Numerical study of crushable granular materials
Wei Hu, Zhenyu Yin, Christophe Dano, Pierre-Yves Hicher

To cite this version:

HAL Id: hal-01008575
https://hal.archives-ouvertes.fr/hal-01008575
Submitted on 30 May 2017

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L’archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d’enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.
Numerical study of crushable granular materials

Hu, W.
State Key Laboratory of Geohazard Prevention and Geoenvironment Protection, Chengdu University of Technology, P. R. China.
Former Research fellow, Research Institute in Civil and Mechanical Engineering, UMR CNRS 6183, Ecole Centrale de Nantes, France, huwei1999@gmail.com

Yin, Z.Y.
Professor, Department of Civil Engineering, Shanghai Jiao Tong University, Shanghai

Dano, C.
Associate professor, Research Institute in Civil and Mechanical Engineering, UMR CNRS 6183, Ecole Centrale de Nantes, France.

Hicher, P.Y.
Professor, Research Institute in Civil and Mechanical Engineering, UMR CNRS 6183, Ecole Centrale de Nantes, France.

ABSTRACT: The paper aims to develop an elasto-plastic model considering particle crushing. For this purpose, the relationship between the grain size distributions, the position of the critical state line and the plastic work is investigated using experimental results on Cambria sand. The constitutive equations describe two yield mechanisms (isotropic and deviatoric) accounting for particle crushing, which in return affects the position of the critical state line in the e – log(p’) diagram. The model is then used to simulate triaxial tests under different loading conditions (drained compression and extension) with high stress levels. All comparisons between experimental results and simulations demonstrate that the model can reproduce with good accuracy the mechanical behavior of granular materials with particle crushing.

Keywords: Granular materials; grain breakage; elasto-plasticity; critical state; grain size distribution.

1. INTRODUCTION

Particle crushing occurred along both compression and shearing stress paths, especially under high confining stress (e.g. within earth dams, deep well shafts). This phenomenon is however more intense during shearing. The influence of particle crushing on the mechanical behavior of granular materials has been widely investigated in the past decades.

To account for particle crushing effect, different modeling methods were proposed in the literature. For instance, Russell and Khalili (2004) used a three-segment type critical state line (CSL) to describe the behavior of crushable granular materials. Salim and Indrarana (2004) proposed flow rules and yield functions which integrate the effects of particle crushing. Daoudji et al. (2001) proposed a revisited Cam-Clay yield surface in which the hardening parameter (consolidation effective pressure) is made dependent on the plastic work. It implies a shift of the position of the critical state line in the e – log (p’) diagram where e is the current void ratio and p’ the mean effective stress. Later on, Muir Wood and Maeda (2008) and Muir-Wood et al. (2009) also considered the shift of the CSL with grain gradation, through a grading state index (IG) which changes with the position of a so-called crushing surface.

Here, an elasto-plastic model based on a Mohr-Coulomb yield criterion for the shearing mechanism, in which the void ratio at critical state is a function of plastic work, is developed.

In what follows, an analysis of published experimental results aims at establishing relationships between CSL, particle crushing and energy consumption. The model is finally verified by simulating tests on Cambria sand specimens under different loading conditions and high confining stresses.

2. CONSTITUTIVE MODEL

For the elastic part of the model, the isotropic hypo-elasticity assumption is adopted. For modeling the plastic behavior, the proposed approach uses two yield surfaces for shear and compression components respectively. Thus the framework of the proposed approach is similar to that of the double-hardening or double-yield-surface models by Vermeer (1978).

