Simplified computational strategies for dynamic shear

Panagiotis Kotronis, J. Mazars, Stéphane Grange

ECCOMAS Thematic Conference on Computational Methods
in Structural Dynamics and Earthquake Engineering
M. Papadrakakis, D.C. Charmpis, N.D. Lagaros, Y. Tsompanakis (eds.)
Rethymno, Crete, Greece, 13-16 June 2007

SIMPLIFIED COMPUTATIONAL STRATEGIES FOR DYNAMIC SHEAR

P. Kotronis, J. Mazars, and S. Grange

Laboratoire Sols Solides Structures-Risques (3S-R), Grenoble Universités and R.N.V.O.
Domaine Universitaire B.P. 53, 38041 Grenoble Cedex 9, France
email: Panagiotis.Kotronis@inpg.fr, {Jacky.Mazars, Stephane.Grange}@hmg.inpg.fr

Keywords: Multifiber Beam, Lattice, Wall, Shear.

Abstract. *Non-linear dynamic analysis of complex civil engineering structures based on a detailed finite element model requires large-scale computations and involves delicate solution techniques. In earthquake engineering, the necessity to perform parametric studies due to the stochastic characteristic of the input accelerations imposes simplified numerical modeling in order to reduce the computational cost. The purpose of this work is to propose two simplified numerical strategies to simulate dynamic shear. The first one is an enhanced multifiber Timoshenko beam element with higher order interpolation functions in order to avoid any shear locking phenomena. The second one is the Equivalent Reinforced Concrete model (ERC) using lattice meshes for concrete and reinforcement bars. For both strategies, advanced constitutive laws are used based on continuum damage mechanics and plasticity. Verification is provided using experimental results on reinforced concrete walls subjected to severe dynamic loading. Both methods are computationally efficient and easy to use for engineering purposes.*

1 INTRODUCTION

In earthquake engineering, the need to perform parametric studies and the stochastic nature of the input accelerations necessitate simplified numerical modelling in order to reduce computational cost. An optimum idealization is needed i.e. one that is sufficiently fine and yet not too costly.

In order to simulate the non linear dynamic behaviour of a reinforced concrete (R/C) structure, a multifiber Timoshenko beam element with higher order interpolation functions has been developed. The element is free of shear locking phenomena, takes into account deformations due to shear and can be coupled with 2D or 3D constitutive laws.

Nevertheless, when dealing with structures with a slenderness ratio far from the classical beam theory a more reliable representation of shear deformations and shear stresses has to be provided. One possibility in that respect - always within the family of simplified modelling strategies - is to adopt the Equivalent Reinforced Concrete model (ERC) that makes use of lattice meshes for concrete and reinforcement bars.

For both methods, the constitutive law used for concrete is based on damage mechanics. It is able to take into account complex phenomena such as decrease in material stiffness due to cracking, stiffness recovery that occurs at crack closure and inelastic strains concomitant to damage. A modified version of the classical Menegoto-Pinto model with an isotropic hardening is used for steel.

Comparisons with experimental results on R/C walls tested on shaking table show, for both cases, the advantages but also the limitations of the approach.

2 A multifiber Timoshenko beam element

In order to simulate - in a simplified manner - the non-linear behaviour of a R/C wall under dynamic loading, a multifiber Timoshenko beam element has been recently developed [9], [14], [15] and [19]. The difference with other multifiber Timoshenko beam elements usually found in the literature - [23] and [24] - is that the element is displacement-based and has higher order interpolation functions depending on the material's properties. It can be implemented to any general-purpose finite element code without major modifications. The user defines at each fibre a material and the appropriate constitutive law (see Figure 1).

