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To cite this version:
Lu Wang, Laurent Albera, Amar Kachenoura, Huazhong Shu, Lotfi Senhadji. CP decomposition of semi-nonnegative semisymmetric tensors based on QR matrix factorization. The eighth IEEE Sensor Array and Multi-Channel Signal Processing Workshop, Jun 2014, A Coruna, Spain. 4 p., 2014. <hal-01012125>

HAL Id: hal-01012125
https://hal.archives-ouvertes.fr/hal-01012125
Submitted on 25 Jun 2014

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CP decomposition of semi-nonnegative semi-symmetric tensors based on QR matrix factorization

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Abstract—The problem of Canonical Polyadic (CP) decomposition of semi-nonnegative semi-symmetric three-way arrays is often encountered in Independent Component Analysis (ICA), where the cumulant of a nonnegative mixing process is frequently involved, such as the Magnetic Resonance Spectroscopy (MRS). We propose a new method, called JD\textsubscript{QR}, to solve such a problem. The nonnegativity constraint is imposed by means of a square change of variable. Then the high-dimensional optimization problem is decomposed into several sequential rational subproblems using QR matrix factorization. A numerical experiment on simulated arrays emphasizes its good performance. A BSS application on MRS data confirms the validity and improvement of the proposed method.

I. INTRODUCTION AND PROBLEM FORMULATION

Canonical Polyadic (CP) decomposition of a multi-way array [1]–[3] plays an important role in Blind Source Separation (BSS), particularly in Independent Component Analysis (ICA) [4]. In this paper, we consider the following semi-nonnegative semi-symmetric CP decomposition problem:

Problem 1. The semi-nonnegative semi-symmetric CP decomposition of a 3-way array \( \mathbf{C} \in \mathbb{R}^{N\times N\times K} \), is the minimal linear combination of rank-1 3-way arrays that yields \( \mathbf{C} \) exactly:

\[
\mathbf{C} = \sum_{p=1}^{P} \mathbf{a}_p \circ \mathbf{a}_p \circ \mathbf{d}_p
\]

subject to \( \mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_P] \in \mathbb{R}^{N\times P} \) having nonnegative components, where \( \circ \) denotes the outer product. \( \mathbf{A} \) and \( \mathbf{D} = [\mathbf{d}_1, \cdots, \mathbf{d}_P] \in \mathbb{R}^{K\times P} \) are called the loading matrices of \( \mathbf{C} \). \( P \) is then the rank of \( \mathbf{C} \).

The decomposition is considered to be essentially unique when the uniqueness is guaranteed up to scaling and permutation indeterminacies. This problem is often encountered in ICA when a nonnegative mixing matrix is considered. For example, in Magnetic Resonance Spectroscopy (MRS), the mixing matrix contains the positive concentrations of the source metabolites. Then the 3-way array built by stacking the matrix slices of a cumulant is both nonnegative and symmetric in two modes. Equation (1) can also be described by using the frontal slices of \( \mathbf{C} \): \( \mathbf{C}^{(k)} = \mathcal{F}_{:,k} = \mathbf{A} \mathbf{D}^{(k)} \mathbf{A}^T, \forall k \in \{1, 2, \cdots, K\} \), where \( \mathbf{D}^{(k)} \in \mathbb{R}^{P\times P} \) is a diagonal matrix whose diagonal contains the elements of the \( k \)-th row of \( \mathbf{D} \), and \( \mathbf{C}^{(k)} \in \mathbb{R}^{N\times N} \) is the \( k \)-th frontal slice \( \mathbf{C} \). In this paper, we focus on computing the square matrix \( \mathbf{A} \), where \( N = P \). In order to compute \( \mathbf{A} \), we can resort to solve the following nonnegative Joint Diagonalization by Congruence (JDC) problem:

Problem 2. Given a 3-way array \( \mathbf{C} \in \mathbb{R}^{N\times N\times K} \) with \( K \) symmetric frontal slices \( \mathbf{C}^{(k)} \in \mathbb{R}^{N\times N} \), find a matrix \( \mathbf{A} \in \mathbb{R}^{N\times N} \) and \( K \) diagonal matrices \( \mathbf{D}^{(k)} \in \mathbb{R}^{N\times N} \) such that:

\[
\forall k \in \{1, 2, \cdots, K\}, \quad \mathbf{C}^{(k)} = \mathbf{A} \mathbf{D}^{(k)} \mathbf{A}^T
\]

subject to \( \mathbf{A} \) having nonnegative components.

