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Absolute and convective instabilities of a swirling jet/wake shear layer

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The absolute (AI)/convective (CI) nature of the instability is determined in the family of swirling jet/wake shear layers considered by Martin and Meiburg [Phys. Fluids **6**, 424 (1994)] and Lim and Redekopp [Eur. J. Mech. B/Fluids **17**, 165 (1998)]. This idealized model includes as essential ingredients both the centrifugal instability associated with the swirl difference and the Kelvin–Helmholtz instability associated with the swirl and axial velocity differences between the core and the outer flow. Centrifugally stabilizing or destabilizing swirl differences are found to promote AI, but a centrifugally destabilizing configuration is more effective in triggering such a transition. For sufficiently large swirl differences, both co-flowing jets and wakes may become AI. In the case of *jets*, a centrifugally destabilizing swirl difference first brings about AI via the axisymmetric mode $m=0$ in a large range of mean swirl values. By contrast, a centrifugally stabilizing swirl difference triggers AI via the helical mode $m=-\infty$. In the case of *wakes*, a centrifugally destabilizing swirl difference leads to AI via the bending mode $m=1$ whereas a centrifugally stabilizing swirl difference triggers AI via various negative helical modes $m=-1,-2$, etc. © *2000 American Institute of Physics.* [S1070-6631(00)01102-8]

I. INTRODUCTION

The pervasiveness of vortical flows in nature makes their description and control an important issue in fluid mechanics research. Understanding such phenomena as water-spouts and tornadoes in meteorology, leading-edge and wing-tip vortices in aerodynamics has motivated numerous studies dealing with the structure, stability, and breakdown of swirling flows. In swirling jets and wakes, dynamical modifications may be ascribed to a change in the nature of the instability from convective to absolute, i.e., to the emergence of an instability wave with a zero-group velocity.[1,2] More precisely, such a transition may explain some of the features associated with the onset of vortex breakdown. In this spirit, previous studies have addressed the absolute/convective instability problem for two specific swirling flow models: The Batchelor vortex[3–5] and the Rankine vortex with axial flow.[6] In both instances, the circulation of the base flow monotonically increases with radial distance so that, according to the Rayleigh criterion,[7,8] centrifugal instabilities are quenched. This is not necessarily the case in many practical or experimental situations, i.e., wing-tip vortices[9] or swirling jet experiments,[10] where the total circulation remains zero at large radial distances. In such cases, the circulation decreases with radial distance over part of the flow, a feature which favors the onset of centrifugal instabilities. In the present article, the interaction and competition between axial shear and centrifugal instabilities are qualitatively assessed on the basis of a simple analytical model. We thus determine the absolute/convective instability characteristics of a family of swirling jet/wake shear layers consisting of a straight vortex filament of circulation Γ_c surrounded by a cylindrical vortex sheet of strength $\Gamma_c-\Gamma_\infty$, where Γ_∞ is the external circulation. A plug flow axial jet/wake velocity profile (Fig. 1) is added to this purely swirling cylindrical vortex sheet. According to Hall[11] the azimuthal velocity radial structure of a leading-edge vortex can be approximated as a thin viscous sub-core embedded within an otherwise inviscid irrotational outer flow. The base flow in Fig. 1 should be viewed as a simple canonical model of such physical situations.

Martin and Meiburg[12] have determined the linear temporal instability properties of this family of profiles: For *centrifugally stabilizing* flows ($\Gamma_c-\Gamma_\infty<0$), the instability is of Kelvin–Helmholtz-type, feeding on both azimuthal and axial vorticity, and counter-rotating helical waves ($m<0$) are the most unstable. When the flow is *centrifugally destabilizing* ($\Gamma_c-\Gamma_\infty>0$), the most unstable modes consist of co-rotating helical waves ($m>0$). These properties are in qualitative agreement with physical intuition: Kelvin–Helmholtz instability generates cross-stream vortices with a wave vector (k,m) perpendicular to the local shear, which therefore, take the shape of counter-rotating spirals ($m<0$), whereas centrifugal instability generates locally streamwise vortices spiralling with the flow ($m>0$). It should be made clear that this model does not take into consideration the finite thickness of the cylindrical shear layer: Arbitrarily large streamwise and azimuthal wave numbers remain unstable. Under such conditions, the temporal instability framework does not single out a most amplified mode.

Kang and Lin[13] and Lian and Lin[14] have investigated the spatial instability characteristics of a swirling or nonswirling liquid jet emanating in a nonswirling or swirling gas in order to gain a better understanding of the break-up process. Their velocity fields correspond to the limiting cases $\Gamma_\infty=0$ in Ref.

