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Efficient implicit schemes for the treatment of the contact between deformable bodies: Application to shock-absorber devices

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Abstract - Classical and non-classical implicit time integration schemes are presented for the transient response of highly nonlinear problems of large deformation analysis exhibiting important material behaviour. These schemes have shown to lead to excellent convergence properties that make them cost efficient alternative to explicit schemes generally advocated as the best choice for these problems. To qualify the theory, examples are presented, whereby performance of the crushed shock-absorber devices, as well as their resolution based on the dynamic buckling of thin steel cylinder phenomenon are given. In these examples, it is shown that implicit schemes are more efficient (less CPU-cost for a given precision of the contact situation and capable of reaching higher precision) than the explicit scheme, despite that both use the Penalty method for the treatment of contact problems.

NOTATION

x	vector of nodal positions
$\dot{x}$	vector of nodal velocities
$\ddot{x}$	vector of nodal accelerations
M	mass matrix
F^{int}	vector of internal forces
F^{ext}	vector of external forces
R	residual (out-of-balance) vector
σ	Cauchy stress tensor
b	body force per unit mass
f	surface forces
ρ	current mass density of the material
ρ_0	initial mass density of the material
N	matrix of FEM shape functions
B	matrix of the derivatives of the FEM shape functions
γ_s	safety parameter in $[0,1]$ allowing for non-linearities
L^e	geometric factor depending on the space discretization (e is the element number)
C_p	pressure wave velocity in the material
β	first Newmark parameter
γ	second Newmark parameter
α_M	first free parameter balancing sampling time around $[t_n, t_{n+1}]$ for averaging inertia terms
α_F	second free parameter balancing sampling time around $[t_n, t_{n+1}]$ for averaging forces
K_T	tangent stiffness matrix
C_T	tangent damping matrix
g_n	normal penetration
ε_n	Penalty coefficient for penetration
ε_t	Penalty coefficient for friction
μ	friction coefficient
$S(t)$	current surface of the body
$V(t)$	current volume of the body
S_0	initial surface of the body
V_0	initial volume of the body
Δt	time increment: $\Delta t = t_{n+1} - t_n$
t	time

INTRODUCTION

When dealing with dynamics of highly nonlinear problems (i.e. problems with coupled material and geometrical non-linearities as encountered in crash analysis or metal forming processes simulation), one is often faced with the choice of a time-stepping algorithm: explicit or implicit. For those problems, the current trend in Computational Mechanics is as follows [1]: for dynamics where low frequencies dominate the response, implicit algorithms of the Newmark family (i.e. Newmark; Hilber,

Hughes & Taylor; ...) are generally felt to be more appropriate. On the contrary, wave propagation problems (shock response from impact or explosion e.g.) are commonly supposed to be best solved, at lower CPU-cost, by explicit techniques.

However these assertions are mainly based on linear analysis. For example, stability and/or accuracy properties are determined from a linearized problem (in a spatial neighbourhood of a given configuration) and for vanishingly small time steps. If the properties of the time-stepping algorithms predicted by this technique are reliable for explicit problems (where time steps are effectively very small), it's not the case for implicit algorithms where the time steps can be much higher and the assertions predicted can be completely false for general nonlinear problems even if they are true for linear problems.

NUMERICAL INTEGRATION OF TRANSIENT PROBLEMS

Equations of motion

FEM (space) semi-discretization of the equations of motion of a nonlinear structure leads to the coupled set of second order nonlinear differential equations [1,2,3,4,5]:

$$R = M \ddot{x} + F^{int}(x, \dot{x}) - F^{ext}(x, \dot{x}) = 0 \quad [1]$$

This set is completed by two sets of given initial conditions, for both

$$x_0 = x(t_0) \text{ and } \dot{x}_0 = \dot{x}(t_0) \quad [2]$$

Internal and external forces expressions can be written:

$$F^{int}(x, \dot{x}) = \int_{V(t)} [B]^T \{\sigma\} dV \quad [3]$$

$$F^{ext}(x, \dot{x}) = \int_{V(t)} \rho [N]^T \{b\} dV + \int_{S(t)} [N]^T \{f\} dS \quad [4]$$