The element takes into account deformations due to shear and uses cubic and quadratic Lagrangian polynomials for the transverse and rotational displacements respectively in order to avoid any shear locking phenomena. The interpolation functions take the following form [2]:

$$\{\mathrm{U_s}\} = [N]\,\{U\} \tag{1}$$

$$\{\mathrm{U_s}\}^{\mathrm{T}} = \left\{ \begin{array}{cccccc} u_s(x) & v_s(x) & w_s(x) & \theta_{sx}(x) & \theta_{sy}(x) & \theta_{sz}(x) \end{array} \right\} \tag{2}$$

$$\{\mathrm{U}\}^{\mathrm{T}} = \left\{ \begin{array}{cccccccccccc} u_1 & v_1 & w_1 & \theta_{x1} & \theta_{y1} & \theta_{z1} & u_2 & v_2 & w_2 & \theta_{x2} & \theta_{y2} & \theta_{z2} \end{array} \right\} \tag{3}$$

and the matrix containing the interpolation functions is equal to:

$$[\mathrm{N}] = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_3 & 0 & 0 & 0 & N_4 & 0 & N_5 & 0 & 0 & 0 & N_6 \\ 0 & 0 & N_3^* & 0 & -N_4^* & 0 & 0 & 0 & N_5^* & 0 & -N_6^* & 0 \\ 0 & 0 & 0 & N_1 & 0 & 0 & 0 & 0 & 0 & N_2 & 0 & 0 \\ 0 & 0 & -N_7^* & 0 & N_8^* & 0 & 0 & 0 & -N_9^* & 0 & N_{10}^* & 0 \\ 0 & N_7 & 0 & 0 & 0 & N_8 & 0 & N_9 & 0 & 0 & 0 & N_{10} \end{bmatrix} \tag{4}$$

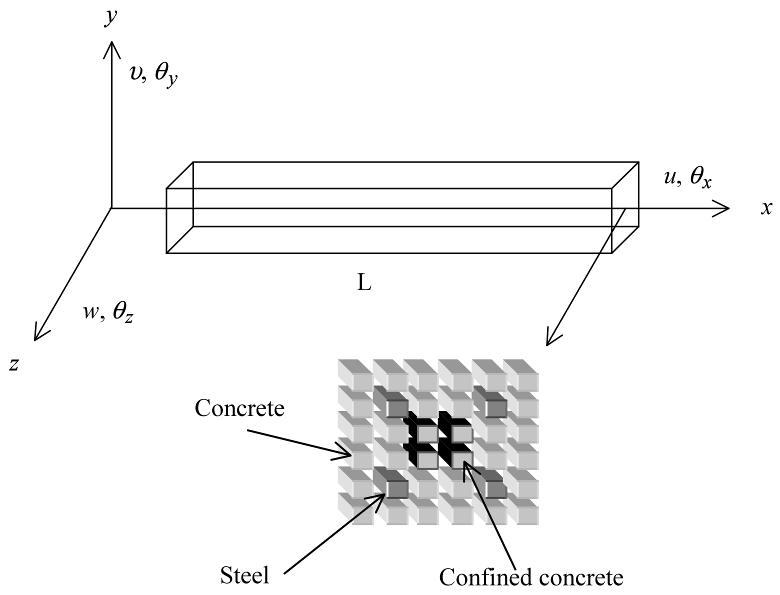

Figure 1: Multifiber beam element for R/C structures

$$
\begin{aligned}
\mathrm{N}_1 &= 1-\frac{x}{L} \\
\mathrm{N}_2 &= \frac{x}{L} \\
\mathrm{N}_3 &= \frac{1}{1+\phi}\{2(\frac{x}{L})^3-3(\frac{x}{L})^2-\phi(\frac{x}{L})+1+\phi\} \\
\mathrm{N}_4 &= \frac{L}{1+\phi}\{(\frac{x}{L})^3-(2+\frac{\phi}{2})(\frac{x}{L})^2+(1+\frac{\phi}{2})(\frac{x}{L})\} \\
\mathrm{N}_5 &= \frac{-1}{1+\phi}\{2(\frac{x}{L})^3-3(\frac{x}{L})^2-\phi(\frac{x}{L})\} \\
\mathrm{N}_6 &= \frac{L}{1+\phi}\{(\frac{x}{L})^3-(1-\frac{\phi}{2})(\frac{x}{L})^2-\frac{\phi}{2}(\frac{x}{L})\} \\
\mathrm{N}_7 &= \frac{6}{(1+\phi)L}\{(\frac{x}{L})^2-(\frac{x}{L})\} \\
\mathrm{N}_8 &= \frac{1}{1+\phi}\{3(\frac{x}{L})^2-(4+\phi)(\frac{x}{L})+(1+\phi)\} \\
\mathrm{N}_9 &= \frac{-6}{(1+\phi)L}\{(\frac{x}{L})^2-(\frac{x}{L})\} \\
\mathrm{N}_{10} &= \frac{1}{1+\phi}\{3(\frac{x}{L})^2-(2-\phi)(\frac{x}{L})\}
\end{aligned}
\tag{5}
$$

