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To cite this version:
Miloud Frikel, Boubekeur Targui, Said Safi, Mohammed M’Saad. Bearing detection of noised wideband sources for geolocation. 18th IEEE Mediterranean Conference on Control and Automation, MED’10, Jun 2010, Marrakech, Morocco. 2010. <hal-01057691>
Bearing detection of noised wideband sources for geolocation

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Abstract—In this paper, we propose a method for the localisation of wideband sources in order to locate mobile station. Indeed, directions-of-arrival of wideband sources are estimated by coherent subspace methods, which use the focusing operators, in the presence of a correlated noise and, then, the angulation is used to determine the mobile’s position. Practical methods of the estimation of the coherent signal subspace are given and are compared for mobile system location.

I. INTRODUCTION

In many problems in signal processing, the received data can be considered as a superposition of a finite number of elementary source signals and an additive noise. Generally, in a multi-sensor environment application, such as sonar, underwater acoustics and seismology, the objectives are the estimation of the number and the direction of arrival (DOA), or bearing, of the radiating sources. Over, the last decade, eigenstructure based methods [1], [2], [3], [4], [5], [6] yield resolution have been proposed to the problem of wideband sources bearing estimation.

In the last decade, there has been a great deal of interest in developing mobile location systems for wireless communication systems [7], [8], [9]. The motivation is the interest in applications for indoor/outdoor geolocation systems [8]. In psychiatric hospitals and jails there is a growing need to identify the location of specific patients or inmates. In laboratories, and hospitals there is a need to identify the location of mobile.

Most conventional location techniques use the signal being transmitted by a mobile to determine its location [7]. Generally, the mobile’s signal is received at several receivers with known positions. After reception, some characteristics of that signal are combined with the known positions of the receivers to solve the mobile’s position. This could be the angle of arrival, or the time of arrival of the signal. For practical reasons, the reception points are usually existing base stations. This minimizes the extra equipment that has to be added to the network to implement location [7], [8]. The only need is the use of an array of sensors at the base station (spatial diversity).

In this paper, we focus on the problem of the localisation of wideband sources in order to locate mobile system. However, to estimate the direction of arrival of a mobile, the line of sight (LOS) is required, in the case of non-line of sight (NLOS), the accuracy, drastically, decreases.

Usually in the classical array processing (narrow-band signals), the parameter of interest is the direction of arrival of the radiating sources from the recorded data. However, there are no wide-band array processing methods for DOA, two approaches were developed in the nineties: some methods sample the frequency spectrum to create narrow-band signals, then at each frequency bin a narrow-band signal methods are used to estimate the direction of arrival; that is the incoherent method. The other approach is the coherent signal subspace method, the cross-spectral matrices at different frequency bins are combined to form an average cross-spectral matrix. Then, the high resolution algorithm, such as Music [10], is used to estimate the DOA. In the coherent signal subspace method, the combination of the narrow-band samples is done through the observation vector or the cross-spectral matrices, this is called focusing. The focusing operator is a matrix that transforms the location matrix of the array at a sampling frequency to the location matrix at the focusing frequency. An improved version of the coherent signal subspace method is also reported in the [6] that uses unitary focusing matrices. A two-sided transformation is applied on the data spectral matrices.

To locate mobile station, the DOA geolocation method [9] uses simple angulation to locate the transmitter. The receiver measures the direction of received wide-band signals from the target transmitter using antenna arrays. DOA measurements at two receivers will provide a position fix but the accuracy of the position estimation depends on the transmitter location with respect of the two receivers, multipath propagation etc. As a result, more than two receivers are normally needed to improve the location accuracy. In an indoor environment the LOS signal path is usually blocked by surrounding objects or walls. DOA can provide acceptable location accuracy but dramatically large location errors will occur if the LOS signal path is blocked and the DOA of a reflected or a scattered signal component is used for estimation. If we wish to use this geolocation method in an indoor environment we would need to combine it with other methods.

II. PROBLEM FORMULATION

The estimation of the angle of arrival for a known signal is the main functions for location systems. The conventional schemes for estimating the DOA in a wireless communication system are based on transmitting a known signal, such as a pulse (wide-band signal), and performing correlation or parametric estimations of DOA.