Note that expression (4) collects all types of loading (applied through local or distributed actions, in a follow-up way or not, reactions to imposed displacements and contact situations) and that the consistent mass matrix reads

$$M = \int_{V(t)} \rho [N]^T [N] dV = \int_{V_0} \rho_0 [N]^T [N] dV_0 \quad [5]$$

Explicit scheme

This is the most advocated scheme [1,2] for integrating (1) in case of wave propagation and impact problems, i.e. high speed dynamics. It reads for a time step $\Delta t = t_{n+1} - t_n$:

$$\dot{x}_{n+\frac{1}{2}} = \dot{x}_{n-\frac{1}{2}} + \Delta t \ddot{x}_n \quad [6]$$

$$x_{n+1} = x_n + \Delta t \dot{x}_{n+\frac{1}{2}} \quad [7]$$

$$\ddot{x}_{n+1} = M^{-1} (F_{n+1}^{ext} - F_{n+1}^{int}) \quad [8]$$

and is subject to unstable behaviour unless [1] time steps are below a critical value (see equation 9) where the minimum is searched for all elements of the mesh.

$$\Delta t_{crit} = \gamma_s \min_{\text{for all } e} (L^e / C_\rho^e) \quad [9]$$

Implicit schemes: The generalized- α trapezoidal scheme

Implicit schemes are classically designed for vibrations and low speed dynamics of structures. The most general scheme for implicit integration of (1) is a generalized trapezoidal scheme [6] where updating of positions and velocities are based on “averaged” accelerations stemming from associated values between t_n and t_{n+1} . It reads for instance

$$\dot{x}_{n+1} = \dot{x}_n + (1 - \gamma) \Delta t \ddot{x}_n + \gamma \Delta t \ddot{x}_{n+1} \quad [10]$$

$$x_{n+1} = x_n + \Delta t \dot{x}_n + \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{x}_n + \beta \Delta t^2 \ddot{x}_{n+1} \quad [11]$$

or equivalently

$$\ddot{x}_{n+1} = \frac{1}{\beta \Delta t^2} \left[x_{n+1} - x_n - \Delta t \dot{x}_n - \left(\frac{1}{2} - \beta\right) \Delta t^2 \ddot{x}_n \right] \quad [12]$$

$$\dot{x}_{n+1} = \frac{\gamma}{\beta \Delta t} \left[x_{n+1} - x_n + \left(\frac{\beta}{\gamma} - 1\right) \Delta t \dot{x}_n + \left(\frac{\beta}{\gamma} - \frac{1}{2}\right) \Delta t^2 \ddot{x}_n \right] \quad [13]$$

The discretized motion equations (1) become:

$$R_{n,n+1} = \frac{1 - \alpha_M}{1 - \alpha_F} M \ddot{x}_{n+1} + \frac{\alpha_M}{1 - \alpha_F} M \ddot{x}_n + (F_{n+1}^{int} - F_{n+1}^{ext}) + \frac{\alpha_F}{1 - \alpha_F} (F_n^{int} - F_n^{ext}) = 0 \quad [14]$$

This general form was introduced, for linear problems, by Chung and Hulbert [6]. Particular choice of the parameters leads to well known [1,2,6] schemes such as

- $\alpha_M = \alpha_F = 0$ for Newmark scheme
- $\alpha_M = 0$ for Hilber-Hughes-Taylor scheme
- $\alpha_F = 0$ for Wood-Bossak-Zienkiewicz scheme

Iterative solution of the nonlinear system (14) requires first the elimination of acceleration and velocity at time t_{n+1} with the help of (12) and (13) and, secondly, the writing of the Hessian matrix of the system, i.e.

