Numerical study of crushable granular materials

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ABSTRACT: The paper aims to develop an elasto-plastic model considering particle crushing. For this purpose, the relationship between the grain size distributions, the position of the critical state line and the plastic work is investigated using experimental results on Cambria sand. The constitutive equations describe two yield mechanisms (isotropic and deviatoric) accounting for particle crushing, which in return affects the position of the critical state line in the e – log(p') diagram. The model is then used to simulate triaxial tests under different loading conditions (drained compression and extension) with high stress levels. All comparisons between experimental results and simulations demonstrate that the model can reproduce with good accuracy the mechanical behavior of granular materials with particle crushing.

Keywords: Granular materials; grain breakage; elasto-plasticity; critical state; grain size distribution.

1. INTRODUCTION

Particle crushing occurred along both compression and shearing stress paths, especially under high confining stress (e.g. within earth dams, deep well shafts). This phenomenon is however more intense during shearing. The influence of particle crushing on the mechanical behavior of granular materials has been widely investigated in the past decades.

To account for particle crushing effect, different modeling methods were proposed in the literature. For instance, Russell and Khalili (2004) used a three-segment type critical state line (CSL) to describe the behavior of crushable granular materials. Salim and Indraranatna (2004) proposed flow rules and yield functions which integrate the effects of particle crushing. Daouadji et al. (2001) proposed a revisited Cam-Clay yield surface in which the hardening parameter (consolidation effective pressure) is made dependent on the plastic work. It implies a shift of the position of the critical state line in the e – log (p') diagram where e is the current void ratio and p' the mean effective stress. Later on, Muir Wood and Maeda (2008) and Muir-Wood et al. (2009) also considered the shift of the CSL with grain gradation, through a grading state index (I_G) which changes with the position of a so-called crushing surface.

Here, an elasto-plastic model based on a Mohr-Coulomb yield criterion for the shearing mechanism, in which the void ratio at critical state is a function of plastic work, is developed. .

In what follows, an analysis of published experimental results aims at establishing relationships between CSL, particle crushing and energy consumption. The model is finally verified by simulating tests on Cambria sand specimens under different loading conditions and high confining stresses.

2. CONSTITUTIVE MODEL

For the elastic part of the model, the isotropic hypo-elasticity assumption is adopted. For modeling the plastic behavior, the proposed approach uses two yield surfaces for shear and compression components respectively. Thus the framework of the proposed approach is similar to that of the double-hardening or double-yield-surface models by Vermeer (1978).

2.1 Elastic part

The elastic strain increments were deducted by using elastic law:

$$\mathrm{d}\varepsilon_v^e = \frac{\mathrm{d}p}{K}; \qquad \mathrm{d}\varepsilon_d^e = \frac{\mathrm{d}q}{3G} \tag{1}$$

where G and K are the hypo-elastic shear and bulk moduli respectively and are defined as follows (Richart et al. [20]):

$$G = G_0 \frac{(2.97-e)^2}{(1+e)}\left(\frac{p'}{p_{at}}\right)^n \quad (2)$$

$$K = K_0 \frac{(2.97-e)^2}{(1+e)}\left(\frac{p'}{p_{at}}\right)^n \quad (3)$$

where G_0, K_0 and n are elastic parameters; p_{at} is the atmospheric pressure used as reference pressure (p_{at} = 101.325 kPa) and p' the mean effective stress. For Cambria sand, K_0 = 26.3 MPa and n = 0.4 were calibrated from isotropic compression test, and G_0 = 35 MPa was calibrated from the initial part of the stress strain curve (e.g., ε_1 < 0.1 %) resulting from drained triaxial compression tests.

2.1 Shear yield criterion

As many models relevant for sand behavior, the shear mechanism is represented by a linear yield function in the (p'-q) plot where q is the deviatoric stress. The shear yield function is written as follows:

$$f_S = \eta - H \quad (4)$$

where η is the stress ratio q/p'; H is the hardening parameter defined by a hyperbolic function in the H-ε_d^p plane, similarly to Yin et al. (2010), given as:

$$H = \frac{M_p \varepsilon_d^p}{\frac{M_p p'}{G_p K} + \varepsilon_d^p} \quad (5)$$

where G_p is used to control the initial slope of the hyperbolic curve η-ε_d^p. Eq. (5) guarantees that the stress ratio will approach the peak value M_p.