We consider an array of N sensors which receives the wavefield generated by P wide-band sources, with M fre-
frequency bins, in the presence of an additive noise. The received signal vector, in the frequency domain, is given by:

\[ \mathbf{r}(f_n) = \mathbf{H}(f_n)\mathbf{s}(f_n) + \mathbf{n}(f_n), \quad n = 1, \ldots, M \]  

(1)

Where \( \mathbf{r}(f_n) \) is the Fourier transform of the array output vector, \( \mathbf{s}(f_n) \) is the \( P \times 1 \) vector of complex signals of wavefronts: \( \mathbf{s}(f_n) = [s_1(f_n), s_2(f_n), \ldots, s_P(f_n)]^T \). \( \mathbf{n}(f_n) \) is the \( N \times 1 \) vector of additive noise

\[ \mathbf{n}(f_n) = [n_1(f_n), n_2(f_n), \ldots, n_N(f_n)]^T, \quad \text{and} \quad \mathbf{H}(f_n) \]

is the \( N \times P \) transfer matrix of the source-sensor array systems with respect to some chosen reference point.

\[ \mathbf{H}(f_n) = [\mathbf{h}(f_n, \theta_1), \mathbf{h}(f_n, \theta_2), \ldots, \mathbf{h}(f_n, \theta_P)] \]

\( \mathbf{h}(f_n, \theta_i) \) is the steering vector of the array toward the direction \( \theta_i \). For example, the steering vector of a linear uniform array with \( N \) sensors is given by:

\[ \mathbf{h}(f_n, \theta_i) = \left[ 1, e^{j\varphi_1}, e^{j\varphi_2}, \ldots, e^{j(N-1)\varphi} \right]^T \]

where \( \varphi_i = 2\pi f_n d \sin(\theta_i) \), \( d \): sensor spacing, \( \theta_i \): direction of arrival (DOA) of the \( i \)th source as measured from the array broadside, \( c \): velocity wave propagation, \( f_n \): analysis frequency.

Assume that the signals and the additive noise are stationary and ergodic zero mean complex valued random processes. In addition, the noises are assumed to be uncorrelated between sensors, and have different variances \( \sigma_i^2(f_n) \) at each sensor. It follows from these assumptions that the spatial \( (N \times N) \) cross-spectral matrix of the observation vector at frequency \( f_n \) is given by:

\[ \mathbf{D}(f_n) = \mathbb{E}[\mathbf{r}(f_n)\mathbf{r}^+(f_n)] \]

where \( \mathbb{E}[\cdot] \) denotes the expectation operator, the superscript * represents conjugate transpose, \( \mathbf{D}_s(f_n) = \mathbb{E}[\mathbf{s}(f_n)\mathbf{s}^+(f_n)] \) is the \( P \times P \) sources cross-spectral matrix, and \( \mathbf{D}_n(f_n) = \mathbb{E}[\mathbf{n}(f_n)\mathbf{n}^+(f_n)] \) is the \( P \times P \) noise cross-spectral matrix.

Several parametric noise models have appeared in the literature recently [11]. In many applications when uniform linear array is used, it is reasonable to assume that the correlation is decreasing along the array. This is a widely used model for colored noise[11]. If we assume that the correlation length is \( L \) sensors, the noise between sensors that are separated with a distance more than \( Ld \), where \( d \) is the sensor separation, is considered uncorrelated, we obtain the following model [11]:

\[ \mathbf{D}_n(f_n) = \begin{pmatrix}
\sigma_1^2 & \rho_{12} & \cdots & 0 \\
\rho_{21} & \sigma_2^2 & \ddots & \vdots \\
\ddots & \ddots & \ddots & \rho_{N-1,N} \\
0 & \cdots & \rho_{N,N-1} & \sigma_N^2
\end{pmatrix} \]  

(2)

where \( \sigma_i^2 \) are the noise variances at sensor \( i \), \( \rho_{mi} = \rho_{mi} + j\beta_{mi}, i = 1, \ldots, L, m = 1, \ldots, N \), \( \rho_{mi} \) are complex variables represent the correlation between sensor \( m \) and \( i \), \( j^2 = -1 \).