$$S = \left[\frac{1}{\beta \Delta t^2} \left(\frac{1 - \alpha_M}{1 - \alpha_F} \right) M + \frac{\gamma}{\beta \Delta t} C_T + K_T \right] \quad [15]$$

where K_T , C_T are respectively the tangent stiffness and damping matrices defined by

$$K_T = \frac{\partial}{\partial x} (F^{int} - F^{ext}) \quad [16]$$

$$C_T = \frac{\partial}{\partial \dot{x}} (F^{int} - F^{ext}) \quad [17]$$

Unconditional stability and second order accuracy of the scheme for linear problems [1,2,6] require that

$$\begin{aligned}\gamma &\geq 1/2 - \alpha_M + \alpha_F \\ \alpha_M &\leq \alpha_F \leq 1/2 \\ \beta &\geq 1/4 + (\alpha_F - \alpha_M)/2\end{aligned}\quad [18]$$

Associated rules for Newmark scheme are

$$\begin{aligned}\gamma &\geq 1/2 \\ \beta &\geq \frac{1}{4}(\gamma + 1/2)^2\end{aligned}\quad [19]$$

It's worth pointing out that though classical schemes require $0 < \alpha_F < 1/2$ (i.e. sampling the force in the second half of $[t_n, t_{n+1}]$), here, no such rule is followed for α_M , the sampling parameter for the inertia terms: it might be negative for instance, thus leading to an interpolation at t_{n+1} instead of an extrapolation.

Implicit schemes : The generalized- θ mid-point scheme (GMP)

An alternative to the above is a generalized midpoint scheme with constant acceleration over the time step [3,5]. In this case, the equations of motion (1) are solved at the sampling time:

$t_{n+\theta} = t_n + \theta(t_{n+1} - t_n)$ with $\theta > 0$, i.e.

$$R_\theta = M \ddot{x}_{n+\theta} + F^{int}(x_{n+\theta}, \dot{x}_{n+\theta}) - F^{ext}(x_{n+\theta}, \dot{x}_{n+\theta}) = 0 \quad [20]$$

where

$$\ddot{x}_{n+\theta} = \frac{2}{(\theta \Delta t)^2} [x_{n+\theta} - x_n - \theta \Delta t \dot{x}_n] \quad [21]$$

$$\dot{x}_{n+\theta} = \frac{2}{\theta \Delta t} \left[x_{n+\theta} - x_n - \frac{\theta \Delta t}{2} \dot{x}_n \right] \quad [22]$$

Iterative solution of the nonlinear system (20) requires the evaluation of the Hessian matrix of the system i.e.

$$S = \left[\frac{2}{(\theta \Delta t)^2} M + \frac{2}{\theta \Delta t} C_T + K_T \right] \quad [23]$$

Let some features of the θ -GMP scheme be with respect to α -family:

- all forces, even contact ones, are exactly estimated at time $t_{n+\theta}$ instead of being averaged between the values at t_n and t_{n+1} as in (14).
- the present scheme is $\ddot{x}_n$ -independent, yielding thus the final acceleration, as a post-treatment result given by (21)

$$\ddot{x}_{n+1} = \frac{\dot{x}_{n+1} - \dot{x}_n}{\Delta t} = \dot{x}_{n+\theta} \quad [24]$$

- since, for $\theta = 1$, it corresponds to the Newmark scheme with $\gamma = 1$ and $\beta = 0.5$, can be stated that it is conditionally stable and exhibits strong numerical damping. However, this scheme has proved to be

very efficient provided that θ is taken to be bigger than unity, i.e. choosing a sampling point out of range $[t_n, t_{n+1}]$, thus producing some backward evaluation of the final status of the system at the end of the time step, with respect to the sampling point for residual evaluation.

