

Comparative analysis of stochastic and non-stochastic methods for the evaluation of uncertainties in simulated and experimental data

Sami Daouk, François Louf, Laurent Champaney, Olivier Dorival

► To cite this version:

Sami Daouk, François Louf, Laurent Champaney, Olivier Dorival. Comparative analysis of stochastic and non-stochastic methods for the evaluation of uncertainties in simulated and experimental data. XIXth Symposium Vibrations, SHocks and NOise, VISHNO 2014, 2014, Aix-en-Provence, France. hal-01023549

HAL Id: hal-01023549

<https://hal.archives-ouvertes.fr/hal-01023549>

Submitted on 14 Jul 2014

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Comparative analysis of stochastic and non-stochastic methods for the evaluation of uncertainties in simulated and experimental data

Sami Daouk^{1*}, François Louf¹, Laurent Champaney², Olivier Dorival³

¹ LMT-Cachan, (ENS Cachan / Université Paris 6 / C.N.R.S.), 61, avenue du Président Wilson, 94230 Cachan, France

² Ecole Nationale Supérieure d'Arts et Métiers, 151 Bd de l'Hôpital, 75013 Paris, France

³ Université de Toulouse; Institut Clément Ader (ICA); INSA, UPS, Mines Albi, ISAE, 135 av. de Rangueil, 31077 Toulouse Cedex

Short Abstract.

Model validation of real structures remains a major issue, not only because of their complexity but also due to their uncertain behavior. Thus, in order to build accurate models and take uncertainties into account, many numerical stochastic and non-stochastic methods have been developed. This work aims to provide engineers with useful information on modeling data uncertainties in structural mechanics. It starts with a state-of-the-art review of some widely used techniques, followed by a presentation of the Lack-Of-Knowledge (LOK) theory. The study of a case representative of a booster pump of thermal units shows the ability of the LOK theory to assess the propagation of numerous small and large uncertainties in simulated and experimental data.

Keywords. *Uncertainty propagation, Structural dynamics, Lack of knowledge, Large uncertainties*

EXTENDED ABSTRACT

Industrial structures are mainly assemblies of many parts with complex geometries and non-linear characteristics. Friction and prestress in joints added to fabrication imperfections lead to a substantial gap between numerical models and real structures. Mechanical systems are commonly analyzed assuming that the mathematical models are deterministic and the input data is precisely defined. Nevertheless, in most cases, parameters of the mathematical-mechanical model linked to geometry, boundary conditions and material properties can neither be identified nor modeled accurately. The need to address data uncertainties is now clearly recognized, and over the past three decades there has been a growing interest in stochastic modeling and application of probabilistic methods [1].

In order to evaluate the random response of a model with uncertain parameters, a first approach is based on the study of the deterministic problem. The Monte Carlo method is the simplest method to calculate the response variability of a structure but its convergence is time-consuming. Therefore, alternative methods yielding faster and accurate results were developed. The combination of finite elements with probabilities resulted in the development of the stochastic finite element method (SFEM). This approach considers randomness as one of the problem dimension and adds it to the spatial variables resulting from the FE discretization. A first variant is the "perturbation method", accurate in the case of small uncertainties, where random fields are expanded using a Taylor series around their mean value.

Accurate representation of real structures usually leads to a huge number of degrees of freedom. For the purpose of reducing computation costs, an interesting numerical approach is to simplify or reduce the studied model itself. Model reduction techniques aim to approximate the response of a complex model by the response of a surrogate model that is built through the projection of the initial complex model on a low-dimensional reduced-order basis. The differences between reduction methods are in the way the reduced basis is defined. Techniques referred to as *a posteriori* require a first evaluation of the solution of the reference problem in order to build the basis up, such as the Proper Orthogonal Decomposition and the Subspace Reduction Method. The problem under study is firstly solved for few values of the uncertain parameters. These solutions, called "snapshots", constitute the basis vectors on which the original problem will be projected. A statistical approximation of the solution can be then obtained by solving the reduced problem for many values of the uncertain parameters. Otherwise the technique is referred to as *a priori*, where the solution is built through the resolution of a set of separated problems, dependent of the uncertain parameters and the spatial variables.

All these techniques are used to analyze uncertainty propagation in the case of system parameters that can be treated as random variables following known probability laws. Nevertheless, uncertainties are sometimes due to imprecise information, and that's why non-stochastic approaches were developed. The first intuitive non-stochastic method describes uncertainties with their range of variation, and is known as the interval theory. Even though this description is less precise than a stochastic approach, it is sufficient in many cases where the engineer's interest lies in identifying the bounds of the interval in which the output of interest varies. However, the major drawback of the interval theory is that it does not take into account the dependency of occurrences of each random variable. The interval propagation could be very pessimistic, which leads to an overestimation of the bounds. During the last years, efforts have been made to develop a method taking into account different sources of uncertainties. The "Lack-Of-Knowledge" (LOK) theory has been proposed to address the structural uncertainties in an ingenious way, by combining the advantages of stochastic and non-stochastic methods [2]. The concept of lack-of-knowledge is based on the idea of globalizing all sources of uncertainty for each substructure through scalar parameters (named LOK variables) that belong to an interval whose boundaries are random variables. This

*Corresponding author.

approach can be seen as an extension of the interval theory where the endpoints are random variables, which reduces the overestimation of the uncertainty propagation. It does not only qualify, but also quantifies the difference between numerical models and real structures.