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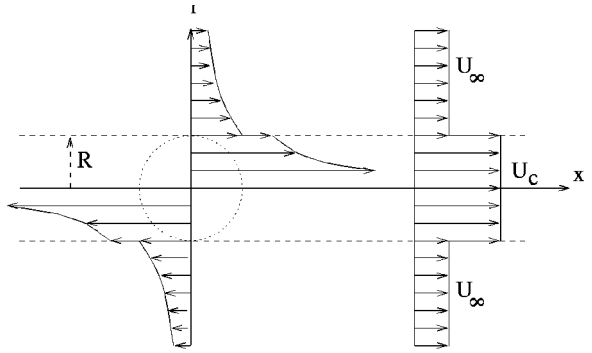


FIG. 1. Sketch of basic flow azimuthal and axial velocity profiles.

13 and $\Gamma_c=0$ in Ref. 14 of the present flow model. A destabilizing (resp. stabilizing) effect of swirl was identified for the centrifugally destabilizing (resp. stabilizing) configuration $\Gamma_\infty=0$ (resp. $\Gamma_c=0$). An increasing gas-to-liquid ratio was determined to be destabilizing in the case of a swirling jet.[13] It was found to be stabilizing or destabilizing in the case of a liquid jet in a swirling gas,[14] depending on the respective magnitudes of the density ratio and Weber number. In the present study we concentrate our attention on the instability properties of a homogeneous fluid without surface tension and density variations.

The absolute/convective instability characteristics of several families of swirling jets and wakes have recently been examined by analytical and numerical means. The centrifugally stabilizing continuous basic flow commonly referred to as the Batchelor vortex is known to provide an approximation of the velocity field within trailing line vortices. Olendraru *et al.*,[3,5] have applied the zero-group velocity criterion of Briggs and Bers[1,2] to the numerically determined dispersion relation of the Batchelor vortex. These issues have also been examined for the same basic flow by direct numerical simulations of the linear impulse response (Delbende, Chomaz, and Huerre[4]). According to these studies, swirl greatly enhances the range of absolute instability to the extent that, for intermediate swirl values, co-flowing jets and wakes become absolutely unstable. The transitional modes are all negative but take distinct values for jets and wakes. The piecewise continuous Rankine vortex with superimposed axial plug flow constitutes an idealized representation of the velocity field prevailing close to the nozzle exit in swirling jets in the absence of a centrifugally destabilizing region. As in the previous case, Loiseleux Chomaz and Huerre[6] have shown that the application of swirl promotes the AI of negative helical modes of increasingly higher order. The absence of a finite shear layer thickness then prevents the quenching of large streamwise and azimuthal wave numbers.

The absolute/convective instability characteristics of two idealized models of centrifugally destabilizing swirling flows have recently been analyzed by Lim and Redekopp.[15] In the first model, a modified Rankine vortex with superimposed axial flow is allowed to exhibit a centrifugally destabilizing azimuthal velocity discontinuity. The increasing magnitude of the discontinuity is then shown to very significantly enhance the absolute growth rate of the axisymmetric mode. Decreasing the core density, with respect to the external density, also contributes to increase the absolute growth rate. The second model is the same as in the present study: In the uniform density case, jets are shown to undergo absolute instability via the $m=0$ mode as the ratio of the external to core swirl is decreased, thereby enhancing the centrifugal instability. Furthermore, decreasing the core density, with respect to the external density, tends to promote AI. When this second model is analyzed in the wake configuration, it is concluded that the negative helical modes are more absolutely unstable than their positive counterparts. It is argued that centrifugally destabilizing swirl promotes AI whereas centrifugally stabilizing swirl inhibits it.

The goal of the present study, which solely focuses on the uniform density case, is to systematically determine the absolute/convective instability boundary of the basic flow displayed in Fig. 1 by locating the absolute/convective transition curves of all positive and negative helical modes. A complete state diagram is obtained for co-flowing or counterflowing jets and wakes and for centrifugally destabilizing or stabilizing swirl differences across the cylindrical vortex sheet. As summarized in the conclusion, the results very significantly extend those obtained by Lim and Redekopp[15] for their second model. The analysis of higher order positive or negative helical modes is crucial to the determination of the absolute/convective instability boundary of the flow and the modes examined by Lim and Redekopp[15] are found not to be the critical ones in several basic flow configurations.