EXPLICIT VERSUS IMPLICIT SCHEME

Relative merits of explicit and implicit schemes have long been advocated in a dichotomic way [1]:

Table 1. Explicit vs. implicit schemes [1] (linear case)

Feature	Explicit	Implicit
Stability (Δt)	Bounded	Unbounded
Cost per Step	Low	Large (Iterative)
Memory	Low	Large
Implementation	Easy	Less easy
Numerical Damping	None	Variable

More important are the reference problems to which they apply best. Most authors agree to state that:

- high speed dynamics involving high frequency response are best treated through explicit schemes (wave propagation, impact, ...)
- inertia problems involving basic (low) frequency modes are best treated via implicit schemes (vibrations, fluid dynamics except for shock waves, seismic analysis,...)

The present paper is aimed at stressing that for particular nonlinear problems (short duration high-frequency wave propagation) unclassical choice, i.e. implicit schemes, may reveal cost efficient.

Though dedicated mathematical studies are lacking at this stage of our research to prove the evidence, two types of reasons may be invoked:

- first, high numerical (artificial) damping associated with these implicit schemes is not coercitive with respect to the own damping properties of the material, thus preventing the actual response to be inhibited by the scheme. On the contrary, this added numerical damping proves to stabilize greatly the transient response of spurious components.
- second, standard stability and accuracy analysis are based on linear theory applied to ideal oscillators (elastic materials). They might thus lead to inaccurate statements in the nonlinear range.

The key point of the newly interpolated implicit schemes presented is indeed in the way of sampling the α - or θ - family outside the classical range $[t_n, t_{n+1}]$, which could not be inferred by the linear theory. Hence, looking at the solution at times later than t_{n+1} , before stepping backwards for the solution at t_{n+1} (thus producing an effective interpolation of the solution at t_{n+1} between those at t_n and $t_{n+\theta}$ or $t_{n+\alpha_{\Delta t}}$), seems to be the reason for their success as demonstrated in the following examples.

NUMERICAL EXAMPLES

Introduction

As a numerical example of these concepts, the behaviour of two types of shock-absorber devices has been investigated. The finite element package METAFOR [7] was used. This finite element package is able to simulate quasi-static and transient large deformation problems with complex material behaviour, as well as frictional contact.

During the numerical experiments, time steps size for implicit schemes has not been guided by physical concepts related to frequency contents of the response but mainly by convergence purposes linked with the limited radius of convergence of the Newton-Raphson algorithm.

Shock absorber device

This first example deals with the numerical modelling of a shock absorber device. It is based on the turning inside-out of a thin walled ductile metal tube. This is generally called an "invertube" device. In this case, a plain tube is going round a hard curved die to produce the inversion, figure 1.

Figure 1: Axisymmetric shock absorber device. Initial geometry. All dimensions are in mm. The shaded area is considered to be rigid

This inversion, in turn, produces very large plastic strains which form an efficient energy absorbing mechanism during impact. In this way, the kinetic energy of the impacting bodies is dissipated through plastic deformation, in a controlled fashion at an acceptable rate. The yield limit of the material keeps the transmitted force below an acceptable upperbound. Hence, the deceleration is slower and less harmful for the people inside the car.

Numerical modelling of the collapse of such energy dissipating structures requires not only to take into account the plastic behaviour of the tube material, as well as inertial forces, but also to consider very large strains and large amplitude rigid body motions that develop and also, in this case, the accurate prediction of frictional forces. Thus a great number of advanced code capabilities are tested by running this kind of problems.

Similar problems were investigated by Beltran and Goicolea [8], by Garcia-Garino [9] with an explicit scheme and by Ponthot & Hogge [4,5] who compared the performances of explicit and implicit algorithms for impact problems. However, all the previous references dealt with frictionless contact. In the present paper, both explicit and implicit schemes have been used to integrate the equations of

motion in time. The material consists of an aluminum tube of 50.8 mm outside diameter times 63.5 mm length times 1.63 mm wall thickness. The material is supposed to behave like a J2 elastic-plastic material with linear isotropic hardening. The material parameters are given in table 2 and numerical parameters for the time marching algorithms are given in table 3. For this simulation, the following penalty parameters have been used: $\varepsilon_N = 10^7$ N/mm & $\varepsilon_T = 10^6$ N/mm