The eigendecomposition of the noiseless data cross-spectral matrix \( \mathbf{D}_{NS}(f_n) \) yields:

\[ \mathbf{D}_{NS}(f_n) = \mathbf{D}(f_n) - \mathbf{D}_n(f_n) = \sum_{i=1}^{N} \lambda_i(f_n) \mathbf{v}_i(f_n)\mathbf{v}_i^+(f_n) \]  

(3)

where \( \lambda_i(f_n), i = 1, \ldots, N \) \((\lambda_1 \geq \lambda_2 \geq \lambda_P > \lambda_{P+1} \geq \ldots \geq \lambda_N \approx 0)\), and \( \mathbf{v}_i(f_n) \) are the \( i \)th eigenvalue and \( i \)th corresponding eigenvector.

The first step is to estimate \( \mathbf{D}_n(f_n) \) at each frequency \( f_n \), for that we use our algorithm developed in the article [11].

Our aim is to estimate the angles \( \theta_i, i = 1, \ldots, P \), from the noiseless received data. We assume that the number of the sources \( P \) is supposed known.

For locating the wide-band sources several solutions have been proposed in the literature and are summarized as following:

- The incoherent subspace methods: the analysis bandwidth is divided into several frequency bins and then at each frequency the treatment is applied and obtained results are combined to obtain the final result.
- The coherent subspace methods: the different subspaces are transformed in a predefined subspace using the focusing matrices [1], [2], [5], [6].

III. FOCUSING OPERATORS

The focusing matrices \( \mathbf{F}(f_o, f_n)'s \) are the solutions of the equations: \( \mathbf{F}(f_o, f_n)\mathbf{H}(f_n) = \mathbf{H}(f_o), \; \forall f_n \in L \). Where \( f_o \) is the focusing frequency and \( \mathbf{H}(f_o) \) is the focusing location matrix.

The matrices \( \mathbf{H}(f_o) \) and \( \mathbf{H}(f_n) \) are functions of the DOA’s \( \theta \). An ordinary beam-forming pre-process gives an estimate of the angles-of-arrival that can be used to form \( \mathbf{H}(f_o) \). Using the focusing matrices \( \mathbf{F}(f_o, f_n) \), the observation vectors at different frequency bins are transformed into the focusing subspace.

A. Coherent signal subspace method

Hung and Kaveh [1] have shown that the focusing is lossless if \( \mathbf{F}(f_o, f_n)'s \) are unitary transformations. Specially, they proposed the use of the transformation matrices obtained by the constrained minimization problem:

\[ \min_{\mathbf{F}(f_o, f_n)} \| \mathbf{H}(f_o) - \mathbf{F}(f_o, f_n)\mathbf{H}(f_n) \| \]

\[ \mathbf{F}^+(f_o, f_n)\mathbf{F}(f_o, f_n) = \mathbf{I} \]  

(4)

The focusing matrix \( \mathbf{F}(f_o, f_n) \) that solves (4) is given by Hung and Kaveh [1]: \( \mathbf{F}(f_o, f_n) = \mathbf{V}(f_n)\mathbf{W}^+(f_n) \)

where the singular value decomposition of \( \mathbf{H}(f_n)\mathbf{H}^+(f_n) \) is represented by \( \mathbf{V}(f_n)\Sigma(f_n)\mathbf{W}^+(f_n) \).
The average cross-spectral matrix is, then, given by:
\[ B = \text{cross-spectral of the received data} \]

almost the same performance than the TCT method. Let
\( X \) be the frequency (CFO) based on the rotation of the source subspace at frequency
\( D. \) Coherent Focusing Operators based on the Projection matrix. Where
\( \Pi \) is the eigenvector matrix of the cross-spectral matrix at the frequencies
\( D \) is the diagonal eigenvalue matrix of \( D(f_n) \).

C. TCT method

In [6], the TCT approach is based on transformation of the matrices:
\[ \mathbf{P}(f_n) = \mathbf{H}(f_n)\mathbf{D}_S(f_n)\mathbf{H}^+(f_n). \]

Where \( \mathbf{P}(f_n) \) is the cross-spectral matrix of the received data at the \( n \)th frequency bin in a noise-free environment.

