Goal-oriented control of finite element models: recent advances and performances on 3D industrial applications

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DRAFT: GOAL-ORIENTED CONTROL OF FINITE ELEMENT MODELS: RECENT ADVANCES AND PERFORMANCES ON 3D INDUSTRIAL APPLICATIONS

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ABSTRACT

In this work, we present two recent developments in goal-oriented error estimation applied to finite element simulations. The first one is a non-intrusive enrichment of the adjoint solution using handbook techniques, inserting locally the (generalized) Green functions associated with the quantity of interest under study. The second one is a new bounding technique, based on homothetic domains, that is an alternative to classical Cauchy-Schwarz boundings. Technical aspects and capabilities of the resulting verification tool will be shown on 3D numerical experiments.

INTRODUCTION

During the last five years, a general method has been set up in order to address robust goal-oriented error estimation for a large class of mechanical problems solved using the finite element framework [1]. This method uses the concept of Constitutive Relation Error (CRE) [2] and associated admissible fields, as well as a classical adjoint-based procedure devoted to the quantity of interest under study. Several works [3–5] illustrated the performances of this method for controlling discretizations when dealing with (visco-)elasticity problems, fracture mechanics problems solved with XFEM, transient dynamics problems, or nonlinear (visco-)plasticity problems.

Here, we introduce two new ingredients to the method in order to optimize the quality of error bounds. On the one hand, the adjoint solution is enriched using handbook techniques, i.e. inserting locally and explicitly, by means of the PUM, some (generalized) Green's functions associated with the quantity of interest [6]. This technique provides for an accurate adjoint solution at low cost, as the residual solution which is added to the enrichment function can be approximated with a relatively coarse finite element mesh. It also enables to address pointwise quantities of interest. On the other hand, a new procedure to obtain error bounds is developed as an alternative to classical, and sometimes coarse, global Cauchy-Schwarz boundings. For that, we derive relations between error contributions over homothetic domains and use these relations to set up sharp bounds on the quantity of interest.

REFERENCE PROBLEM AND CLASSICAL ERROR ESTIMATION STRATEGY

We consider a linear elasticity problem, with displacement-stress solution $(\mathbf{u}, \sigma)$, and assume it is in practice solved using the finite element method which provides for the approximate solution $(\mathbf{u}_h, \sigma_h)$. We are interested in assessing the discretization error on a given quantity of interest of the problem, denoted $I(\mathbf{u})$, which is a linear functional of the displacement field $\mathbf{u}$. Defining the associated adjoint problem, with solution $(\tilde{\mathbf{u}}, \tilde{\sigma})$, the error on

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I can be strictly bounded as [6]:

$$|I(\mathbf{u}) - I(\mathbf{u}_h) - I_{hh}| \leq E_{CRE}(\hat{\mathbf{u}}_h, \hat{\sigma}_h) \times E_{CRE}(\hat{\tilde{\mathbf{u}}}_h, \hat{\tilde{\sigma}}_h) \quad (1)$$

where I_{hh} is a computable correcting term, $(\hat{\mathbf{u}}_h, \hat{\sigma}_h)$ (resp. $(\hat{\tilde{\mathbf{u}}}_h, \hat{\tilde{\sigma}}_h)$) is an admissible solution for the reference (resp. adjoint) problem, and E_{CRE} is the CRE functional. The construction of admissible solutions is detailed in [2]. Let us note that no Galerkin orthogonality property is used to obtain (1), so that independent meshes can be used to solve reference and adjoint problems.

ENRICHMENT OF THE ADJOINT SOLUTION

We propose to search the adjoint displacement field under the form $\tilde{\mathbf{u}} = \tilde{\mathbf{u}}^{hand}\Phi + \tilde{\mathbf{u}}^{res}$ where:

- $\tilde{\mathbf{u}}^{hand}$ is an enrichment (or handbook) function that corresponds to the (generalized) Green's solution in a (semi-)infinite domain. It is obtained analytically [7] or numerically (see Fig. 1 for a 3D example);
- Φ is a weighting function, with bounded support, so that the enrichment is introduced locally. It is in practice constructed by means of the PUM defined by linear shape functions;
- $\tilde{\mathbf{u}}^{res}$ is a residual term that enables to verify boundary conditions of the adjoint problem.

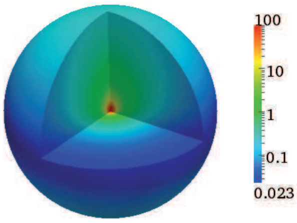

FIGURE 1. EXAMPLE OF 3D GREEN'S FUNCTION.

In practical applications, $\tilde{\mathbf{u}}^{res}$ is a smooth solution (high gradients of $\tilde{\mathbf{u}}$ are captured by the enrichment term) and can be accurately approximated with the initial (and potentially coarse) mesh. Furthermore, the CRE term $E_{CRE}(\hat{\tilde{\mathbf{u}}}_h, \hat{\tilde{\sigma}}_h)$ of the adjoint problem becomes very low, providing for high-quality error bounds on I at reasonable cost and in a non-intrusive way.

NEW BOUNDING PROCEDURE

The bounding given in (1) uses the Cauchy-Schwarz inequality; it may be a rather coarse bounding, particularly when discretization errors of reference and adjoint problems are concentrated over different domains. This is often the case as the error on the adjoint problem usually concentrates around the area in which the quantity of interest is defined. To avoid coarse boundings, the error on the reference problem is split into two parts: (i) a part E_1 defined over the zone in which the error on the adjoint problem concentrates; (ii) a part E_2 defined over the remainder of the physical domain. Mathematical arguments based on relations among errors defined over homothetic domains enable to relate part E_1 with the error on a larger domain, so that a more precise and still guaranteed bounding can be obtained. Fig. 2 shows comparisons between classical (resp. new) bounds ξ_{inf} and ξ_{sup} (resp. ζ_{inf} and ζ_{sup}) on I with respect to the number of elements used in the adjoint mesh.

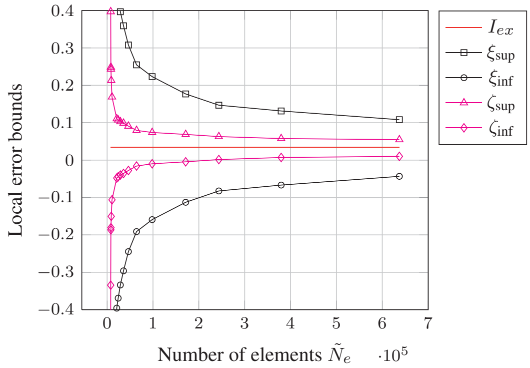

FIGURE 2. COMPARISON BETWEEN CLASSICAL AND NEW BOUNDINGS.

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