Many existing CP algorithms handle the symmetry and the nonnegativity separately, such as in [5]–[7]. Several methods consider the combination of both constrains [8], [9], but they aim at solving different problems rather than problem 1. Only a few methods were proposed to solve the nonnegative JDC problem [10], [11]. In this paper, we propose a new algorithm, called JD\textsubscript{QR}, based on minimizing the following indirect least square criterion [6], [12]:

\[
J_1(\mathbf{A}) = \sum_{k=1}^{K} \| \text{off}(\mathbf{A}^{-1} \mathbf{C}^{(k)} \mathbf{A}^{-T}) \|_F^2
\]

where \( \text{off}(\cdot) \) vanishes the diagonal components of the input matrix, the superscript \( ^{-T} \) denotes the inverse of the transposed matrix, and \( \| \cdot \|_F \) computes the Frobenius norm. The nonnegativity constraint is imposed by means of a square change of variable. The QR matrix factorization of the Hadamard square root of \( \mathbf{A} \) decomposes the high-dimensional optimization problem into a sequential rational subproblems. In addition, the rotation matrix and the unit triangular matrix of the QR factorization have unit determinants, therefore the resulting matrix \( \mathbf{A} \) is nonsingular. A numerical experiment on simulated arrays emphasizes its good performance. A BSS application on MRS data confirms the validity and improvement of the proposed method.

II. THE JD\textsubscript{QR} METHOD

In order to avoid the inverse of \( \mathbf{A} \) in cost function (3), let us consider the following assumptions: i) \( \mathbf{A} \in \mathbb{R}^+_{\times N} \) is nonsingular; ii) \( \mathbf{D} \in \mathbb{R}^{K\times N} \) is nonsingular and does not contain zero entries. Then each frontal slice of \( \mathbf{C} \) is nonsingular and its inverse can be expressed as follows:

\[
(\mathbf{C}^{(k)})^{-1} = \mathbf{A}^{-T} (\mathbf{D}^{(k)})^{-1} \mathbf{A}^{-1}
\]

In practice, only the sufficiently well-conditioned matrix \( \mathbf{C}^{(k)} \) is chosen when its condition number is below a predefined
We use $C^{(k-1)}$ to denote $(C^{(k)})^{-1}$ for simplicity. Equation (4) shows that $C^{(k-1)}$ is jointly diagonalizable by $A$. Then $A$ can be estimated by minimizing the following modified criterion of (3) directly:

$$J_2(A) = \sum_{k=1}^{K} \| \text{off}(A^T C^{(k-1)} A) \|^2_F.$$  

(5)

The nonnegativity constraint on $A$ can be imposed by a square change of variable: $A = B \boxtimes B = B^{\boxtimes 2}$, where $B \in \mathbb{R}^{N \times N}$ and where $\boxtimes$ denotes Hadamard product [13, 14]. Then we can find $A \in \mathbb{R}^{N \times N}$ by estimating $B \in \mathbb{R}^{N \times N}$, such that $A = B^{\boxtimes 2}$, and $B$ is the global minimum of the following cost function:

$$J_2(B) = \sum_{k=1}^{K} \| \text{off}(B^{\boxtimes 2}C^{(k-1)} B^{\boxtimes 2}) \|^2_F.$$  

(6)

Now let us recall the following definitions and lemmas:

**Definition 1.** A unit upper triangular matrix is an upper triangular matrix whose main diagonal entries are $1$.

**Definition 2.** An elementary upper triangular matrix $R^{(i,j)}(r_{i,j})$ is equal to an identity matrix except the $(i,j)$-th entry, which is equal to $r_{i,j}$.

**Definition 3.** A Givens rotation matrix $Q^{(i,j)}(\theta_{i,j})$ is equal to an identity matrix except the $(i,i)$-th, $(j,j)$-th, $(i,j)$-th and $(j,i)$-th entries, which are equal to $\cos(\theta_{i,j}), \cos(\theta_{j,i}), -\sin(\theta_{i,j})$, and $\sin(\theta_{j,i})$, respectively.

**Lemma 1.** Any $(N \times N)$ unit upper triangular matrix can be factorized as a product of $N(N-1)/2$ elementary upper triangular matrices.

**Lemma 2.** Any $(N \times N)$ orthonormal matrix can be factorized as a product of, at most, $N(N-1)/2$ Givens rotation matrices.