In order to take into account the dilation or contraction during the shear sliding, the Roscoe-type stress-dilatancy equation is used:

$$\frac{d\varepsilon_v^p}{d\varepsilon_d^p} = D\left(M_{pt} - \eta\right) \quad (6)$$

where D is a soil parameter. M_{pt} is the slope of the phase transformation line in the (p' – q) diagram.

2.2 Normal compression criterion

In order to describe the compressible behavior of crushable granular materials, a second yield surface is added. The second yield function is assumed to be as follows:

$$f_N = p' - p_y \quad (7)$$

where p_y is the hardening parameter controlling the size of the second yield surface. The yield surface expands with the plastic volumetric strain. The hardening rule of the Modified Cam Clay model is adopted:

$$dp_y = p_y \frac{d\varepsilon_v^p}{c_p} \quad (8)$$

An associated flow rule is adopted for the normal compression.

In order to interpolate M between its values M_c (for compression) and M_e (for extension) by means of the Lode angle $\frac{-\pi}{6} \le \theta = \frac{1}{3}\sin^{-1}\left(\frac{-3\sqrt{3}J_3}{2J_2^{3/2}}\right) \le \frac{\pi}{6}$ (Sheng et al., 2000), the following relationships is assumed:

$$M_\mu = M_c \left[\frac{2c^4}{1+c^4+\left(1-c^4\right)\sin 3\theta}\right]^{\frac{1}{4}} \quad (9)$$

where c=(3-sinϕ_μ)/(3+sinϕ_μ) assuming the same constant volume friction angle ϕ_μ for compression and extension conditions. $J_2 = \left(I_1^2 + 2I_3\right)/3$ and $J_3 = \left(2I_1^3 + 9I_1I_2 + 27I_3\right)/27$ are respectively the second and third invariants of the deviator stress tensor with $I_1=\sigma'_1+\sigma'_2+\sigma'_3$, $I_2=\sigma'_1\sigma'_2+\sigma'_2\sigma'_3+\sigma'_3\sigma'_1$, $I_3=\sigma'_1\sigma'_2\sigma'_3$ under axi-symmetric triaxial conditions.

2.2 Density state effect

The density state of soil is defined as the ratio e_c/e, where e_c is the critical void ratio and e is the current void ratio of soil at current p'.

According to Biarez & Hicher (1994), the peak friction angle ϕ_p (relating to M_p = 6sinϕ_p/(3-sinϕ_p) for triaxial compression) relates to the intrinsic friction angle ϕ_μ (relating to the critical state value M=6sinϕ_μ/(3-sinϕ_μ) for triaxial compression) and density state of soil (e_c/e):

$$e\tan\phi_p = e_c \tan\phi_\mu \quad (10)$$

The parameter M_p is therefore obtained through the critical state M and density state e_c/e. Eq. (10) means that in a loose structure, the "peak" frictional angle ϕ_p is smaller than ϕ_μ. On the other hand, a dense structure provides a higher degree of interlocking. Thus, the peak frictional angle ϕ_p is greater than ϕ_μ. When the loading stress reaches the peak frictional angle ϕ_p, the dense structure dilates and the degree of interlocking relaxes. As a consequence, the peak frictional angle is reduced, which results in a strain-softening phenomenon.

M_{pt} is the slope of phase transformation line for sand which is assumed as follows:

$$e_c \tan\phi_{pt} = e\tan\phi_\mu \quad (11)$$

Eq. (11) suggests that the dense packing has smaller phase transformation angle than the loose packing, which results in the same effect than the one formulated by Muir Wood et al. (2009).

3. BREAKAGE ANALYSIS

3.1 Definition of breakage index

Hardin (1985) proposed a breakage index to quantify the amount of particle crushing. The index is based on the changes in particle size as the key measure. Einav (2007) modified the breakage index, termed as B_r, based on the changes on the overall grain size distribution as follows:

$$B_r = \frac{B_p}{B_t} = \frac{\int_{d_m}^{d_M} (F(d) - F_0(d)) d(\log(d))}{\int_{d_m}^{d_M} (F_u(d) - F_0(d)) d(\log(d))} \quad (12)$$

where B_p is the area between the curves of original size distribution and current grain size distribution; B_t is the total area between the curves of original size distribution and an assumed limit grain size distribution of fractal form. F_0(d) and F_u(d) represent the initial gradation before particle crushing and the fractal distribution with high quantities of particle crushing respectively; F(d) is the current gradation during loading during which particle crushing occurs; d is the grain size; d_M and d_m are respectively the maximum grain size and the minimum grain size. The gradation can be expressed as:

$$F(d) = (d / d_M)^{\alpha} \quad (13)$$

where α is a material constant.