2.1 Elastic part

The elastic strain increments were deducted by using elastic law:

\[ d\varepsilon^e = \frac{dp}{K}; \quad d\varepsilon^p = \frac{dg}{3G} \]  \hspace{1cm} (1)
where \( G \) and \( K \) are the hypo-elastic shear and bulk moduli respectively and are defined as follows (Richart et al. [20]):

\[
G = G_0 \left( \frac{2.97 - e}{1 + e} \right)^n \left( \frac{P'}{p_{at}} \right)^{m}
\]

(2)

\[
K = K_0 \left( \frac{2.97 - e}{1 + e} \right)^n \left( \frac{P'}{p_{at}} \right)^{m}
\]

(3)

where \( G_0, K_0 \) and \( n \) are elastic parameters; \( p_{at} \) is the atmospheric pressure used as reference pressure \( (p_{at} = 101.325 \, \text{kPa}) \) and \( p' \) the mean effective stress. For Cambria sand, \( K_0 = 26.3 \, \text{MPa} \) and \( n = 0.4 \) were calibrated from isotropic compression test, and \( G_0 = 35 \, \text{MPa} \) was calibrated from the initial part of the stress strain curve (e.g., \( e_1 < 0.1 \% \) ) resulting from drained triaxial compression tests.

2.1 Shear yield criterion

As many models relevant for sand behavior, the shear mechanism is represented by a linear yield function in the \((p'-q)\) plot where \( q \) is the deviatoric stress. The shear yield function is written as follows:

\[
f_y = \eta - H
\]

(4)

where \( \eta \) is the stress ratio \( q/p' \); \( H \) is the hardening parameter defined by a hyperbolic function in the \(H-E_\eta\) plane, similarly to Yin et al. (2010), given as:

\[
H = \frac{M_p e''}{M_p p' + e''}
\]

(5)

where \( G_p \) is used to control the initial slope of the hyperbolic curve \( \eta-E_\eta \). Eq. (5) guarantees that the stress ratio will approach the peak value \( M_p \).

In order to take into account the dilation or contraction during the shear sliding, the Roscoe-type stress-dilatancy equation is used:

\[
\frac{de''}{de''} = D(M_{tp} - \eta)
\]

(6)

where \( D \) is a soil parameter. \( M_{tp} \) is the slope of the phase transformation line in the \((p' - q)\) diagram.

2.2 Normal compression criterion

In order to describe the compressible behavior of crushable granular materials, a second yield surface is added. The second yield function is assumed to be as follows:

\[
f_y = p' - p,
\]

(7)

where \( p_0 \) is the hardening parameter controlling the size of the second yield surface. The yield surface expands with the plastic volumetric strain. The hardening rule of the Modified Cam Clay model is adopted:

\[
dp_y = p_r \frac{de''}{c_p}
\]

(8)

An associated flow rule is adopted for the normal compression.

In order to interpolate \( M \) between its values \( M_c \) (for compression) and \( M_e \) (for extension) by means of the Lode angle \( -\pi/6 \leq \theta = \frac{1}{3} \sin^{-1} \left( \frac{-3\sqrt{I_1 J_2}}{2J_3^{1/2}} \right) \leq \pi/6 \) (Sheng et al., 2000), the following relationships is assumed:

\[
M_\mu = M_c \left[ \frac{2c^4}{1 + c^4 + (1-c^4)\sin 3\theta} \right]^{1/2}
\]

(9)

where \( c = (3-\sin \phi)/(3+\sin \phi) \) assuming the same constant volume friction angle \( \phi_c \) for compression and extension conditions.

\[
J_2 = \left( I_1^2 + 2I_2 \right)/3 \quad \text{and} \quad J_3 = \left( 9I_1I_2 + 27I_3 \right)/27
\]

are respectively the second and third invariant of the deviator stress tensor with \( I_1 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 \), \( I_2 = \sigma_1^2\sigma_2^2 + \sigma_2^2\sigma_3^2 + \sigma_3^2\sigma_1^2, \) \( I_3 = \sigma_1^2\sigma_2^2\sigma_3^2 \) under axi-symmetric triaxial conditions.

2.2 Density state effect

The density state of soil is defined as the ratio \( e/e_c \), where \( e_c \) is the critical void ratio and \( e \) is the current void ratio of soil at current \( p' \).