Let \( \mathbf{P}(f_0) \) be the focusing noise-free cross-spectral matrix. The TCT focusing matrices are found by minimising:
\[
\min_{\mathbf{F}(f_0,f_n)} \| \mathbf{P}(f_0) - \mathbf{F}(f_0,f_n)\mathbf{P}(f_n)\mathbf{F}^+(f_0,f_n) \|
\]
\[
\mathbf{F}^+(f_0,f_n)\mathbf{F}(f_0,f_n) = \mathbf{I} \quad (5)
\]

It is shown [6] that the optimal solution of 5 is given by the eigenvectors of the cross-spectral matrix at the frequencies \( f_0 \) and \( f_n \). The solution of the equation system (5) is [6]:
\[
\mathbf{F}_{TCT}(f_0,f_n) = \mathbf{X}(f_0)\mathbf{X}^+(f_n) \quad (6)
\]

Where \( \mathbf{X}(f_0) \) and \( \mathbf{X}(f_n) \) are the eigenvector matrices of \( \mathbf{P}(f_0) \) and \( \mathbf{P}(f_n) \), respectively:
\[ \mathbf{P}(f_n) = \mathbf{X}(f_n)\Pi(f_n)\mathbf{X}^+(f_n), \]

with \( \Pi(f_n) \) is the eigenvalue diagonal matrix.

D. Coherent Focusing Operators based on the Projection Matrices

In this section, we propose a coherent focusing operator (CFO) based on the rotation of the source subspace at the frequency \( f_0 \) to the source subspace at the focusing frequency \( f_n \). This limitation to the transformation of the signal subspace reduces the computational load, and has, almost, the same performance than the TCT method. Let the partition of the eigenvector matrix \( \mathbf{X}(f_0) \):
\[
\mathbf{X}(f_0) = [\mathbf{X}_S(f_n) | \mathbf{X}_B(f_n)].
\]

Where \( \mathbf{X}_S(f_n) \) is \( (N \times P) \) of \( P \) largest eigenvectors, and \( \mathbf{X}_B(f_n) \) is \( N \times (N-P) \) of \( (N-P) \) smallest eigenvectors of the cross-spectral matrix \( \mathbf{P}(f_n) \), the eigenvalues of the cross-spectral of the received data \( \mathbf{P}(f_n) \) is:
\[
\Pi_S(f_n) = \begin{bmatrix} \Pi_S(f_n) & 0 \\ 0 & \Pi_B(f_n) \end{bmatrix} \quad (7)
\]

Where \( \Pi_S(f_n) \) is \( P \times P \) of \( P \) largest eigenvalues, and \( \Pi_B(f_n) \) is \( (N-P) \times (N-P) \) of \( (N-P) \) smallest eigenvalues of \( \mathbf{P}(f_n) \). The proposed focusing operator is [5]:
\[
\mathbf{H}_{CFO}(f_0,f_n) = \mathbf{X}_S(f_0)\mathbf{X}_S^+(f_n) \quad (8)
\]

The average cross-spectral matrix is, then, given by:
\[
\bar{\mathbf{P}}(f_0) = \frac{1}{M}\sum_{n=1}^{M}\mathbf{H}_S(f_0,f_n)\mathbf{P}(f_n)\mathbf{H}_S^+(f_0,f_n). \]

It is shown [10] that the noise and signal subspaces are orthogonal, we have:
\[ \mathbf{X}_S^+(f_n)\mathbf{X}_S(f_n) = \mathbf{I} \]

Using the above properties, we have:
\[
\bar{\mathbf{P}}(f_0) = \mathbf{X}_S(f_0)[\frac{1}{M}\sum_{n=1}^{M}\Pi_S(f_n)]\mathbf{X}_S^+(f_n) \quad (9)
\]

This formula shows that the proposed operator focuses the signal subspace into the focusing frequency \( f_0 \), all the power of the different signal subspaces of the analysis band.

The eigendecomposition of \( \mathbf{P}(f_0) \) is:
\[ \mathbf{P}(f_0) = \mathbf{X}(f_0)\Pi(f_0)\mathbf{X}^+(f_0). \]

Let the partition of the eigenvector matrix \( \mathbf{X}(f_0) \):
\[ \mathbf{X}(f_0) = [\mathbf{X}_S(f_0) | \mathbf{X}_B(f_0)]. \]

We have: \( \mathbf{X}_S(f_0) \) and \( \mathbf{X}_B(f_0) \) are orthogonal, this property is used to estimate the DOA [10].

Remark: We can also extracted the focusing matrices from the received cross-spectral matrix \( \mathbf{D}(f_n) \) [5]: The partition of the eigenvector matrix \( \mathbf{V}(f) \) gives:
\[ \mathbf{V}(f) = [\mathbf{V}_S(f_n) | \mathbf{V}_B(f_n)]\], where \( \mathbf{V}_S(f_n) \) is \( (N \times P) \) of \( P \) first eigenvectors, and \( \mathbf{V}_B(f_n) \) is \( N \times (N-P) \) of \( (N-P) \) last eigenvectors.