3.2 Influence of plastic work on the evolution of gradation

In this section, results of drained triaxial compression tests on Cambria sand (Yamamuro and Lade, 1996; Lade et al., 1996) were analyzed to investigate the relationship between the breakage index B_r and the modified plastic work w_p representing energy consumption. Cambria sand is a coarse and uniform sand consisting of sub-angular to well-rounded grains. The diameter of the grains varies from 0.83 to 2 mm. The maximum void ratio is 0.792 and the minimum void ratio is 0.503. The specific gravity is 2.69.

The modified plastic work is expressed as follows:

$$w_p = \int p' \langle d\varepsilon_v^p \rangle + q d\varepsilon_d^p \quad (14)$$

where p' = (σ'$_1$+2σ'$_3$)/3 with σ'$_1$ and σ'$_3$ representing principle stresses; q is the deviatoric stress: q = σ'$_1$-σ'$_3$; $d\varepsilon_v^p$ and $d\varepsilon_d^p$ are the volumetric and deviatoric plastic strain increments respectively: $d\varepsilon_v^p = d\varepsilon_1^p + 2d\varepsilon_3^p$ and $d\varepsilon_d^p = 2(d\varepsilon_1^p - d\varepsilon_3^p)/3$; <F> is the MacCauley function such as: <F> = 0 for F < 0 and <F> = F for F>0. Using this MacCauley function, the shear induced dilation ($d\varepsilon_v^p$ <0) is not accounted in the modified plastic work. As a result, the gradation cannot be influenced by dilation when using this modified plastic work to link with.

The values of breakage index B_r for different tests were measured. For each test, the modified plastic work w_p was also calculated based on the stress strain curves. The breakage index B_r is then plotted against w_p in Figure 1(a) showing that the value of B_r increases with the modified plastic work. Based on Figure 1(a), the hyperbolic function is proposed for the relationship between B_r and w_p, as follows:

$$B_r = \frac{w_p}{\chi + w_p} \quad (15)$$

where χ is a material constant controlling the evolution rate of gradation. For Cambria sand, χ = 15000 is obtained.

3.3 Influence of gradation on the position of critical state line

The critical state void ratio e_c is a function of the mean effective stress p'. The relationship has traditionally been written as Eq. (16). The location of the CSL in the *e* –log(*p'*) plane depends on three parameters e_{ref}, λ and p'_{ref}:

$$e_c = e_{ref} - \lambda \ln\left(\frac{p'}{p_{ref}}\right) \quad (16)$$

where p_{ref} = 100 kPa is assigned. The couple (e_{ref} ; p'_{ref} determine a reference point in the *e* –log(*p')*' plane, and λ determines the slope of the CSL.

According to Biarez and Hicher (1994), the position of CSL moves down in the *e*-log(*p')* plan with the increase of the coefficient of uniformity C_u (d_{60}/d_{10}) based on experimental tests on sub-angular Hostun sand. The movement of CSL has also been discussed by Muir Wood and Maeda (2008) using discrete element analysis which agrees with Biarez and Hicher (1994). However, there is no relationship available between the position of CSL and the gradation based on experiments. From the concept of CSL (at critical state, the material remains at a constant volume while it is subjected to a continuous distortion), if the CSL can move, it is no longer CSL. We note that, in this paper, we still use this term but representing the temporary position of CSL for a grading curve.

Drained compression tests on Cambria sand up to failure by Yamamuro and Lade (1996) were used to investigate the movement of CSL with the gradation. For each drained compression test, the void ratio at failure with the mean effective stress p' was measured and considered as a temporary critical state corresponding to a gradation. The reference void ratio e_{ref} representing the temporary position of CSL was then obtained by Eq. (16). Based on all drained compression tests, e_{ref} is plotted against the breakage index B_r, as shown in Figure 1(b) from which a hyperbolic curve is raised as follows:

$$e_{ref} = e_{ref\,0} + \left(e_{refu} - e_{ref\,0}\right)\frac{B_r}{\rho + B_r} \quad (17)$$

where e_{ref0} and e_{refu} is the initial and ultimate reference critical state void ratios respectively; ρ is a material constant relating to the movement rate of CSL with breakage index due to particle crushing. For Cambria sand, e_{ref0} = 0.59 and λ = 0.006 were obtained from drained triaxial compression tests under low confining stresses (less than 1 MPa) for which Yamamuro and Lade (1996) indicated that only slight grain crushing occurred. e_{refu} = 0.13 and ρ = 0.16 were obtained from Figure 1(b).