According to Biarez & Hicher (1994), the peak friction angle \( \phi_p \) (relating to \( M_p = 6\sin \phi_p/(3-\sin \phi_p) \) for triaxial compression) relates to the intrinsic friction angle \( \phi_0 \) (relating to the critical state value \( M = 6\sin \phi_0/(3-\sin \phi_0) \) for triaxial compression) and density state of soil \( (e/e_c) \):

\[
e \tan \phi_p = e_c \tan \phi_0
\]

(10)

The parameter \( M_p \) is therefore obtained through the critical state \( M \) and density state \( e/e_c \). Eq. (10) means that in a loose structure, the “peak” frictional angle \( \phi_p \) is smaller than \( \phi_0 \). On the other hand, a dense structure provides a higher degree of interlocking. Thus, the peak frictional angle \( \phi_p \) is greater than \( \phi_0 \).

When the loading stress reaches the peak frictional angle \( \phi_p \), the dense structure dilates and the degree of interlocking relaxes. As a consequence, the peak frictional angle is reduced, which results in a strain-softening phenomenon.

\( M_p \) is the slope of phase transformation line for sand which is assumed as follows:

\[
e_c \tan \phi_p = e \tan \phi_0
\]

(11)

Eq. (11) suggests that the dense packing has smaller phase transformation angle than the loose
packing, which results in the same effect than the one formulated by Muir Wood et al. (2009).

3. BREAKAGE ANALYSIS

3.1 Definition of breakage index

Hardin (1985) proposed a breakage index to quantify the amount of particle crushing. The index is based on the changes in particle size as the key measure. Einav (2007) modified the breakage index, termed as $B_r$, based on the changes on the overall grain size distribution as follows:

$$B_r = \frac{B_p}{B_t} = \int_{d_M}^{d_m} \frac{F(d) - F_c(d)}{d(\log(d))} \, \log \left( \frac{d}{d_M} \right)$$

(12)

where $B_p$ is the area between the curves of original size distribution and current grain size distribution; $B_t$ is the total area between the curves of original size distribution and an assumed limit grain size distribution of fractal form. $F_0(d)$ and $F(d)$ represent the initial gradation before particle crushing and the fractal distribution with high quantities of particle crushing respectively; $F_0(d)$ is the current gradation during loading while during which particle crushing occurs; $d$ is the grain size; $d_M$ and $d_m$ are respectively the maximum grain size and the minimum grain size. The gradation can be expressed as:

$$F(d) = (d/d_a)^\alpha$$

(13)

where $\alpha$ is a material constant.

3.2 Influence of plastic work on the evolution of gradation

In this section, results of drained triaxial compression tests on Cambria sand (Yamamuro and Lade, 1996; Lade et al., 1996) were analyzed to investigate the relationship between the breakage index $B_r$ and the modified plastic work $w_p$ representing energy consumption. Cambria sand is a coarse and uniform sand consisting of sub-angular to well-rounded grains. The diameter of the grains varies from 0.83 to 2 mm. The maximum void ratio is 0.792 and the minimum void ratio is 0.503. The specific gravity is 2.69.

The modified plastic work is expressed as follows:

$$w_p = \int \frac{\sigma'}{\lambda} \, d\varepsilon^{p}_{v} + q \, d\varepsilon^{p}_{v}$$

(14)

where $\sigma' = (\sigma_1' + 2\sigma_3')/3$ with $\sigma_1'$ and $\sigma_3'$ representing principle stresses; $q$ is the deviatoric stress: $q = \sigma_1' - \sigma_3'$; $d\varepsilon^{p}_{v}$ and $d\varepsilon^{p}_{v}$ are the volumetric and deviatoric plastic strain increments respectively; $d\varepsilon^{p}_{v} = d\varepsilon^{p}_{v} + 2d\varepsilon^{p}_{v}$ and $d\varepsilon^{p}_{v} = 2(d\varepsilon^{p}_{v} + d\varepsilon^{p}_{v})/3$; $<F>$ is the MacCauley function such as: $<F> = 0$ for $F < 0$ and $<F> = F$ for $F > 0$. Using this MacCauley function, the shear induced dilation ($d\varepsilon^{p}_{v} < 0$) is not accounted in the modified plastic work. As a result, the gradation cannot be influenced by dilation when using this modified plastic work to link with.