II. PROBLEM FORMULATION AND LINEAR DISPERSION RELATION

In the entire study, the flow is assumed to be inviscid and incompressible. As sketched in Fig. 1, the basic flow under consideration consists of a straight vortex filament aligned along the x axis surrounded by a co-axial cylindrical vortical sheet of radius R, upon which is superimposed a plug flow axial velocity profile. Let U and W denote the axial and azimuthal velocity components in the usual $x-r-\theta$ cylindrical coordinate system. The basic velocity field is then given by

$$U(r)=U_c, \quad W(r)=\frac{\Gamma_c}{2\pi r}, \quad \text{if } r<R, \tag{1a}$$

$$U(r)=U_\infty, \quad W(r)=\frac{\Gamma_\infty}{2\pi r}, \quad \text{if } r>R, \tag{1b}$$

where U_c, Γ_c are the core axial velocity and circulation, and U_∞, Γ_∞ the external axial velocity and circulation. The basic flow effectively depends on three nondimensional control parameters, for instance, the external flow a and the normalized core and external circulations γ_c and γ_∞ defined as

$$a\equiv\frac{U_\infty}{\Delta U}, \quad \gamma_c\equiv\frac{\Gamma_c}{R\Delta U}, \quad \gamma_\infty\equiv\frac{\Gamma_\infty}{R\Delta U}, \tag{2}$$

where $\Delta U=U_c-U_\infty$ represents the axial shear. This set of parameters based on the reference scales ΔU and R is well suited to discuss the effects of rotation on the instability

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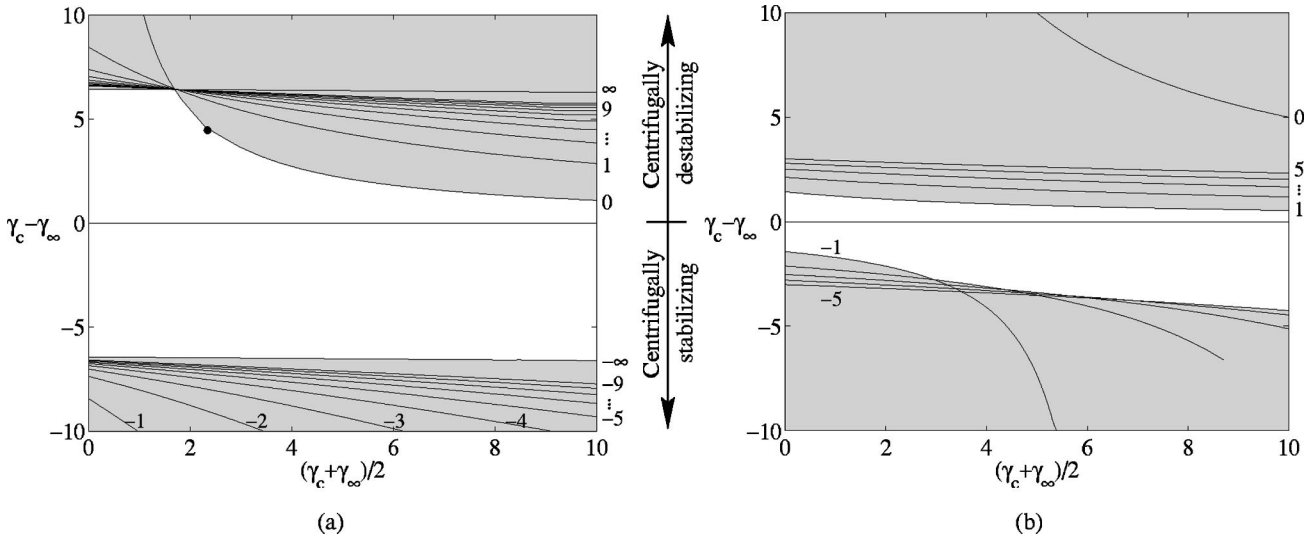


FIG. 2. AI/CI domains in the $(\gamma_c+\gamma_\infty)/2-(\gamma_c-\gamma_\infty)$ plane for (a) a zero-external axial flow jet ($a=0$) and (b) a zero-centerline velocity wake ($a=-1$). The shaded area corresponds to the absolutely unstable region, and the clear area to the convectively unstable region. The line $\gamma_c-\gamma_\infty=0$, separates the centrifugally stabilizing region ($\gamma_c-\gamma_\infty<0$) and the centrifugally destabilizing region ($\gamma_c-\gamma_\infty>0$). The solid dot in (a) corresponds to the AI/CI transition point $\gamma_c=4.65$, $\gamma_\infty=0$ calculated in Ref. 15.

characteristics of plug flow axial jets and wakes. *A contrario*, if one is interested in the effect of axial shear and advection on the instability characteristics of cylindrical azimuthal vortex sheets, it is more appropriate to use as reference scales $\Delta\Gamma=\Gamma_c-\Gamma_\infty$ and R. An alternate set of nondimensional control parameters may then be defined as:

$$\delta\gamma\equiv\frac{\Delta\Gamma}{\Gamma_\infty},\quad \delta u\equiv\frac{R\Delta U}{\Delta\Gamma},\quad \overline{u}\equiv\frac{R(U_c+U_\infty)}{2\Delta\Gamma}. \tag{3}$$

In most of the study, results are presented in terms of the set of parameters (2). The set (3) is briefly used at the end of the paper to illustrate the influence of axial shear δu and advection $\overline{u}$.