According to experimental investigations, Eq. (15) and Eq. (17) were proposed and can be used for Eq. (16). Therefore, particle crushing can influence directly the position of CSL, which results in the change of the density state e_c/e. All terms relating to e_c/e (e.g., ϕ_p, ϕ_{pt}, H etc.) are then influenced. The particle crushing is then incorporated into the model. The effect of particle crushing or the influence of some selected parameters are shown in Figure 2.

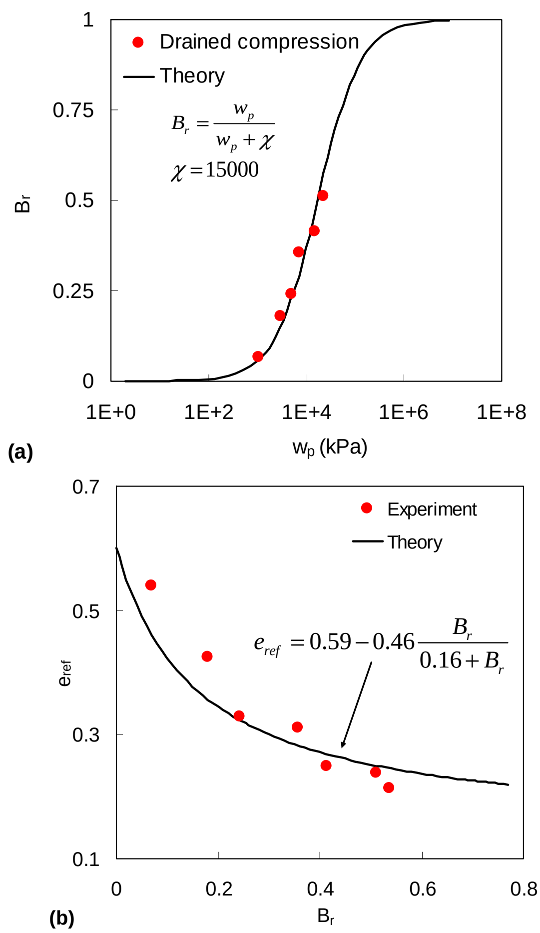


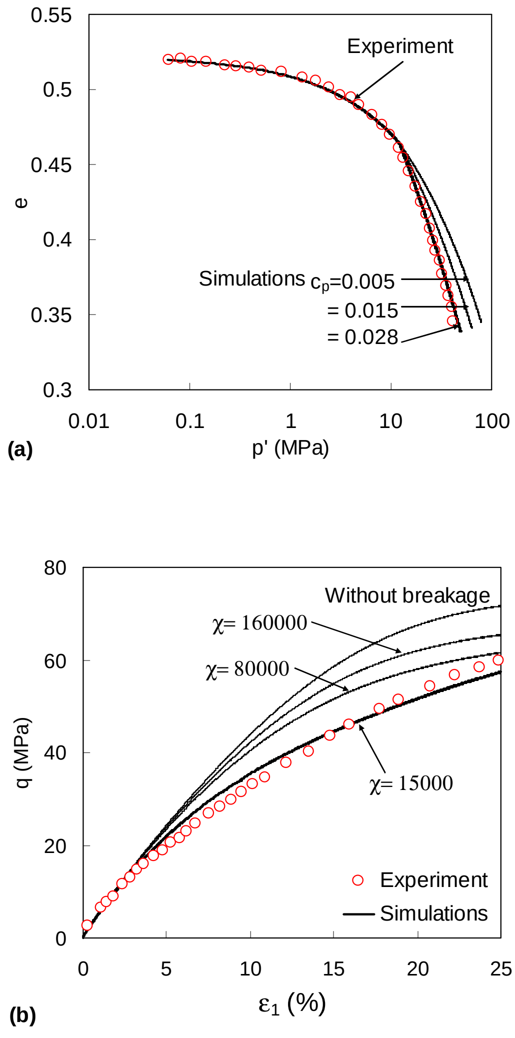
Figure 1. Particle crushing effect: (a) Evolution of breakage index versus modified plastic work, and (b) Evolution of reference critical state void ratio versus breakage index

4. TEST SIMULATIONS

All determined values of constitutive parameters are summarized in Table 1, and will be subsequently used for simulations along different stress paths.

Table 1. Values of the constitutive parameters for Cambria sand specimens.

