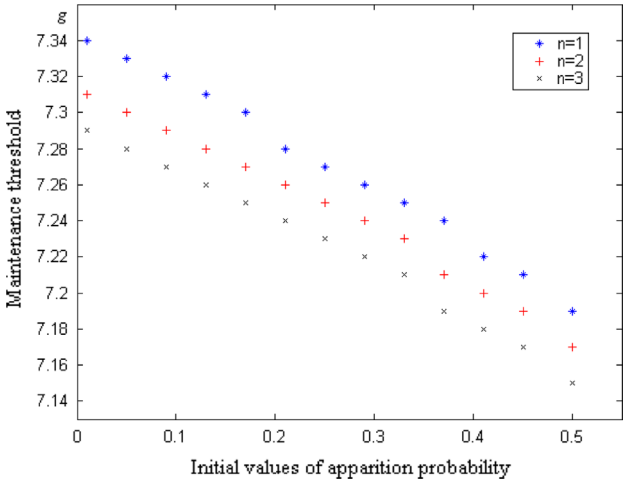


Fig. 1. Impact of the new technology appearance probability on the maintenance threshold for the three first decision periods.

In the obsolescence case, new technology has appeared rendering existing technology obsolete (Table 3), the new technology has appeared with purchase price c_1, and we find that the higher the purchase price is, the lower the replacement threshold in new technology is (Theorem 3). If c_1 is very high, the model tends to take advantage of the current system by extending its useful life through maintenance.

5.2. *Impact of the new technology appearance probability on the optimal policy*

With the parameters of Table 1, we consider the probability process of the new technology's appearance time with the same increasing rate (κ=0.9) and different initial values δ. Fig. 1 shows the impact of the appearance probability of a new technology on the maintenance threshold for the three first decision periods. We find that this threshold is non-increasing when the probability is increasing. However, this impact is less significant than the case of replacement threshold (Fig. 2). In fact, if the initial value is increasing in the interval $[0.01, 0.5]$, the maintenance thresholds in the first decision period decrease in the interval $g = [7.34, 7.19]$ while the replacement thresholds reduce from g=5.59 to g=3.28.

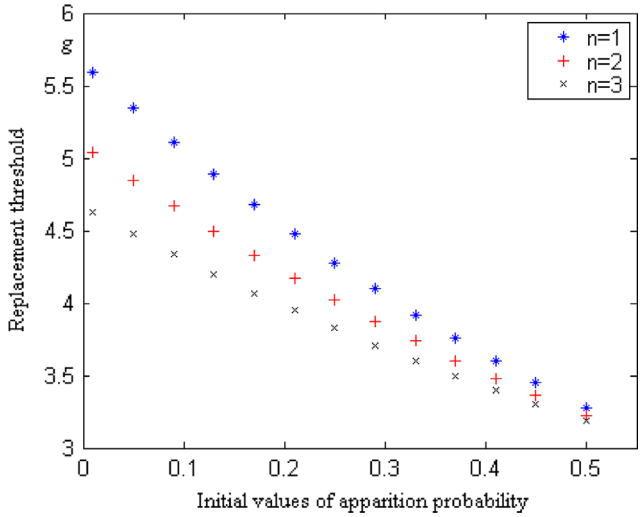


Fig. 2. Impact of the new technology appearance probability on the replacement threshold for the three first decision periods.

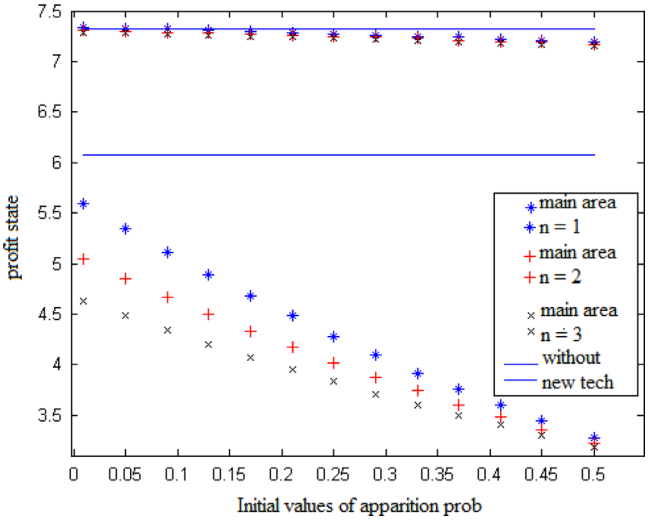


Fig. 3. Comparison of maintenance areas for the three first decision periods.

Moreover, the significance of the maintenance option value increases with the appearance probability of new technology. Indeed, in Fig. 3, we find that the maintenance area (the interval of revenue rate state where maintenance is performed) expands in the initial value of the new technology appearance probability.