As discussed for instance in Loiseleux, Chomaz and Huerre (see Fig. 2 of Ref. 8), distinct axial velocity configurations are obtained for specific ranges of a. The external and core streams are co-flowing when $a<-1$ or $a>0$ and counter-flowing when $-1<a<0$. Jets correspond to $a>-0.5$ and wakes to $a<-0.5$. The particular values $a=0$ and $a=-1$ are associated with the zero external flow jet and zero centerline velocity wake respectively.

Following Martin and Meiburg,[1] the nondimensional linear dispersion relation is readily found to be

$$\begin{aligned}\omega&=\Omega(k;a,\gamma_c,\gamma_\infty,m)\\&=ak+\frac{1}{\alpha-\beta}\left(\frac{m}{2\pi}(\alpha\gamma_c-\beta\gamma_\infty)+k\alpha\right)\\&\quad\pm i\left[\frac{1}{4(\alpha-\beta)}\left(\frac{4mk\alpha\gamma_c}{\pi}+\frac{m^2}{\pi^2}(\alpha\gamma_c^2-\beta\gamma_\infty^2)+4k^2\alpha\right.\right.\\&\quad\left.\left.-\frac{\gamma_\infty^2-\gamma_c^2}{\pi^2}\right)-\frac{1}{(\alpha-\beta)^2}\left(\frac{m}{2\pi}(\alpha\gamma_c-\beta\gamma_\infty)+k\alpha\right)^2\right]^{1/2},\end{aligned} \tag{4}$$

where

$$\alpha(k)\equiv\frac{I_m(k)}{kI_m'(k)}\quad\text{and}\quad\beta(k)\equiv\frac{K_m(k)}{kK_m'(k)}. \tag{5}$$

III. ABSOLUTE/CONVECTIVE INSTABILITY CHARACTERISTICS

As extensively discussed in the literature (see for instance Bers[2] and Huerre and Monkewitz[16]), the absolute/convective nature of the instability is determined by the sign of the absolute growth rate $\omega_{0,i}$, associated with the saddle points of the dispersion relation satisfying

$$\begin{cases}\dfrac{d\Omega}{dk}(k_0;a,\gamma_c,\gamma_\infty,m)=0\\ \omega_0=\Omega(k_0;a,\gamma_c,\gamma_\infty,m).\end{cases} \tag{6}$$

In addition the saddle points should emerge from the pinching of two spatial branches issuing from the upper and lower parts of the complex k-plane. If $\omega_{0,i}>0$ ($\omega_{0,i}<0$) the flow is absolutely unstable (resp. convectively unstable). Details of the numerical procedure used to determine ω_0 may be found in Loiseleux *et al.*[6]

The diagrams in Fig. 2 document the effect of the swirl difference $\gamma_c-\gamma_\infty$ and of the mean swirl $(\gamma_c+\gamma_\infty)/2$ on the absolute/convective nature of the instability in zero-external axial flow jets [$a=0$, Fig. 2(a)] and zero-centerline velocity wakes [$a=-1$, Fig. 2(b)].

Consider first the case $a=0$ [Fig. 2(a)]. In the absence of rotation ($\gamma_c=\gamma_\infty=0$), zero-external axial flow jets are convectively unstable. In order to discuss the effect of swirl, it is convenient to distinguish in the $(\gamma_c+\gamma_\infty)/2-(\gamma_c-\gamma_\infty)$ plane the centrifugally destabilizing and stabilizing domains $\gamma_c-\gamma_\infty>0$ and $\gamma_c-\gamma_\infty<0$, respectively. The application of a sufficiently large centrifugally destabilizing swirl difference $\gamma_c-\gamma_\infty>0$ is seen to induce transition to AI via the axisymmetric mode $m=0$ whenever the mean swirl (γ_c