with $N_i^*=N_i(\phi^*)$, ϕ and ϕ^* stiffness ratio due to flexion and shear:

$$
\begin{aligned}
\phi &= \frac{12}{L^2}\,\frac{\int_S Ey^2dS}{\int_S GdS} \\
\phi^* &= \frac{12}{L^2}\,\frac{\int_S Ez^2dS}{\int_S GdS}
\end{aligned}
\tag{6}
$$

If $\{\mathrm{F}\}$ and $\{\mathrm{D}\}$ are the section "generalised" stresses and strains respectively:

$$\{\mathrm{F}\} = [K_s]\,\{D\} \tag{7}$$

$$\{\mathrm{F}\}^{\mathrm{T}} = \left\{\begin{array}{cccccc} N & T_y & T_z & M_x & M_y & M_z \end{array}\right\} \tag{8}$$

$$\{\mathrm{D}\}^{\mathrm{T}} = \left\{\begin{array}{cccccc} u_s'(x) & v_s'(x)-\theta_{sz}(x) & w_s'(x)+\theta_{sy}(x) & \theta_{sx}'(x) & \theta_{sy}'(x) & \theta_{sz}'(x) \end{array}\right\} \tag{9}$$

the section stiffness matrix $[\mathrm{K_s}]$ takes the following form [7]:

$$[\mathrm{K_s}] = \begin{bmatrix} K_{s11} & 0 & 0 & 0 & K_{s15} & K_{s16} \\ & K_{s22} & 0 & K_{s24} & 0 & 0 \\ & & K_{s33} & K_{s34} & 0 & 0 \\ & & & K_{s44} & 0 & 0 \\ & & & & K_{s55} & K_{s56} \\ & & & & & K_{s66} \end{bmatrix} \tag{10}$$

$$\begin{aligned} K_{s11} = \int_S EdS;\ K_{s15} = \int_S EzdS;\ K_{s16} = -\int_S EydS \\ K_{s22} = k_y \int_S GdS;\ K_{s24} = -k_y \int_S GzdS \\ K_{s33} = k_z \int_S GdS;\ K_{s34} = k_z \int_S GydS \\ K_{s44} = \int_S G(k_z y^2 + k_y z^2)dS \\ K_{s55} = \int_S Ez^2 dS;\ K_{s56} = -\int_S EyzdS \\ K_{s66} = \int_S Ey^2 dS \end{aligned} \tag{11}$$

The equation that provides the "generalised" strains as a function of the nodal displacements takes the following form (with α' the derivative of α with respect to the spatial variable x):

$$\{\mathrm{D}\} = [B]\,\{U\} \tag{12}$$

and $[B]$ that takes the following form:

$$\begin{bmatrix} N_1' & 0 & 0 & 0 & 0 & 0 & N_2' & 0 & 0 & 0 & 0 & 0 \\ 0 & N_3' - N_7 & 0 & 0 & 0 & N_4' - N_8 & 0 & N_5' - N_9 & 0 & 0 & 0 & N_6' - N_{10} \\ 0 & 0 & {N'}_3^* - N_7^* & 0 & -{N'}_4^* + N_8^* & 0 & 0 & 0 & {N'}_5^* - N_9^* & 0 & -{N'}_6^* + N_{10}^* & 0 \\ 0 & 0 & 0 & N_1' & 0 & 0 & 0 & 0 & 0 & N_2' & 0 & 0 \\ 0 & 0 & -{N'}_7^* & 0 & {N'}_8^* & 0 & 0 & 0 & -{N'}_9^* & 0 & {N'}_{10}^* & 0 \\ 0 & N_7' & 0 & 0 & 0 & N_8' & 0 & N_9' & 0 & 0 & 0 & N_{10}' \end{bmatrix} \tag{13}$$