For any nonsingular matrix $B \in \mathbb{R}^{N \times N}$, the QR matrix factorization decomposes it as $B = QRA$, where $Q \in \mathbb{R}^{N \times N}$ is an orthonormal matrix, $R \in \mathbb{R}^{N \times N}$ is a unit upper triangular matrix, and $A \in \mathbb{R}^{N \times N}$ is a diagonal matrix. Due to the indeterminacies of the CP decomposition, the matrix $B$ solving (6) can be chosen as $B = QR$ without loss of generality. Moreover, lemma 1 and lemma 2 yield that $B$ can be written as a product of the following matrices:

$$B = \prod_{i=1}^{N} \prod_{j=i+1}^{N} Q^{(i,j)}(\theta_{i,j}) \prod_{i=1}^{N} \prod_{j=i+1}^{N} R^{(i,j)}(r_{i,j}).$$  

(7)

As a consequence, the minimization of (6) with respect to $B$ is converted to the estimation of $N(N-1)$ parameters: $\theta_{i,j}$ and $r_{i,j}$. We propose a Jacobi-like procedure, called JD$_{QR}$, in order to compute these parameters sequentially.

**A. Minimization with respect to $Q^{(i,j)}(\theta_{i,j})$**

Let $\hat{A}$ and $\hat{B}$ denote the current estimate of $A$ and $B$ before estimating $Q^{(i,j)}(\theta_{i,j})$, respectively. Let $\hat{A}^{(\text{new})}$ and $\hat{B}^{(\text{new})}$ stand for $\hat{A}$ and $\hat{B}$ updated by $Q^{(i,j)}(\theta_{i,j})$, respectively. Furthermore, the update of $B$ is defined as follows:

$$\hat{B}^{(\text{new})} = BQ^{(i,j)}(\theta_{i,j}).$$  

(8)

In order to compute $\theta_{i,j}$, the natural way is to minimize criterion (6) with respect to $\theta_{i,j}$ by replacing matrix $B$ by $\hat{B}^{(\text{new})}$. For the sake of convenience, we denote $J_2(\theta_{i,j})$ instead of $J_2(\hat{B}^{(\text{new})})$. $J_2(\theta_{i,j})$ can be expressed as follows:

$$J_2(\theta_{i,j}) = \sum_{k=1}^{K} \text{off} \left( \| \text{off}(\hat{B}^{(\text{new})})^{T}C^{(k-1)}(\hat{B}^{(\text{new})})^{T} \|_F^2 \right).$$  

(9)

The Hadamard square of $\hat{B}^{(\text{new})}$ in (9) can be written as a function of $\theta_{i,j}$ as follows:

$$(\hat{B}^{(\text{new})})^{\boxtimes 2} = \hat{B}^{\boxtimes 2}(Q^{(i,j)}(\theta_{i,j}))^{\boxtimes 2} + \sin(2\theta)(\hat{b}_i \boxtimes \hat{b}_j)(e_i^T - e_j^T)$$  

(10)

where $\hat{b}_i$ denotes the $i$-th column of $\hat{B}$, and $e_i$ is the $i$-th column of the identity matrix $I \in \mathbb{R}^{N \times N}$. Inserting (10) into the cost function (9), we obtain:

$$J_2(\theta_{i,j}) = \sum_{k=1}^{K} \text{off} \left( \| \text{off}(\hat{C}^{(\text{new})})^{T}C^{(k-1)}(\hat{C}^{(\text{new})})^{T} \|_F^2 \right).$$  

$$= \sum_{k=1}^{K} \text{off} \left( \| (\hat{Q}^{(i,j)}(\theta_{i,j}))^{\boxtimes 2}C^{(k-1)}(\hat{Q}^{(i,j)}(\theta_{i,j}))^{\boxtimes 2} + \sin(2\theta) (\hat{Q}^{(i,j)}(\theta_{i,j}))^{\boxtimes 2}T C^{(k-1)}(e_i^T - e_j^T) \right).$$  

(11)

where $\hat{C}^{(k)}(\theta_{i,j}) = \hat{A}^{\boxtimes 2}C^{(k-1)}(\hat{A}^{\boxtimes 2})^{T}$ and $\hat{C}^{(k,2)} = (\hat{b}_i \boxtimes \hat{b}_j)C^{(k-1)}(\hat{b}_i \boxtimes \hat{b}_j)^T$. $\theta_{i,j}$ is the $(i,j)$-th element of $\hat{C}^{(\text{new})}$, instead of minimizing all the off-diagonal entries. This simplified minimization criterion is denoted by $J_2^{(\text{new})}(\theta_{i,j})$. The $(i,j)$-th element of $\hat{C}^{(\text{new})}$ can be expressed as a function of $\theta_{i,j}$ as follows:

$$\hat{C}^{(\text{new})}(\theta_{i,j}) = -\sin^2(2\theta)(\hat{c}_{i,k})^2 + \sin^2(\theta)(\hat{c}_{i,k}^2 \cos(\theta_{i,j}) + \hat{c}_{i,k}^2 \sin(\theta_{i,j})) + \cos^2(\theta)(\hat{c}_{i,k} \cos(\theta_{i,j}) + \hat{c}_{i,k} \sin(\theta_{i,j})) + \sin(2\theta)(\hat{c}_{i,k} \cos(\theta_{i,j}) + \hat{c}_{i,k} \sin(\theta_{i,j})) - \sin(2\theta)(\hat{c}_{i,k} \cos(\theta_{i,j}) + \hat{c}_{i,k} \sin(\theta_{i,j}))$$  

(12)

where $\hat{c}_{i,k}$ is the $(i,j)$-th element of $\hat{C}^{(\text{new})}$, and $\hat{c}_{i,k}$ is the $(i,j)$-th element of vector $\hat{c}^{(\text{new})}$ with $q \in \{1, 2\}$. By using the Weierstrass change of variable: $t_{i,j} = \tan(\theta_{i,j})$, the expression of (12) can be rewritten as follows:

$$\hat{c}^{(\text{new})}(t_{i,j}) = \frac{f_{k}(4) \hat{c}_{i,k} + \hat{f}_{k}(3) \hat{c}_{i,k} + f_{k}(2) \hat{c}_{i,k}^2 + f_{k}(1) \hat{c}_{i,k} + f_{k}(0)}{(1 + \hat{c}_{i,k}^2)^2}$$  

(13)

where $f_{k}(4) = \hat{c}_{i,k}^4, f_{k}(3) = -2\hat{c}_{i,k}^3, f_{k}(2) = \hat{c}_{i,k}^2 + 2c_{i,k}^2 - 4c_{i,k}^2, f_{k}(1) = 2c_{i,k}^2 - c_{i,k}^4$, and $f_{k}(0) = \hat{c}_{i,k}^2$. Equation (13) shows that the sum of the squares of $\hat{c}_{i,k}$ is a rational function in $t_{i,j}$, namely $J_2(t_{i,j})$, where the degrees of the numerator and the denominator are 8 and 8, respectively. The global minimum $t_{i,j}$ can be obtained by computing the roots of its derivative and selecting the one yielding the smallest value of $J_2(t_{i,j})$. Once $t_{i,j}$ is obtained, $\theta_{i,j}$ can be computed by $\theta_{i,j} = \arctan(t_{i,j})$. Then $B$ is updated by (8) and $\hat{A}$ is updated by computing $(\hat{B}^{(\text{new})})^{\boxtimes 2}$. 
B. Minimization with respect to $R^{(i,j)}(r_{ij})$

Let $\hat{A}$ and $\hat{B}$ continue to denote the current estimate of $A$ and $B$ before estimating $R^{(i,j)}(r_{ij})$, respectively. The update of $B$, denoted by $\hat{B}^{(\text{new})}$, is defined as follows:

$$
\hat{B}^{(\text{new})} = \hat{B} R^{(i,j)}(r_{ij})
$$

(14)

By replacing matrix $\hat{B}$ by $\hat{B}^{(\text{new})}$ into criterion (6), the criterion $J_2(r_{ij})$ can be expressed as follows:

$$
J_2(r_{ij}) = \sum_{k=1}^{K} \| \text{off} \left( \left( \left( \hat{B}^{(\text{new})} \right)^{\square 2} \right) \mathcal{C}^{(k,-1)} \right) \|_F^2.
$$

(15)

The Hadamard square of $\hat{B}^{(\text{new})}$ in (15) can be expressed as a function of $r_{ij}$ as follows:

$$
\left( \hat{B}^{(\text{new})} \right)^{\square 2} = \hat{B}^{\square 2} R^{(i,j)}(r_{ij})^2 + \sum_{i,j} \left( \left( \hat{C}^{(k,1)} \right)^T \mathcal{C}^{(k,1)} \right) + \sum_{i,j} \left( \left( \hat{C}^{(k,2)} \right)^T \mathcal{C}^{(k,2)} \right) + \sum_{i,j} \left( \left( \hat{C}^{(k,3)} \right)^T \mathcal{C}^{(k,3)} \right)
$$