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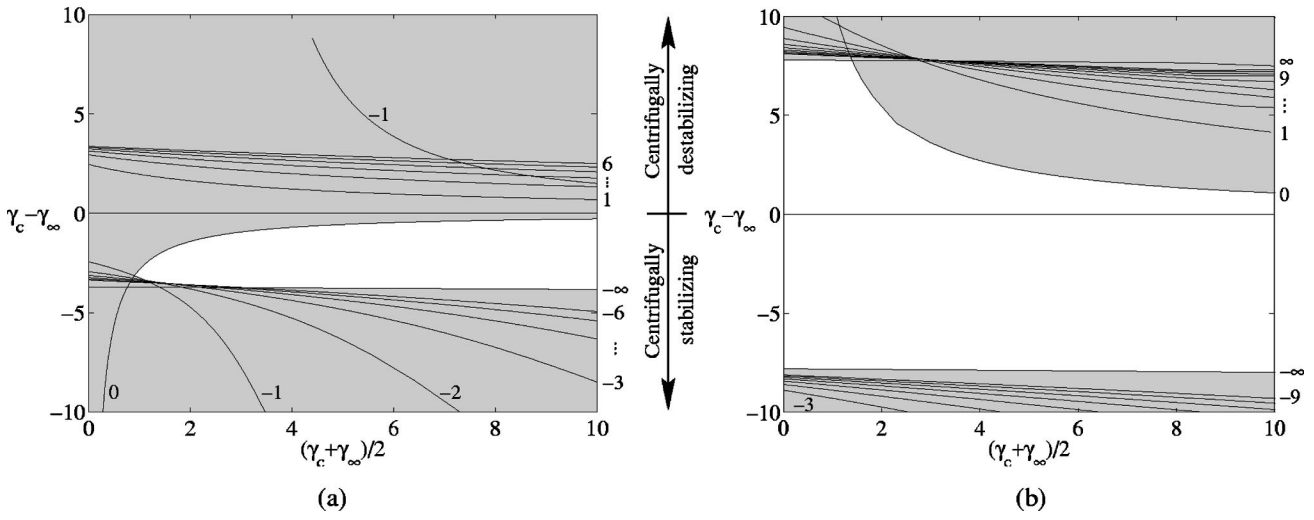


FIG. 3. AI/CI domains in the $(\gamma_c+\gamma_\infty)/2-(\gamma_c-\gamma_\infty)$ plane for (a) an external counterflow jet at $a=-0.2$ and (b) an external co-flow jet at $a=0.1$. The shaded area corresponds to the absolutely unstable region, and the clear area to the convectively unstable region. The line $\gamma_c-\gamma_\infty=0$ separates the centrifugally stabilizing region $(\gamma_c-\gamma_\infty<0)$ and the centrifugally destabilizing region $(\gamma_c-\gamma_\infty>0)$.

$+\gamma_\infty)/2$ exceeds 1.7. When $(\gamma_c+\gamma_\infty)/2$ is lower than 1.7, transition to AI occurs via $m=+\infty$. By contrast, a centrifugally stabilizing swirl difference $\gamma_c-\gamma_\infty<0$ always provokes a transition to AI via the pathological helical mode $m=-\infty$. This unusual feature has been verified numerically: For a fixed $(\gamma_c+\gamma_\infty)/2$, the AI/CI transition value of $\gamma_c-\gamma_\infty$ pertaining to a given negative helical mode m increases continuously towards an asymptotic maximum as m tends to $-\infty$. The locus of such maxima as $(\gamma_c+\gamma_\infty)/2$ is varied constitutes the curve labeled $m=-\infty$ in Fig. 2(a). The numerical value of $m=-\infty$ is taken to be $m=-40$. The same methodology is used to obtain the AI/CI transition curve labeled $m=+\infty$ on the centrifugally destabilizing side $\gamma_c-\gamma_\infty>0$. According to Fig. 2(a), a centrifugally destabilizing swirl difference is more efficient in triggering a transition to AI than a centrifugally stabilizing swirl difference. More importantly, a centrifugally destabilizing swirl difference clearly promotes *axisymmetric* or *positive helical* modes, whereas a centrifugally stabilizing swirl difference promotes *negative* helical modes of gradually decreasing $|m|$. The AI/CI transition point $(\gamma_c=4.65;\ \gamma_\infty=0)$ for the $m=0$ mode in Fig. 2(a) is in satisfactory agreement with the result $\gamma_c=2\pi\times0.74=4.62$ obtained by Lim and Redekopp[2] at $\gamma_\infty=0$.

Consider next the case $a=-1$ [Fig. 2(b)]. In the absence of rotation $(\gamma_c=\gamma_\infty=0)$, the zero-centerline velocity wake is CI. In this instance, a moderately large centrifugally destabilizing swirl difference induces a transition to AI via the *positive* bending mode $m=1$. A centrifugally stabilizing swirl difference clearly promotes *negative* helical modes but the critical transitional mode is sensitive to the value of $(\gamma_c+\gamma_\infty)/2$. As in the previous case, a centrifugally destabilizing swirl difference is more effective in promoting AI. At first sight, the present results for $\gamma_c-\gamma_\infty>0$ do not corroborate those of Lim and Redekopp[15] for $\gamma_c>0$ and $\gamma_\infty=0$: Negative helical modes are found by these authors to be critical whereas the present study clearly singles out positive helical modes. Such a discrepancy is more apparent than real: It is due to the fact that the swirl parameter S_w introduced in Ref. 15 is related to the parameters of the present study by $S_w=\gamma_c/2\pi a$. When $a=-1$, $\gamma_c>0$ corresponds to $S_w<0$ and consequently m is changed into $-m$. Furthermore, the curves $m=-1$ and $m=-2$ in Fig. 17 of Ref. 15 should probably be switched. If such substitutions are made, there is both quantitative and qualitative agreement between both studies.