Finally, the stiffness matrix of the element is given by:

$$K_{elem} = \int_0^L [B]^T\,[K_s]\,[B]\,dx \tag{14}$$

To reproduce correctly the behaviour of concrete under cyclic loading we use a continuous damage model with two scalar damage variables one for damage in tension and one for damage in compression [17]. Inelastic strains are taken into account thanks to an isotropic tensor. A modified version of the classical Menegoto-Pinto model with an isotropic hardening is used for steel [20]. The Timoshenko multifiber beam element and the damage mechanic law have been introduced into FedeasLab, a Matlab finite element toolbox [1] by the 3S-R group.

The previous tools are used hereafter to model a full-scale vertical slice of a seven-story reinforced concrete walls building (benchmark NEES/UCSD performed between October 2005 and January 2006, [6] and [21]) subjected to increasing intensity of uniaxial earthquake ground motions on a shaking table. The structure is composed of 2 main perpendicular walls: the web wall and the flange wall linked with the slabs. The Timoshenko multifiber elements used to model the web wall are divided into 20 concrete fibers whereas those of the flange wall are divided into 8 concrete fibers. The results presented hereafter and the comparisons with the experimental response are "blind" (for more details see [6]).

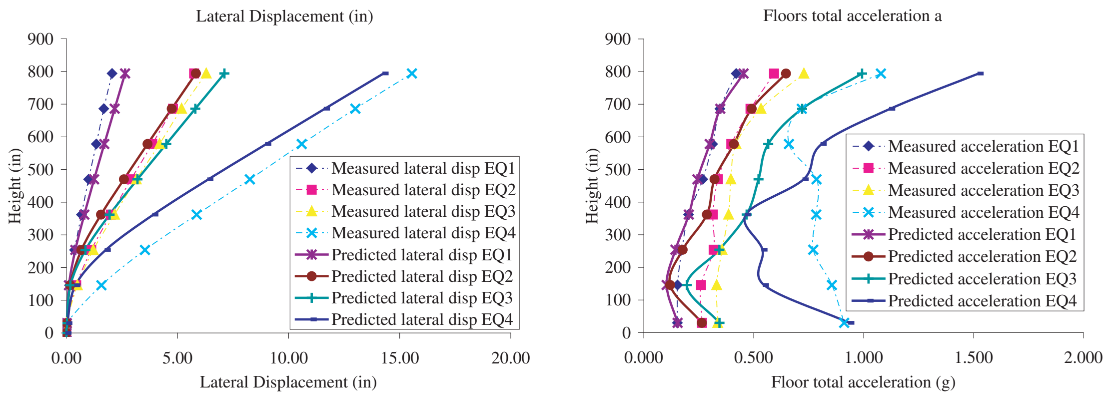

Figure 2: Maximum lateral displacements and accelerations at different levels of the NEES structure for the 4 sequences (EQ1 to EQ4), comparisons between experimental (dotted lines) and numerical results (continuous lines).