G_0 (MPa)	K_0 (MPa)	n	G_p	ϕ_μ	p_{y0} (MPa)
35	26.3	0.4	3.5	37.5°	12
c_p	$e_{ref\,0}$	λ	χ	ρ	e_{refu}
0.028	0.59	0.006	15000	0.16	0.13



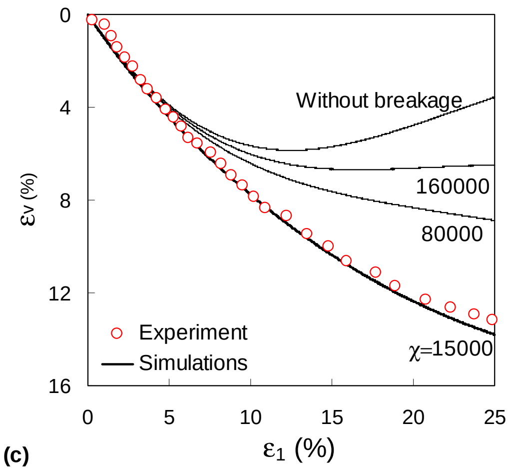


Figure 2. Parametric study for particle crushing related parameters: (a) for isotropic compression test and (b)-(c) deviatoric stress and volumetric strain versus major principle strain respectively for drained compression test with constant confining stress of 26MPa

5. SIMULATIONS OF DRAINED TESTS IN COMPRESSION AND EXTENSION

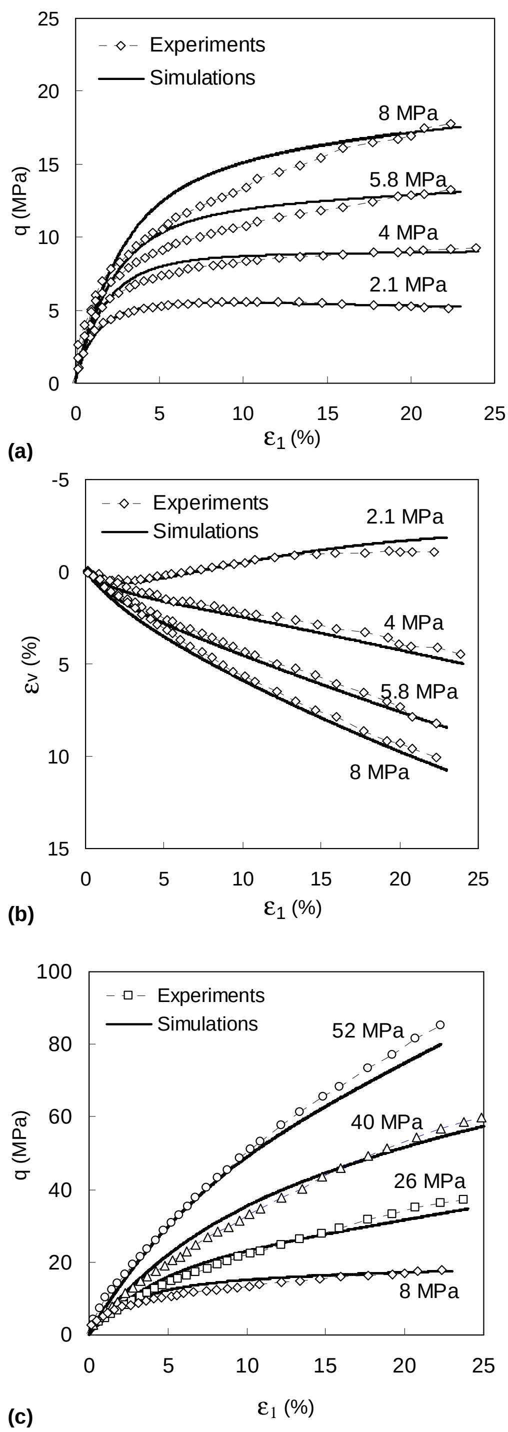
Figure 3 shows comparisons between experimental results and simulations for drained triaxial tests in compression with confining stresses varying from 2.1 to 52 MPa. Good agreement was achieved for all comparisons. The model well captured the mechanical behavior of sand influenced by particle crushing:

(1) Under low confining stress (2.1 MPa), the sample exhibits a dilative behavior.

(2) For higher confining stresses, the samples present a contractive behavior. This disappearance of dilation is related to more particle crushing occurred under high stresses (2.1-26 MPa). For tests under confining stresses from 2.1 to 26 MPa, the contraction increases with the increase of confining stress.