It is worth pointing out the peculiar features which arise along the line $\gamma_c-\gamma_\infty=0$ separating the centrifugally destabilizing and stabilizing domains. Such a configuration corresponds to a single vortex filament embedded in an axial plug flow. The dispersion relation (4) then reduces to

$$\omega=ak+\frac{m\gamma}{2\pi}+k\frac{\alpha\pm\sqrt{\alpha\beta}}{\alpha-\beta}, \tag{7}$$

where $\gamma_c=\gamma_\infty=\gamma$. As a result, the absolute wave number k_0 and the absolute growth rate $\omega_{0,i}$ do not depend on γ. The AI/CI transition values of a are also γ-independent and equal to those prevailing at $\gamma=0$, namely $a_c=-0.15$, $m=0$ on the jet side and $a_c=-0.91$, $|m|=1$ on the wake side.[6] At these specific values of the external axial flow parameter a, AI/CI transition takes place simultaneously along the line $\gamma_c-\gamma_\infty=0$. In other words, a single vortex filament of strength γ does not alter the AI/CI characteristics of plug flow velocity jets or wakes.

A typical external counter-flow jet case $(-0.5<a<0)$ is illustrated in Fig. 3(a). When the counterflow $|a|$ is gradually increased, the CI-domain shrinks to a narrow band on the centrifugally stabilizing side $\gamma_c-\gamma_\infty<0$, and the flow becomes AI over most of the parameter plane. A typical external co-flow jet case $(a>0)$ is illustrated in Fig. 3(b). When the co-flow a is gradually increased, the CI-domain expands and occupies most of the control parameter plane. The same qualitative tendencies hold for counter-flow and co-flow wakes.

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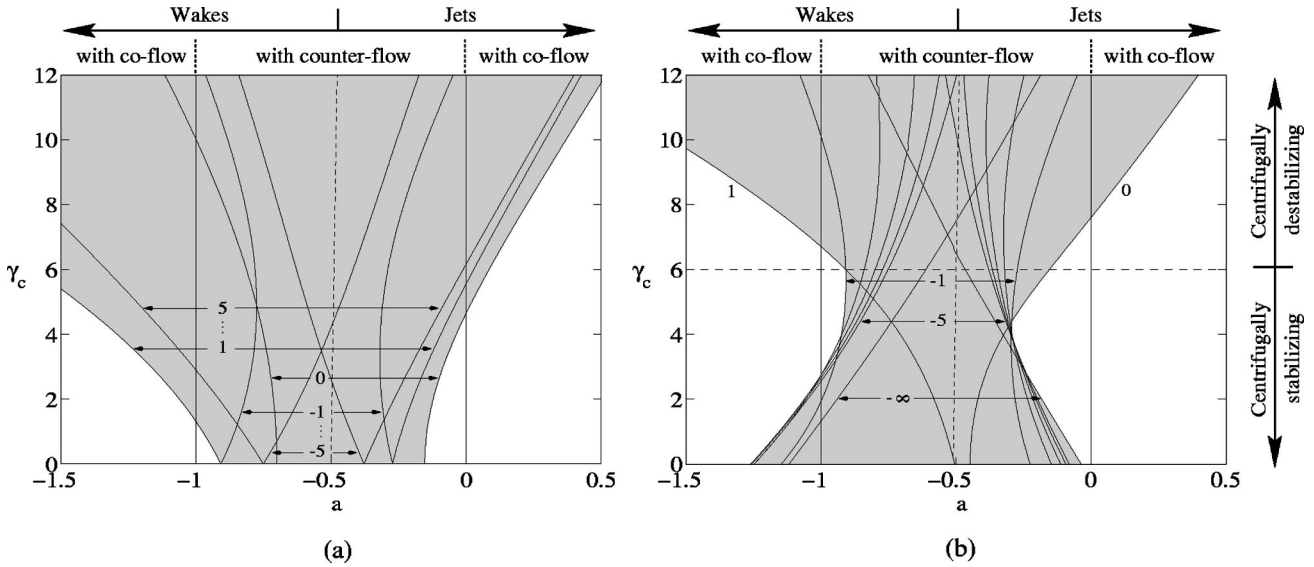


FIG. 4. AI/CI domains in the $a-\gamma_c$ plane for (a) a zero-external swirl ($\gamma_\infty=0$) and (b) a finite external swirl ($\gamma_\infty=6$). The shaded area corresponds to the absolutely unstable region, and the clear area to the convectively unstable region. The dotted line $\gamma_c=\gamma_\infty$ separates the centrifugally stabilizing and destabilizing regions. For clarity, in (b), only the helical modes participating in the delineation of the AI/CI regions have been represented.