5.3. *Impact of maintenance efficiency on the replacement threshold*

With the parameters of Table 1, we examine the effect of maintenance efficiency that changes in the interval $[0.8, 0.86]$ on the replacement threshold. In the non-obsolescence case, the impact of the maintenance efficiency on the replacement threshold is monotone. The replacement threshold decreases with the maintenance efficiency. This tendency is shown clearly in Fig. 4. Consider, for example, that if maintenance efficiency is high, $\varrho = 0.86$, the investment in new technology is not optimal at n=2, 3. At the first decision epoch, while the maintenance efficiency raises from $\varrho = 0.8$ to $\varrho = 0.86$, the replacement threshold decreases from g=5.06 to g=0.14.

In the obsolescence case, we lose the advantage of extending the economic life of the system through maintenance when the decision-maker can take advantage of the new technology. This is illustrated in Fig. 5 with an anticipation of system replacement when maintenance efficiency increases. If the purchase price of

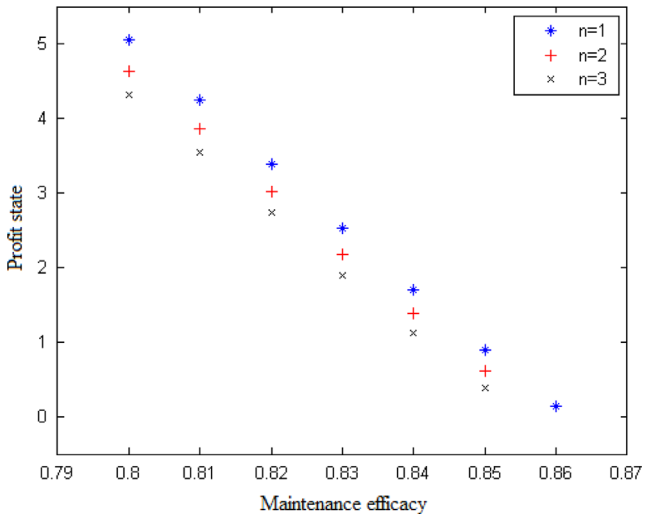


Fig. 4. The effect of maintenance efficacy on the replacement threshold in non-obsolescence case.

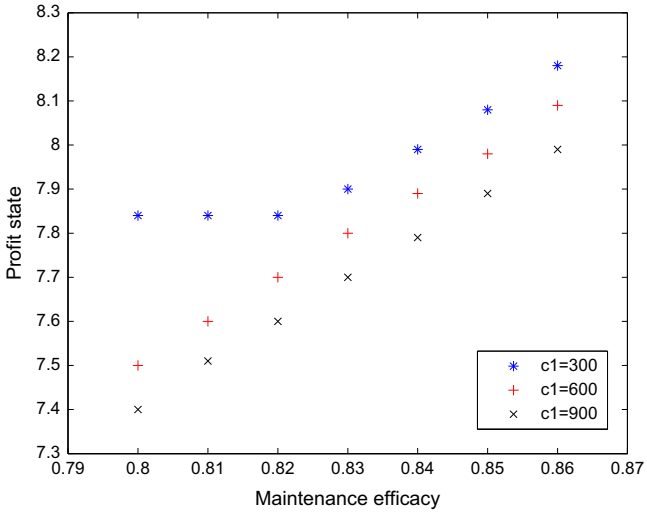


Fig. 5. The effect of maintenance efficiency on replacement threshold in obsolescence case with low purchase price of new technology.

new technology is not very expensive, the ratio of the maintenance cost and its associated benefits is not greater than the expected rewards of new technology. We find that the replacement option demonstrates its dominance in this case. On the contrary, we weigh the benefits of utilizing the available asset and the revenues gained by investment in new technology. Hence, the maintenance area expands in the purchase price of new technology, not in the maintenance efficiency, Fig. 6. This interesting result thus demonstrates that it seems not necessary to improve the efficiency of maintenance; a routine maintenance effort may be sufficient under technological evolution.

6. Conclusion

In general, maintenance is often viewed as a necessary short-term investment by a company for dealing with equipment failures and enhancing system efficiency while investment in a new technology is considered as a part of long-term competitive strategy. In this paper, we have highlighted the importance of considering maintenance actions at the tactical level by integrating them into the problem of technology investment. We have also developed an efficient approach

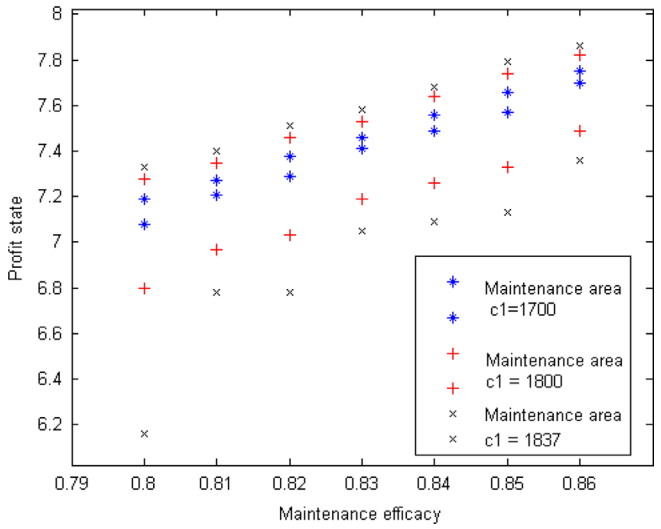


Fig. 6. The maintenance area in the high purchase price cases of new technology.