(16)

where $\hat{b}_i$ denotes the $i$-th column of $\hat{B}$, and $C^{(k,1)}$ is the $j$-th column of the identity matrix $I \in \mathbb{R}^{N \times N}$. Inserting (16) into the cost function (15), we have:

$$
J_2(r_{ij}) = \sum_{k=1}^{K} \text{off} \left( \left( \left( \hat{B}^{(\text{new})} \right)^{\square 2} \right) \mathcal{C}^{(k,1)} \right) \|_F^2.
$$

(17)

where $\hat{C}^{(k,1)} = \hat{A}^T \mathcal{C}^{(k,-1)} \hat{A}$, $\hat{C}^{(k,2)} = \hat{b}^T \mathcal{C}^{(k,-1)} \hat{b}$, $\hat{C}^{(k,3)} = \hat{b}^T \mathcal{C}^{(k,1)} \hat{b}$, $\hat{C}^{(k,4)} = \hat{b}^T \mathcal{C}^{(k,2)} \hat{b}$, $\hat{C}^{(k,5)} = \hat{b}^T \mathcal{C}^{(k,3)} \hat{b}$, and $\hat{C}^{(k,6)} = \hat{b}^T \mathcal{C}^{(k,4)} \hat{b}$ are a matrix, a column vector, a row vector and a scalar of constant values, respectively. (17) shows that just the $j$-th column and row of $\hat{C}^{(\text{new})}$ involve the parameter $r_{ij}$. Therefore, the minimization of the cost function (17) is equivalent to minimizing the sum of the squares of the $j$-th columns of all the symmetric matrices $\hat{C}^{(\text{new})}$ except their $(j,j)$-th elements. These elements can be expressed by a polynomial function of degree 2 in $r_{ij}$ as follows, for every $n$ value different of $j$:

$$
\hat{C}^{(\text{new})}_{n,j} = \hat{C}^{(k,1)}_{n,i} r_{ij}^2 + \hat{C}^{(k,1)}_{n,j} r_{ij} + \hat{C}^{(k,1)}_{n,j}
$$

(18)

where $\hat{C}^{(k,1)}_{n,i}$ is the $(n,i)$-th component of $\hat{C}^{(k,1)}$, and $\hat{C}^{(k,1)}_{n,j}$ is the $n$-th element of $\hat{C}^{(k,1)}$. Then the cost function (17), which is the total sum of squares of (18), is a polynomial function of degree 4 in $r_{ij}$. The global minimum $r_{ij}$ is one of the roots of its derivative, which yields the smallest value of (17). Once the optimal $r_{ij}$ is computed, $\hat{B}$ is updated by (14) and $\hat{A}$ is updated by computing $(\hat{B}^{(\text{new})})^{\square 2}$.

The processing of all the $N(N-1)$ parameters $\theta_{ij}$ and $r_{ij}$, is called a QR sweep. The proposed JD$_{QR}$ algorithm is comprised of several QR sweeps in order to guarantee the convergence. In ICA, when a non-square matrix $A \in \mathbb{R}^{N \times P}$ with $N > P$ is encountered, we can compress it by a matrix $W \in \mathbb{R}^{N \times P}$ such that the resulting matrix $A = W^T \hat{A}$ is a nonnegative square matrix [15]. It is noteworthy that the proposed algorithm is different from the two published nonnegative JDC methods, which are based on the LU matrix factorization [10], [11]. We use QR factorization in this paper. The method in [10] estimates $B$ and $D^{(k)}$ alternately, and its performance is sensitive to the initialization. The algorithm in [11] needs to compute the inverse of $A$ in all the $N(N-1)$ Jacobi-like iterations, leading to a high numerical complexity.

III. SIMULATION RESULTS

In this section, the proposed JD$_{QR}$ algorithm is compared with several existing JDC methods and BSS algorithms. The performance is measured in terms of the error between the true matrix $A$ and its estimate $\hat{A}$, as well as the source $s$ and its estimate $\hat{s}$ when a BSS context is considered. The following scale-invariant and permutation-blind distance is chosen as the preferred measure:

$$
\alpha(A, \hat{A}) = \left( \frac{1}{N} \right) \sum_{n=1}^{N} \min_{\alpha, \beta} \| d(a_n, \hat{a}_n) \|_F
$$

(19)

where $a_n$ and $\hat{a}_n$ are the $n$-th column of $A$ and the $n$-th column of $A$, respectively. $I_2$ is defined recursively by $I_2^2 = \{1, \cdots, N\} \times \{1, \cdots, N\}$, and $I_2^3 = I_2 - J_2^2$, where $J_2 = \arg \min_{n \in I_2} d(a_n, \hat{a}_n)$. In addition, $d(a_n, \hat{a}_n)$ is defined as the pseudo-distance between two vectors [4]:

$$
\| d(a_n, \hat{a}_n) \|_F = 1 - \frac{\| a_n^T \hat{a}_n \|_2^2}{\| a_n \|_2^2 \| \hat{a}_n \|_2^2}
$$

(20)

The smaller the value of (19) is, the better estimation of $\hat{A}$ is achieved.