The values of breakage index $B_r$ for different tests were measured. For each test, the modified plastic work $w_p$ was also calculated based on the stress strain curves. The breakage index $B_r$ is then plotted against $w_p$ in Figure 1(a) showing that the value of $B_r$ increases with the modified plastic work. Based on Figure 1(a), the hyperbolic function is proposed for the relationship between $B_r$ and $w_p$, as follows:

$$B_r = \frac{w_p}{\chi + w_p}$$

(15)

where $\chi$ is a material constant controlling the evolution rate of gradation. For Cambria sand, $\chi = 15000$ is obtained.

3.3 Influence of gradation on the position of critical state line

The critical state void ratio $e_c$ is a function of the mean effective stress $p'$. The relationship has traditionally been written as Eq. (16). The location of the CSL in the $e - \log(p')$ plane depends on three parameters $e_{ref}$, $\lambda$, and $p'_{ref}$:

$$e_c = e_{ref} - \lambda \ln \left( \frac{p'}{p'_{ref}} \right)$$

(16)

where $p'_{ref} = 100$ kPa is assigned. The couple ($e_{ref}$; $p'_{ref}$) determine a reference point in the $e - \log(p')$ plane, and $\lambda$ determines the slope of the CSL.

According to Biarez and Hicher (1994), the position of CSL moves down in the $e - \log(p')$ plane with the increase of the coefficient of uniformity $C_u$ ($d_{10}/d_{60}$) based on experimental tests on sub-angular Hostun sand. The movement of CSL has also been discussed by Muir Wood and Maeda (2008) using discrete element analysis which agrees with Biarez and Hicher (1994). However, there is no relationship available between the position of CSL and the gradation based on experiments. From the concept of CSL (at critical state, the material remains at a constant volume while it is subjected to a continuous distortion), if the CSL can move, it is no longer CSL. We note that, in this paper, we still use this term but representing the temporary position of CSL for a grading curve.

Drained compression tests on Cambria sand up to failure by Yamamuro and Lade (1996) were used to investigate the movement of CSL with the gradation. For each drained compression test, the void ratio at failure with the mean effective stress $p'$ was measured and considered as a temporary critical state corresponding to a gradation. The reference void ratio $e_{ref}$ representing the temporary position of CSL was then obtained by Eq. (16). Based on all drained compression tests, $e_{ref}$ is plotted against the breakage
index $B_n$ as shown in Figure 1(b) from which a hyperbolic curve is raised as follows:

$$e_{ref} = e_{ref0} + \left( e_{refu} - e_{ref0} \right) \frac{B_n}{\rho + B_n}$$  \hspace{1cm} (17)

where $e_{ref0}$ and $e_{refu}$ is the initial and ultimate reference critical state void ratios respectively; $\rho$ is a material constant relating to the movement rate of CSL with breakage index due to particle crushing. For Cambria sand, $e_{ref0} = 0.59$ and $\lambda = 0.006$ were obtained from drained triaxial compression tests under low confining stresses (less than 1 MPa) for which Yamamuro and Lade (1996) indicated that only slight grain crushing occurred. $e_{refu} = 0.13$ and $\rho = 0.16$ were obtained from Figure 1(b).

According to experimental investigations, Eq. (15) and Eq. (17) were proposed and can be used for Eq. (16). Therefore, particle crushing can influence directly the position of CSL, which results in the change of the density state $e_c/e$. All terms relating to $e_c/e$ (e.g., $\phi_p$, $\phi_{pt}$, $H$ etc.) are then influenced. The particle crushing is then incorporated into the model. The effect of particle crushing or the influence of some selected parameters are shown in Figure 2.

4. TEST SIMULATIONS

All determined values of constitutive parameters are summarized in Table 1, and will be subsequently used for simulations along different stress paths.