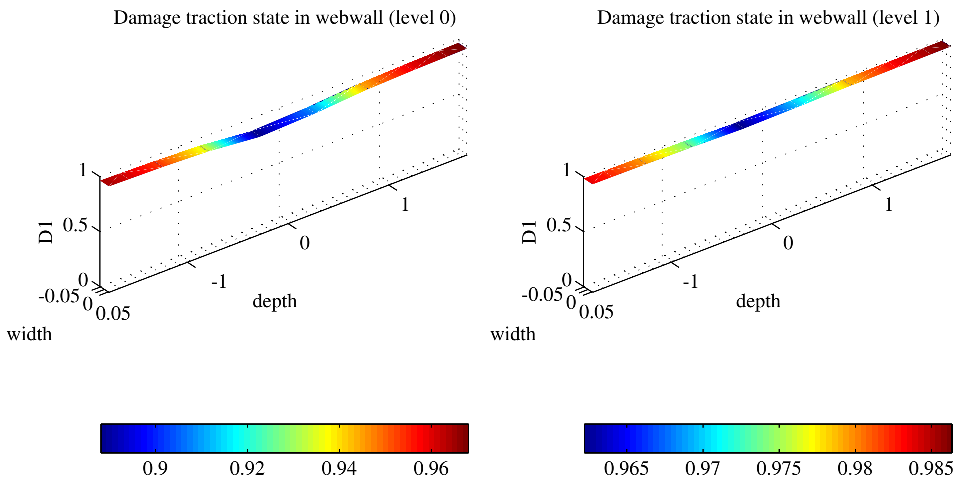

Figure 3: State of damage for the NEES structure at level 0 and level 1 (first sequence, EQ1).

In figures 2 and 3 it is demonstrated that the modelling strategy based on Timoshenko multifiber beam elements and constitutive laws within the framework of damage mechanics and plasticity is able to reproduce with good approximation the global response of the seven story building and qualitatively the distribution of damage. Further validations of this modelling strategy on R/C columns and U-shaped walls tested cyclically or dynamically can be found in [9], [14], [15] and [19].

3 MODELING OF A R/C WALL WITH A SMALL SLENDERNESS RATIO

The behaviour of A R/C shear wall with a small slenderness is controlled primarily by shear. For structures with small slenderness ratio (less than 1) a model based on beam theory has difficulties in reproducing satisfactory the shear deformations and stresses [18]. An alternative simplified method is the so-called Equivalent Reinforced Concrete model (ERC model), [9], [16] and [18]. The model uses a lattice mesh for predicting the non-linear behaviour of shear walls under dynamic loading and is inspired on the Framework Method [8]. The basic idea consists in using the patterns of the Framework Method for 2D or 3D problems coupled with continuous damage mechanics in a non-linear context and for a non-homogenous material. The main assumptions of the proposed strategy are (see figure 4):

- An elementary volume of reinforced concrete (EV) can be separated into a concrete element (C) and a horizontal and a vertical reinforcement bar (SH and SV respectively). Concrete and steel are then modelled separately using two different lattices,
- The sections of the bars simulating concrete are derived from the Framework Method,
- A lattice composed by horizontal and vertical bars coupled with a classical uniaxial plasticity model with or without hardening simulates steel. The section and position of the bars coincide with the actual section and position of the reinforcement. In order to simplify the mesh the method of distribution is used, where the sections of bars are defined proportional to a corresponding surface area. In that way the mesh is independent of the geometry of the specimens,
- Perfect bond is assumed between concrete and steel,
- Geometrical symmetry of the pattern is required for cyclic and transient dynamic loading,
- For at least the type of structure tested hereafter, where the stress field is quite homogeneous, the number of elements that simulate concrete or steel does't have a great influence on the result [9]. Therefore a “macroscopic” model can be used instead of the “equivalent lattice”.

The performance of the ERC model is evaluated on the NUPEC specimen (a shear wall with a slenderness ratio equal to 0.7) tested on the shaking table at the Tadotsu Engineering Laboratory [22]. The specimen is excited with six horizontal acceleration signals parallel to its plane. The rotation at the top of the specimen is free. The pattern presented in figure 5 and the following equations of the Framework Method (valid for plane stress conditions) are used to calculate the lattice simulating concrete:

$$A_{\mathrm{v}} = \frac{3}{8}\ \frac{3k^2-1}{k}\ \alpha t \tag{15}$$

$$A_{\mathrm{h}} = \frac{3}{8}\ (3-k^2)\ \alpha t \tag{16}$$

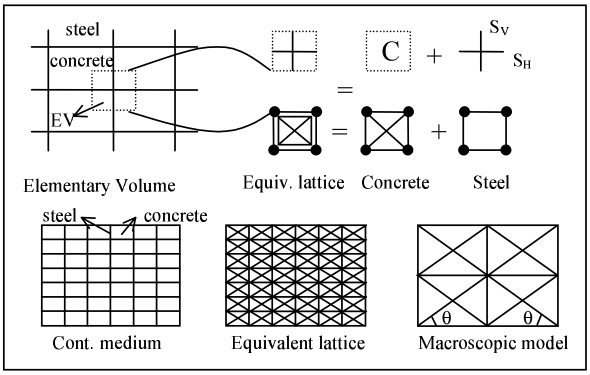

Figure 4: The Equivalent Reinforced Concrete (ERC) model.

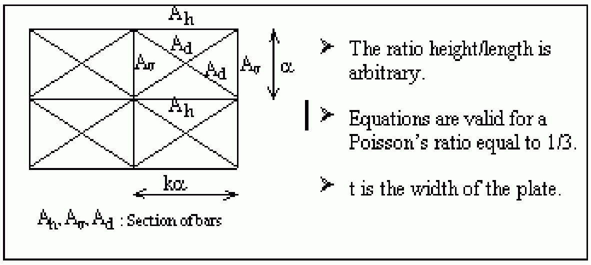

Figure 5: Framework Method - Pattern for plane stress.

$$A_{\rm d} = \frac{3}{16}\ \frac{(1+k^2)^{\frac{3}{2}}}{k}\ \alpha t, \tag{17}$$

where k the ratio between the length and the height α of the pattern and t is the width of the plate.

A zoom at the last two sequences (figure 6) shows that the ERC model predicts correctly the global behaviour of the NUPEC specimen in terms of maximum values and frequency content even under severe loading (just before collapse). Further verification of the method with experimental results on less slender R/C walls with particular boundary conditions (rotations at the top prohibited) can be found in [9], [16] and [18] .

4 CONCLUSIONS

In order to simulate correctly but also quickly the behaviour of R/C structures under severe ground motions one has to find the right compromise for an optimum idealization i.e. one that is sufficiently fine and yet not too costly. Two simplified modelling strategies are presented in this work:

1. A Timoshenko multifiber element with higher order interpolations functions, free of shear locking. The element can be easily implemented into any general-purpose finite element code without major modifications. Numerical simulations of a seven-story R/C building tested on a seismic table are presented to prove the efficiency of the method. Nevertheless, for the calculations presented throughout this work, shear and torsion are considered linear - the 1D version of the constitutive continuous damage law is used. In order to reproduce correctly the behaviour at local level (for example in case of important warping)