To compare the present results with those pertaining to the Rankine vortex with axial flow (Ref. 6), it is enlightening to display the AI/CI domains at a fixed setting of γ_∞ as shown in Fig. 4. Such diagrams are the equivalent of Fig. 20 in Ref. 6. In the absence of external swirl [$\gamma_\infty=0$, Fig. 4(a)], the effect of a finite γ_c is always centrifugally destabilizing. The a-interval of AI is seen to increase continuously with the core swirl γ_c. The transitional modes $m=0$ and $m=1$ are clearly in evidence on the jet and wake sides, respectively. For a finite external swirl [$\gamma_\infty=6$, Fig. 4(b)], the effect of a finite γ_c is centrifugally stabilizing below $\gamma_c=\gamma_\infty$ and centrifugally destabilizing above $\gamma_c=\gamma_\infty$. However, in both cases, the a-interval of AI expands with increasing swirl difference $|\gamma_c-\gamma_\infty|$. Furthermore, in the centrifugally destabilizing domain $\gamma_c>\gamma_\infty$, the AI range is bounded by the $m=0$ and $m=1$ modes on the jet and wake sides, respectively. In the centrifugally stabilizing domain $\gamma_c<\gamma_\infty$, various negative helical waves partake in the delineation of the AI domain.

Finally, the effect of axial shear and advection on the instability characteristics of purely swirling vortex sheets is more clearly brought out by considering the alternate set of control parameters defined in Eq. (3). Two typical state diagrams in the $\bar{u}-\delta u$ plane are illustrated in Fig. 5 for different centrifugally destabilizing settings of $\delta\gamma$. Consider first $\delta\gamma=1.5$ [Fig. 5(a)]. When $\delta u=0$, there is a narrow AI interval of $\bar{u}$ necessarily associated with finite counterflow. As axial shear δu is increased, this AI-interval continuously widens: Larger magnitudes of the advection parameter $\bar{u}$ are then required to return to CI. When $\delta\gamma=0.01$ [Fig. 5(b)], the

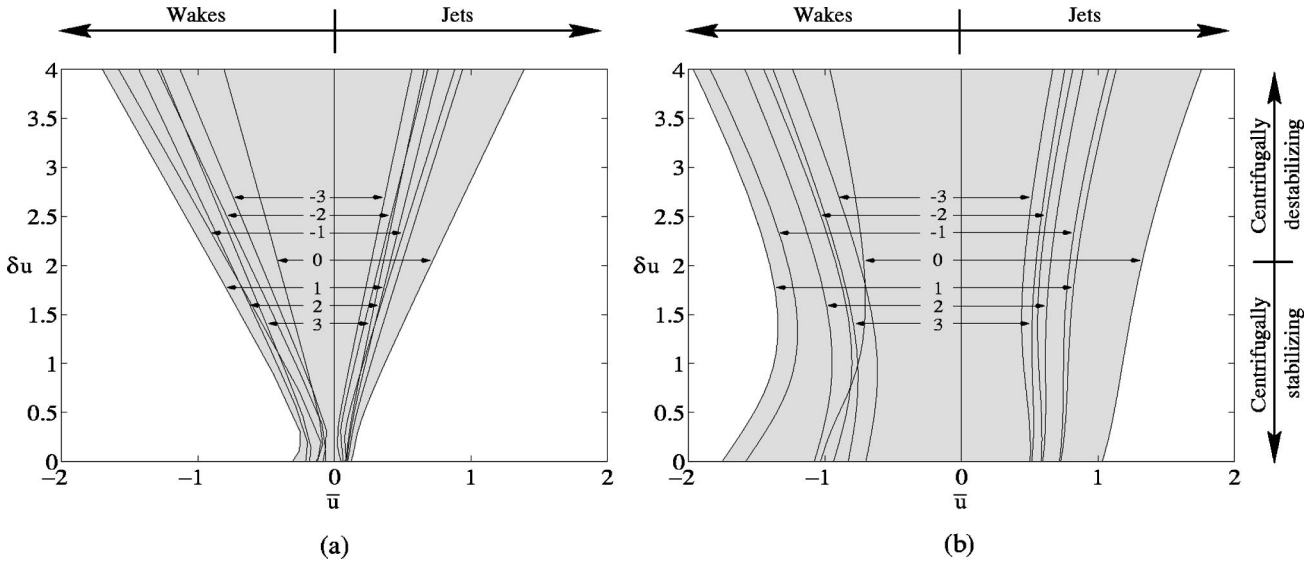


FIG. 5. AI/CI domains in the $\bar{u}-\delta u$ plane for (a) $\delta\gamma=1.5$ and (b) $\delta\gamma=0.01$. The shaded area corresponds to the absolute unstable region, and the clear area to the convectively unstable region.