After a brief overview of the theoretical background and then validating the implementation on a simple academic example, the Monte Carlo (MC) method, the perturbation method (PSFEM), the subspace reduction method (SRM) and the LOK theory were used to model low uncertainty propagation in the dynamical study of a 3D assembly. The model was inspired from the geometry of the booster pump studied in the framework of the international benchmark SICODYN [3]. Figure 1 shows the shapes of the first three eigenmodes for the deterministic reference studied structure.

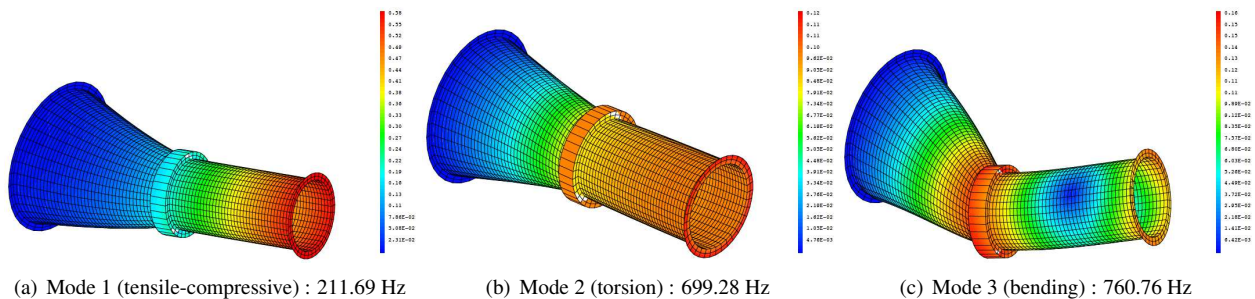


Figure 1. The shapes of the first three eigenmodes of the deterministic 3D assembly

The material stiffness was considered as an uncertain parameter of uncertainty. The Young's modulus of the material of each substructure was assumed to be a random quantity such as $E = \bar{E} (1 + \delta \eta)$ with $\delta = 0.08$ and η is a uniform random variable defined in the interval $[-1; +1]$. For 20000 samples of Young's moduli, Table 1 shows a comparison of the methods in terms of quality of results (MC taken as the reference), intrusiveness, computation time and extensibility to high values of uncertainties, where 1 is attributed to the best case and 4 to the worst.

Table 1. Comparison in terms of quality, intrusiveness, computation time and extensibility to large uncertainties

Method	Quality	Intrusiveness	Computation time	Extensibility
MC	–	1	4 (51h)	1
PSFEM	$[-3.8 ; +18.1].10^{-2}\%$	4	2 (20s)	4
SRM	$[+4.7 ; +8.6].10^{-2}\%$	2	3 (10h)	1
LOK	$[+10.0 ; -32.6].10^{-2}\%$	2	1 (14s)	2

Focusing on the LOK theory, it seems to be conservative inducing higher precision. Unlike the other uncertainty propagation methods, this modeling technique only provides a stochastic interval that bounds the response without any further information about it. It is considerably quick and can be extended to high values of uncertainties without a great computation time cost. Its main advantage is the globalization of all sources of uncertainties, related to data and model, which reveals to be very handy for modeling real industrial assemblies [4].

In the framework of the SICODYN Project [5], initiated in 2012 and to be carried out till 2015, these methods will be evaluated in the dynamical analysis of a one-stage booster pump of thermal units studied within its industrial environment. One of the main goals is to evaluate the ability of the LOK theory, to quantify, not only data (parameter) uncertainties, but also model uncertainties, in the cases of low and high values of uncertainties.

REFERENCES

- [1] G. Schuëller, A state-of-the-art report on computational stochastic mechanics, Probabilistic Engineering Mechanics 12 (1997) 197–321.
- [2] P. Ladevèze, G. Puel, T. Romeuf, Lack of knowledge in structural model validation, Computer Methods in Applied Mechanics and Engineering 195 (2006) 4697–4710.
- [3] S. Audebert, SICODYN international benchmark on dynamic analysis of structure assemblies: variability and numerical-experimental correlation on an industrial pump, Mechanics & Industry 11(6) (2010) 439–451.
- [4] S. Daouk, F. Louf, O. Dorival, L. Champaney, S. Audebert, Modelling of uncertainties on industrial structures : a review of basic stochastic and other methods, and application to simple and complex structures, Mechanics & Industry (Submitted in 2013).
- [5] S. Audebert, F. Fall-Lo, Uncertainty Analysis on a Pump Assembly using Component Mode Synthesis, in: Proceedings of 4th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, COMPDYN, Kos Island, Greece, 2013.