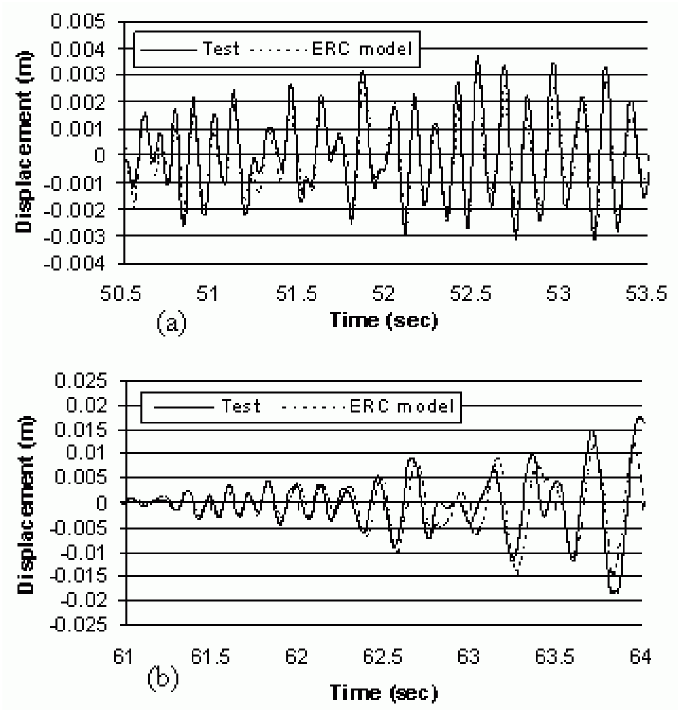

Figure 6: NUPEC specimen (ERC model) - Displacement time history analysis: (a) RUN-4, (b) RUN-5.

the implementation of a 3D robust constitutive law for concrete under cyclic loading is a necessary step. Another possibility in that respect is to consider a warping - conduction analogy method [19].

2. For structures with small slenderness ratio, a model based on the Timoshenko beam theory has difficulties in reproducing satisfactory the shear deformations and stresses. A solution - always within the family of simplified models - is to use the ERC model that privileges the use of two separate lattices meshes one for concrete and one for steel. A crucial parameter for the success of the non-linear simulation is the angle that the diagonals of the concrete lattice form with the horizontal bars [9], [16] and [18]. Other limitations of the method are that perfect bond is assumed between concrete and steel and the stress field must be quite homogeneous. Finally, although the extension of the method seems possible in 3D problems, its feasibility has still to be proven.

Our current work concerns also the development of a 3D macro-element for Soil Structure Interaction ([3], [4] and [5]) and the development of second gradient models to assure the objectivity of the numerical results ([10], [11] and [13]) using advanced following path methods [12].

5 ACKNOWLEDGEMENTS

The authors are grateful for the financial support of the European Contract LESSLOSS, Risk Mitigation for Earthquakes and Landslides of the Sixth Framework Programme (Project No.: GOCE-CT-2003-505488). They would also like to congratulate the organizers of the Seven-story building-slice earthquake blind prediction contest and to thank them for their financial support that made possible for the 3S-R group to participate to the NEES/UCSD Workshop and

Seminar “Analytical Model of Reinforced Concrete Walls” held in San Diego the 15th and 16th of December 2006.

REFERENCES

[1] Filippou FC. and M. Constantinides, Fedeaslab getting started guide and simulations examples, Tech. report, Dpt of civil and env. Engng. UC Berkeley, 2004.

[2] Z. Friedman and J. B. Kosmatka, An improved 2-node timoshenko beam finite-element, *Computers and Structures* **47**, no. 3, 473–481, 19, 1993.

[3] S. Grange, P. Kotronis, and J. Mazars, Advancement of simplified modelling strategies for 3d phenomena and/or boundaries conditions for base-isolated buildings or specific soil-structure interactions. project lessloss risk mitigation for earthquakes and landslides, Tech. report, (http://hal.archives-ouvertes.fr/hal-00102513), 2006.

[4] S. Grange, P. Kotronis, and J. Mazars, A 3d macro-element for soil-structure interaction, International *Journal for Numerical and Analytical Methods in Geomechanics*, submitted to publication, 2007.

[5] S. Grange, P. Kotronis, and J. Mazars, 3d macro element for soil structure interaction, *4th International Conference on Earthquake Geotechnical Engineering*, Thessaloniki, Grèce, June 25-28, 2007.

[6] S. Grange, J. Mazars, and P. Kotronis, Seven-story building-slice earthquake blind prediction contest: a simplified modeling using multifiber timoshenko beams, *Workshop and Seminar on Analytical Modeling of Reinforced Concrete Walls for Earthquake Resistance*, San Diego, USA-California, December 15-16, 2006.

[7] J. Guedes, P. Pégon, and A. Pinto, A fibre timoshenko beam element in castem 2000, Tech. report, Special publication Nr. I.94.31, J.R.C., I-21020, Ispra, Italy, 1994.

[8] A. Hrennikoff, Solution of problems of elasticity by the framework method, *J. of Applied Mechanics*, A169–175, 1941.