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basic flow is only marginally centrifugally destabilizing but the AI-domain extends over a wider range of $\overline{u}$. The same qualitative tendencies are observed in both cases: The critical transitional modes are $m=0$ and $m=1$ on the jet and wake sides, respectively.

IV. CONCLUDING REMARKS

These results complement those of Lim and Redekopp[15] in several important respects: A systematic study of the AI/CI transition curves pertaining to all helical modes has been conducted for both jets *and* wakes and for both centrifugally destabilizing *and* stabilizing swirl distributions.

In the case of *jets* ($a>-0.5$), both investigations find that the axisymmetric mode $m=0$ first becomes AI for centrifugally destabilizing swirl configurations. The present analysis further establishes that $m=0$ is replaced by $m=+\infty$ at low mean swirl $(\gamma_c+\gamma_\infty)/2$. Furthermore, this study demonstrates that centrifugally stabilizing swirl triggers a transition to AI via $m=-\infty$.

In the case of *wakes* ($a<-0.5$), Lim and Redekopp[15] find that a centrifugally destabilizing swirl induces AI via the $m=-2$ retrograde mode, whereas we determine that AI first takes place through the $m=+1$ cograde mode, as discussed in Sec. III. Furthermore, the present study shows that centrifugally stabilizing swirl leads to AI via various negative helical modes.

In Ref. 15, the onset of AI in swirling flows is primarily related to the vortex core size: ''The more concentrated the vortical core of the swirling flow compared to the jet diameter, the greater the tendency toward absolute instability'' (p. 184 of Ref. 15). In the present context, we prefer to relate the onset of AI to the centrifugally stabilizing or destabilizing nature of the swirl distribution. More specifically, it has been found illuminating to choose as pertinent control parameters the swirl difference $\gamma_c-\gamma_\infty$, the sign of which determines centrifugal instability or stability, and the mean swirl $(\gamma_c+\gamma_\infty)/2$. As demonstrated in Fig. 4(b) *AI is promoted by increasing* $|\gamma_c-\gamma_\infty|$ *on both the centrifugally stabilizing and destabilizing sides. But the centrifugally stabilizing or destabilizing nature of the swirl, i.e., the sign of* $\gamma_c-\gamma_\infty$, *leads to radically distinct transitional helical modes*.

According to Fig. 4(b), rotation whether centrifugally stabilizing or destabilizing leads to a widening of the AI region, to the extent that co-flow jets ($a>0$) and wakes ($a<-1$) may become AI for sufficiently large $|\gamma_c-\gamma_\infty|$. Thus, the same general tendency is observed as in the case of a solid body rotation, for instance the centrifugally stabilizing Rankine vortex (Fig. 20 of Ref. 6) or the centrifugally stabilizing Batchelor vortex (Fig. 12 of Ref. 4 and Fig. 7 of Ref. 5). Note, however, that the critical transitional modes are radically different. In the *centrifugally stabilizing* region $\gamma_c-\gamma_\infty<0$ of Fig. 4(b), AI/CI transition is dominated by *negative helical* modes ($m=-1$, $m=-2$, $m=-3,\ldots$) on both the jet and wake sides, as in the Rankine and Batchelor vortices. In the *centrifugally destabilizing* case [$\gamma_c-\gamma_\infty>0$ of Fig. 4(b)], AI/CI transition solely involves the *axisymmetric* mode $m=0$ on the jet side and the *bending* mode $m=1$ on the wake side.

The present study further confirms that, in contrast with purely temporal analyses, AI/CI transition criteria lead to the selection of specific helical modes in simple swirling flow profiles. The peculiar $m=-\infty$ transitional mode on the centrifugally stabilizing jet side [$\gamma_c-\gamma_\infty<0$, $a>-0.5$ in Fig. 4(b)] is likely to be quenched for more realistic basic flow distributions exhibiting a finite shear layer thickness.

The broadening of the AI region and the triggering of an absolute instability above a critical swirl level is a common robust feature of all swirling flow models that have so far been investigated, whether centrifugally stabilizing or not. In the same way, vortex breakdown is also observed in a wide variety of configurations (leading-edge and wing-tip vortices, swirling jets, etc.) but a direct causal link with absolute instability remains to be established on experimentally measured velocity profiles.

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