(3) However, for tests under confining stresses from 26 to 52 MPa, volumetric strain decreases with the increase of confining stress. Yamamuro and Lade (1996) indicated that this effect is caused by the increase in the amount of volumetric contraction and particle crushing during isotropic consolidation. Lower void ratios are obtained with increasing confining stress, which in turn allows less volumetric contraction during shearing. This trend was well captured by the model incorporating particle crushing. The increase of particle crushing during isotropic loading results in a low amount of crushable sand left during shearing stage (see Fig.1a: the change of B_r becomes stable for high plastic work). As a result, the contraction of sample caused by particle crushing is reduced.

Using all parameters determined from drained compression tests, the model has also been applied to simulate the drained extension tests on Cambria sand with confining stress varying from 6 to 42MPa by Bopp and Lade (2005). Good agreement was achieved between experimental results and simulations, as shown in Figure 4. The model well captured the stress strain response and the volumetric strain response for different confining stress levels.



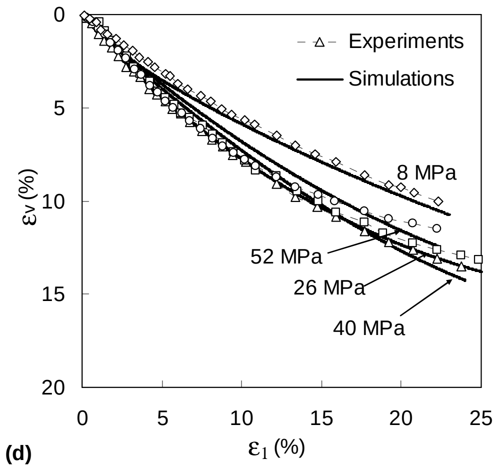
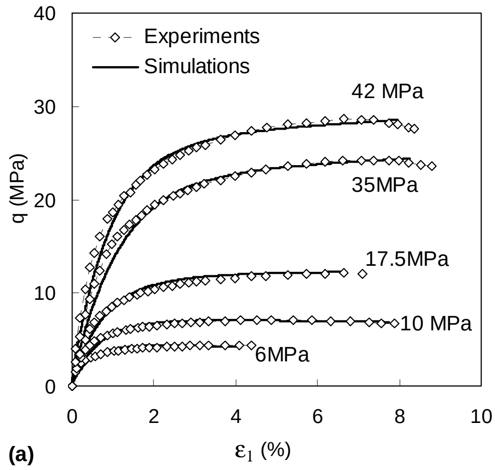


Figure 3. Comparisons between experimental data and simulations for drained triaxial compression tests: (a-c) deviatoric stress versus major principle strain; (b-d) volumetric strain versus major principle strain



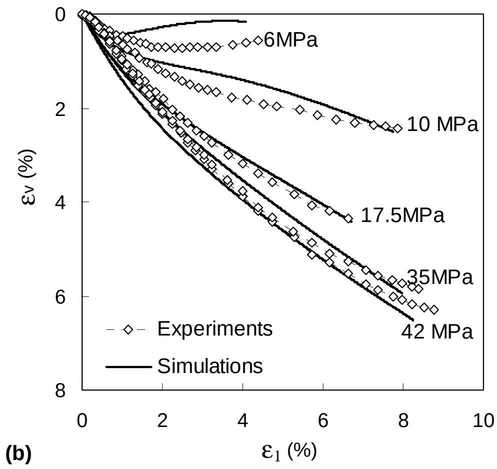


Figure 4. Comparisons between experimental results and simulations for drained triaxial extension tests: (a) deviatoric stress versus major principle strain, and (b) volumetric strain versus major principle strain

Conclusions

Experimental investigations were carried out on the evolution of gradation with the modified plastic work and on the evolution of the position of CSL with the gradation. Based on this, two constitutive equations were proposed for the relationship between the breakage index, the modified plastic work and the reference critical state void ratio. Relating these two equations with the function of CSL, a double-yield surface model accounting for the influence of particle crushing has been proposed.

Triaxial tests on Cambria sand were used to validate the proposed model. The model parameters can be easily determined from one isotropic compression test and several drained compression tests. Using determined parameters, different tests including drained tests in compression and extension, undrained tests in compression and extension were simulated by the proposed model. The grain size distributions for all tests were also predicted. All comparisons between experiments and simulations demonstrate that the model can accurately describe the mechanical behavior of granular materials with particle crushing, as well as the evolution of grain size distribution during loading.

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