The proposed focusing operator is then given by:
\[ \mathbf{F}_{CFO}(f_0,f_n) = \mathbf{V}_S(f_n)\mathbf{V}_S^+(f_n) \]

IV. LOCALIZATION OF DOA AND MOBILE LOCATION

The algorithm based Coherent Focusing Operators is summarized as follows: first, use an ordinary estimator to scan the space and find an initial estimate of the DOA. Form \( \hat{\mathbf{H}}(f_n) \) and estimate:
\[ \hat{\mathbf{D}}_S(f_n) = \left[ \hat{\mathbf{H}}^+(f_n)\hat{\mathbf{H}}(f_n) \right]^{-1}\hat{\mathbf{H}}^+(f_n)\left[ \hat{\mathbf{D}}_S(f_n) \right]^{-1} \]

\[ \bar{\mathbf{A}}(f_n) = \left[ \hat{\mathbf{H}}^+(f_n)\hat{\mathbf{H}}(f_n) \right]^{-1}, \]

where \( \hat{\mathbf{D}}_S(f_n) \) is the noiseless received data. Then, average the source cross-spectral matrices to obtain:
\[ \bar{\mathbf{D}}_S(f_0) = \frac{1}{M}\sum_{n=1}^{M}\hat{\mathbf{D}}_S(f_n). \]

Estimate the cross-spectral matrix of the received data:
\[ \hat{\mathbf{D}}(f_0) = \mathbf{H}(f_0)\hat{\mathbf{D}}_S(f_0)\mathbf{H}^+(f_0) + \sigma^2(f_0)\mathbf{I}. \]

Find \( \hat{\mathbf{P}}(f_0) = \mathbf{H}(f_0)\hat{\mathbf{D}}(f_0)\mathbf{H}^+(f_0) \), and
\[ \hat{\mathbf{P}}(f_0) = \hat{\mathbf{D}}(f_0) - \sigma^2(f_0)\mathbf{I}, n = 1, ..., M. \]

Determine the focusing operator (8). Multiply these matrices by the sample cross-spectral matrices, and average the results:
\[ \bar{\mathbf{P}}(f_0) = \mathbf{X}_S(f_0)\left[ \frac{1}{M}\sum_{n=1}^{M}\Pi_S(f_n) \right]\mathbf{X}_S^+(f_n) \]

Apply a localization method e.g. MUSIC [10] to find the DOA of the sources. Finally, once these DOAs are estimated, the angulation technique is then used to estimate the mobile position.
V. SIMULATION RESULTS

To analyze the performances of the presented focusing operators, the normalized root mean-square error (NRMSE) is employed as a performance tool of the input estimates. A linear array of $N = 5$ omnidirectional sensors at the base station in order to have a spatial diversity with equal inter-element spacing $d = \frac{\lambda}{2}$ is used, where $f_o$ is the center frequency and $c$ is the propagation velocity. The Rayleigh angle resolution limit for this array is about $\theta_r = 2^\circ$. The source signals are temporally stationary zero-mean bandpass white gaussian processes with the center frequency $f_o = 902.5\,MHz$ (GSM frequencies: 890-915MHz), and the same bandwidth $BW = 25\,MHz$. The distance between two sensors at the base station is about $16\,cm$, it can reach $10\lambda$, about $d = 30\,cm$, where $\lambda$ is the wave length, so the total distance of the array is about $1\,m$. The noise is stationary zero-mean bandpass (the same pass-band as that the signals) white gaussian process, independent of the signals, and statistically independent and identical. Three equipowers uncorrelated mobile sources impinging from the angles $10^\circ$, $18^\circ$ and $30^\circ$, with a $SNR$ of 3$dB$ are used for this simulation.

The TCT and Hung’s operator give a high performance than the adaptive and the Coherent Focusing Operators, however this last method is very interesting in term of computational load because only one part of the eigenvectors are used to built the focusing operators.

VI. CONCLUSION

In this paper a class of focusing operators have been proposed, and their numerical performances have been evaluated. The obtained results show the efficiency of the eigenvectors focusing operators in term of accurate angles of arrival. Those operators have been used to estimate the DOAs of a known pulse and finally to determine the position of mobile station.

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