[9] P. Kotronis, Cisaillement dynamique de murs en béton armé. modèles simplifiés 2d et 3d, Ph.D. thesis, ENS Cachan, Ph.D thesis, http://tel.ccsd.cnrs.fr/tel-00074469, 2000.

[10] P. Kotronis, S. Al Holo, P. Bésuelle, and R. Chambon, Shear softening and localization, modelling of the evolution of the width of the shear zone, *Acta Geotechnica*, submitted to publication, 2007.

[11] P. Kotronis, R. Chambon, J. Mazars, and F. Collin, Local second gradient models and damage mechanics: Application to concrete, *11th International Conference on Fracture*, 20-25 March Turin Italy (ICF, ed.), pp. cd,paper no 5712, 2005.

[12] P. Kotronis and F. Collin, Implementation of following path techniques into the finite element code lagamine, Tech. report, Rapport Interne GomaC/3S, septembre, 35 pages, http://hal.ccsd.cnrs.fr/ccsd-00100402, 2005.

[13] P. Kotronis, F. Collin, P. Bésuelle, R. Chambon, and J. Mazars, Local second gradient model and damage mechanics : 1d post-localisation studies in concrete specimens, *Bifurcations, Instabilities, Degradation in Geomechanics*. Book, Springer, march, XVI, 470 p. (Vardoulakis Ionannis G Exadaktylos George, ed.), 2007.

[14] P. Kotronis, L. Davenne, and J. Mazars, Poutre 3d multifibre timoshenko pour la modélisation des structures en béton armé soumises à des chargements sévères, *Revue Franaise de Gnie Civil* **8**, no. 2-3, 329–343, 2004.

[15] P. Kotronis and J. Mazars, Simplified modelling strategies to simulate the dynamic behaviour of r/c walls, *Journal of Earthquake Engineering* **9**, no. 2, 285–306, 24, 2005.

[16] P. Kotronis, J. Mazars, and L. Davenne, The equivalent reinforced concrete model for simulating the behavior of walls under dynamic shear loading, *Engineering Fracture Mechanics* **70** (2003), no. 7-8, 1085–1097, 18, 2003.

[17] C. La Borderie, Phénomènes unilatéraux dans un matériau endommageable: modélisation et application à l'analyse des structures en béton., Phd thesis, université Paris 6, 1991.

[18] J. Mazars, P. Kotronis, and L. Davenne, A new modelling strategy for the behaviour of shear walls under dynamic loading, *Earthquake Engineering and Structural Dynamics* **31**, no. 4, 937–954, 24, 2002.

[19] J. Mazars, P. Kotronis, F. Ragueneau, and G. Casaux, Using multifiber beams to account for shear and torsion - applications to concrete structural elements, *Computer Methods in Applied Mechanics and Engineering* **195**, no. 52, 7264–7281, 2006.

[20] M. Menegoto and P. Pinto, Method of analysis of cyclically loaded reinforced concrete plane frames including changes in geometry and non-elastic behaviour of elements under combined normal force and bending, IABSE Symposium on resistance and ultimate deformability of structures acted on by well-defined repeated loads, final report (Lisbon, 328p), 1973.

[21] NEES7story, Nees at ucsd seven-story building-slice earthquake blind prediction contest http://nees.ucsd.edu/7Story.html, 2006.

[22] OECD, Seismic shear wall isp. nupec's seismic ultimate dynamic response test, comparison report ocde/gd(96)188, Tech. report, OECD, 1996.

[23] M. Petrangeli, P. E. Pinto, and V. Ciampi, Fiber element for cyclic bending and shear of rc structures. i: Theory, *Journal of Engineering Mechanics - Asce* **125**, no. 9, 994–1001, 31, 1999.

[24] E. Spacone, F. C. Filippou, and F. F. Taucer, Fibre beam-column model for non-linear analysis of r/c frames .1. formulation, *Earthquake Engineering and Structural Dynamics* **25**, no. 7, 711–725, 38, 1996.