for determining the optimal forecast horizon – a given finite horizon N that is long enough to guarantee the optimal decision over the infinite horizon. As the investment planning in a new technology is the long-term development strategy of a company, the identification of the minimum forecast horizon in order to avoid bad decisions is critical.

Through our mathematical analysis, we have shown the control limit structure of the optimal policy and demonstrated the intuition that the replacement investment in new technology is postponed when its purchase price is high.

Finally, the impact of maintenance actions on the investment strategy in a new technology is demonstrated through the results of numerical examples. Indeed, the maintenance action allows postponing the investment in a new asset in order to wait for better technology. In the obsolescence case, the optimal maintenance policy when applied on new technology would offer a higher profit than that on the current asset. This encourages the firms to consider investment in the new technology.

Several perspectives arise from this study. This model could be extended to consider a sequence of new technologies. This, however, would drastically increase the complexity in the decision by having to choose the most suitable technology. From an optimization perspective, the number of bounds for the differences in option values would need to be extended for defining the new stopping rule. Furthermore, the solution algorithm would also need to be improved in order to identify the forecast horizon for a sequential technology evolution.

References

[2] Bethuyne G. The timing of technology adoption by a cost-minimizing firm. J Econ 2002;76:123–54.
[3] Borgonovo E, Marseguerra M, Zio E. A Monte Carlo methodological approach to plant availability modeling with maintenance, aging and obsolescence. Reliab Eng Syst Saf 2000;67:61–73.
[4] Castanier B, Brenguer C, Grall A. A sequential condition based repair/replacement policy with non-periodic inspections for a system subject to continuous wear. Appl Stoch Models Bus Ind 2003;19:327–47.
[6] Clavareau J, Labeau PE. A Petri net-based modeling of replacement strategies under technological obsolescence. Reliab Eng Syst Saf 2009;94:357–69.
[7] Clavareau J, Labeau PE. Maintenance and replacement policies under technological obsolescence. Reliab Eng Syst Saf 2009;94:370–81.
[9] Hopp WJ, Nair SK. Markovian deterioration and technological change. IIE Trans 1994;26:74–82.
[11] Huisman KJM, Kort PM. Strategic technology adoption taking into account future technological improvement: a real options approach. Eur J Oper Res 2004;159:705–28.
[12] Mannan S. Lees' loss prevention in the process industries. Butterworth Heinemann, UK: Elsevier Inc; 2005 ISBN: 978-0-7506-7555-0.

[13] Mauer DC, Ott SH. Investment under uncertainty: the case of replacement investment decisions. J Financ Quant Anal 1995;30:581–605.
[14] Mercier S. Optimal replacement policy for obsolete components with general failure rates submitted to obsolescence. Appl Stoch Models Bus Ind 2008;24:221–35.
[15] Nair SK. Modeling strategic investment decisions under sequential technological change. Manag Sci 1995;41:282–97.
[16] Nair SK. Identifying technology horizons for strategic investment decision. IEEE Trans Eng Manag 1997;44:227–36.
[17] Nguyen TPK, Yeung TG, Castanier B. Optimal maintenance and replacement decisions under technological change. In: Proceedings of the European safety and reliability conference ESREL 2010, Rhodes, Greece; 2010. p. 1430–7.
[18] Nguyen TPK, Yeung TG, Castanier B. Impact of maintenance on the replacement investment under technological improvement. In: Proceedings of the European safety and reliability conference ESREL 2011, Troyes, France; 2011.
[19] Nicolai RP, Frenk JBG, Dekker R. Modelling and optimizing imperfect maintenance of coatings on steel structures. Econometric Institute report EI 2007-24. Erasmus University Rotterdam, Econometric Institute; 2007.
[20] Puterman ML. Markov decision processes – discrete stochastic dynamic programming. Hoboken, NJ: John Wiley & Sons, Inc.; 2005 ISBN 0-471-72782-2.
[21] Wang H. A survey of maintenance policies of deteriorating systems. Eur J Oper Res 2002;139:469–89.
[22] Yeh RH, Kao KC, Chang WL. Optimal preventive maintenance policy for leased equipment using failure rate reduction. Comput Ind Eng 2009;57:304–9.
[23] Yor M. Exponential functional of Brownian motion and related process. Springer-Verlag, Berlin, Heidelberg, New York: Springer Finance; 2001 ISBN 3-540-65943-9.