<table>
<thead>
<tr>
<th>$G_0$ (MPa)</th>
<th>$K_0$ (MPa)</th>
<th>$n$</th>
<th>$G_p$</th>
<th>$\phi_p$</th>
<th>$p_{cr0}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>35</td>
<td>26.3</td>
<td>0.4</td>
<td>3.5</td>
<td>37.5°</td>
<td>12</td>
</tr>
<tr>
<td>$c_p$</td>
<td>$e_{ref0}$</td>
<td>$\lambda$</td>
<td>$\chi$</td>
<td>$\rho$</td>
<td>$e_{refu}$</td>
</tr>
<tr>
<td>0.028</td>
<td>0.59</td>
<td>0.006</td>
<td>15000</td>
<td>0.16</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 1. Values of the constitutive parameters for Cambria sand specimens.

![Figure 1](image1.png)

Figure 1. Particle crushing effect: (a) Evolution of breakage index versus modified plastic work, and (b) Evolution of reference critical state void ratio versus breakage index.
5. SIMULATIONS OF DRAINED TESTS IN COMPRESSION AND EXTENSION

Figure 3 shows comparisons between experimental results and simulations for drained triaxial tests in compression with confining stresses varying from 2.1 to 52 MPa. Good agreement was achieved for all comparisons. The model well captured the mechanical behavior of sand influenced by particle crushing:

1. Under low confining stress (2.1 MPa), the sample exhibits a dilative behavior.
2. For higher confining stresses, the samples present a contractive behavior. This disappearance of dilation is related to more particle crushing occurred under high stresses (2.1-26 MPa). For tests under confining stresses from 2.1 to 26 MPa, the contraction increases with the increase of confining stress.
3. However, for tests under confining stresses from 26 to 52 MPa, volumetric strain decreases with the increase of confining stress. Yamamuro and Lade (1996) indicated that this effect is caused by the increase in the amount of volumetric contraction and particle crushing during isotropic consolidation. Lower void ratios are obtained with increasing confining stress, which in turn allows less volumetric contraction during shearing. This trend was well captured by the model incorporating particle crushing. The increase of particle crushing during isotropic loading results in a low amount of crushable sand left during shearing stage (see Fig.1a: the change of B becomes stable for high plastic work). As a result, the contraction of sample caused by particle crushing is reduced.

Using all parameters determined from drained compression tests, the model has also been applied to simulate the drained extension tests on Cambria sand with confining stress varying from 6 to 42 MPa by Bopp and Lade (2005). Good agreement was achieved between experimental results and simulations, as shown in Figure 4. The model well captured the stress strain response and the volumetric strain response for different confining stress levels.
Figure 3. Comparisons between experimental data and simulations for drained triaxial compression tests: (a-c) deviatoric stress versus major principle strain; (b-d) volumetric strain versus major principle strain.

Figure 4. Comparisons between experimental results and simulations for drained triaxial extension tests: (a) deviatoric stress versus major principle strain, and (b) volumetric strain versus major principle strain.

Conclusions

Experimental investigations were carried out on the evolution of gradation with the modified plastic work and on the evolution of the position of CSL with the gradation. Based on this, two constitutive equations were proposed for the relationship between the breakage index, the modified plastic work and the reference critical state void ratio. Relating these two equations with the function of CSL, a double-yield surface model accounting for the influence of particle crushing has been proposed.

Triaxial tests on Cambria sand were used to validate the proposed model. The model parameters can be easily determined from one isotropic compression test and several drained compression tests. Using determined parameters, different tests including drained tests in compression and extension, undrained tests in compression and extension were simulated by the proposed model. The grain size distributions for all tests were also predicted. All comparisons between experiments and simulations demonstrate that the model can accurately describe the mechanical behavior of granular materials with particle crushing, as well as the evolution of grain size distribution during loading.

REFERENCES

Salim, W. and Indraratna, B. (2004). “A new elasto-plastic constitutive model for coarse granular aggregates...