A. Simulated semi-nonnegative semi-symmetric arrays

In this part, JD$_{QR}$ is compared with two classic JDC methods, namely ACDC [5] and FFDIAG [6], and one nonnegative JDC method ACDC$_{QR}^{\text{LU}}$ [10] with simulated nonnegative semi-symmetric 3-way arrays $\mathbf{C} \in \mathbb{R}^{3 \times 3 \times 5}$ is generated randomly according to equation (2). The loading matrices $A$ and $D$ are randomly drawn from a uniform distribution between 0 and 1. The pure array $\mathbf{C}$ is perturbed by a semi-symmetric residual noise array $\mathbf{V}$. The loading matrices of $\mathbf{V}$ obey the zero-mean unit-variance Gaussian distribution. The resulting noisy 3-way array can be written by $\mathbf{C} = \mathbf{C} + \sigma_N \mathbf{V} + \sigma_N \mathbf{V}^T$, where $\sigma_N$ is a scalar controlling the noise level. Then the Signal-to-Noise Ratio (SNR) is defined by $\text{SNR} = -20 \log_{10} \sigma_N$. All the algorithms stop either when the relative error of the corresponding criterion between two successive sweeps is less than $10^{-5}$ or when the number of sweeps exceeds 200. We repeat the experiment with SNR ranging from $-10$ dB to $30$ dB with 500 Monte Carlo trials. Figure 1 shows the average curves of $\alpha(A, \hat{A})$ of all the three algorithms as a function of SNR. It shows that ACDC performs better than FFDIAG under higher SNR levels. The nonnegativity constraint obviously helps ACDC$_{QR}^{\text{LU}}$ and JD$_{QR}$ to outperform the classic ones. The proposed JD$_{QR}$ algorithm maintains the best estimation accuracy, especially for the lower SNR levels.

B. BSS application on MRS data

In this section, the BSS performance of JD$_{QR}$ is compared with an effective ICA method CoM2 [16] and a Nonnegative Matrix Factorization (NMF) method based on alternating Non-Negativity Least Squares (NNLS) [17], through an experiment carried out on simulated MRS data. Two metabolites, namely the Choline and Myo-inositol, serve as source signals $s_1(f)$ and $s_2(f)$. 32 observations are generated according to the noisy linear mixing model $x(f) = As(f) + \nu(f)$, where $\nu(f)$ is an additive white Gaussian noise. $\mathbf{A} \in \mathbb{R}^{N \times 2}$ is similarly generated as in the previous section. For an ICA method based
arrays. We proposed a method, called JD\textsuperscript{++} decomposition of semi-nonnegative semi-symmetric 3-way array evolution of ACDC, FFDIAG, ACDC\textsubscript{LU} and JD\textsubscript{QR} as a function of SNR on simulated arrays.

Fig. 1. Average error $\alpha(A, \hat{A})$ evolution of ACDC, FFDIAG, ACDC\textsubscript{LU} and JD\textsubscript{QR} as a function of SNR on simulated arrays.

Fig. 2. Average error $\alpha(\{\hat{s}(f)\}^T, \{\hat{s}(f)\}^T)$ evolution of CoM\textsubscript{2}, NMF and JD\textsubscript{QR} as a function of SNR for BSS of 2 simulated MRS metabolites.

Fig. 3. MRS source metabolites, observations and estimated metabolites by JD\textsubscript{QR}-ICA, CoM\textsubscript{2} and NMF with SNR = 10 dB for one typical realization. The overlapping red lines in figures (c), (d) and (e) indicate the correct sources.

IV. CONCLUSION

In this paper, we have addressed the problem of the CP decomposition of semi-nonnegative semi-symmetric 3-way arrays. We proposed a method, called JD\textsubscript{QR}-ICA, based on the QR factorization of the Hadamard square root of the nonnegative loading matrix. A numerical experiment on simulated arrays highlights its advantage. A BSS application on MRS signals also demonstrates the interest of the proposed method.

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