Table 2. Material properties for Aluminum

Young Modulus	$E = 67000 \text{ N/mm}^2$
Poisson Ratio	$\nu = 0.33$
Density	$\rho = 2700 \text{ kg/m}^3$
Yield Stress	$\sigma_v = 150 \text{ N/mm}^2$
Hardening parameter	$h = 44.7 \text{ N/mm}^2$

Table 3. Numerical parameters for time marching algorithms

Explicit scheme: safety parameter	$\gamma_s = 0.8$
Implicit scheme : Newmark parameters	$\gamma = 0.5$ and $\beta = 0.25$

The tube has been modelled using 300 quadrilateral elements (3×100) with 4 Gauss points and constant pressure to avoid locking. It is driven against a 3.97 mm radius die made of mild steel at a velocity of 44 m/s (144 Km/h). Thus a 50 mm prescribed vertical displacement over a time period of 0.00125 seconds is imposed on the upper nodes of the tube.

Figure 2: Deformed configurations (frictionless case) for $t=0.00$, $t=0.25$, $t=0.5$, $t=0.75$, $t=1.00$ and $t=1.25$ milliseconds

Table 4 displays the performances of the explicit and various implicit algorithms. From this table, it is obvious that if classical implicit scheme (i.e. Newmark), is much slower than the explicit scheme, all alternative implicit schemes, i.e. Hilber-Hughes-Taylor (HHT), Chung-Hulbert (CH) and Generalized Midpoint (GMP) exhibit much better performances.

The history of the deformation (frictionless case) is given in figure 2 and a comparison of the final configurations for the different friction coefficients ($\mu=0.0$; 0.15 & 0.30) is given in figure 3. In

figure 4 are displayed the time/load curves obtained for the three coefficients of friction and table 5 shows the computational effort for the different frictional case (GMP with $\theta=1.1$).

Table 4. Performances of the explicit and various implicit schemes (frictionless case)

Scheme	Steps	Iterations	CPU	CPU-ratio
Explicit	32687		25 min 19 sec	18.8
Implicit				
NEWMARK				
$\alpha_M = \alpha_f = 0$	4448	14334	43 min 30 sec	32.2
HHT($\alpha_M = 0$)				
$\alpha_f = 0.01$	812	2472	7 min 36 sec	5.6
$\alpha_f = 0.05$	328	966	3 min 04 sec	2.3
$\alpha_f = 0.1$	266	778	2 min 27 sec	1.8
CH				
$\alpha_f = .01 \ \alpha_M = -.97$	174	508	1 min 38 sec	1.2
$\alpha_f = .05 \ \alpha_M = -.85$	174	514	1 min 37 sec	1.2
$\alpha_f = .10 \ \alpha_M = -.70$	160	466	1 min 30 sec	1.1
GMP				
$\theta = 1.00$	449	1236	4 min 06 sec	3.0
$\theta = 1.01$	198	575	1 min 58 sec	1.4
$\theta = 1.05$	143	412	1 min 27 sec	1.1
$\theta = 1.10$	142	404	1 min 22 sec	1.0
$\theta = 1.15$	149	431	1 min 28 sec	1.1

Figure 3: Comparison of the final configurations as a function of friction coefficient.
($\mu=0.0$: upper configuration; $\mu=0.15$: intermediate configuration ; $\mu=0.3$: lower configuration)

Table 5. Performances of GMP implicit scheme for various friction coefficients

Friction	Steps	Iterations	CPU
$\mu = 0.00$	142	404	1 min 22 sec
$\mu = 0.15$	143	415	1 min 24 sec
$\mu = 0.30$	147	432	1 min 32 sec

Figure 4: Applied load as a function of time for different friction coefficients

Dynamic buckling of a cylinder

This second example deals with the buckling of a thin steel cylinder by axial compression into a rigid matrix. This problem has been first mentioned by Tod A. Laursen [10] who compares the results with and without friction in a quasi-static situation.

In this axisymmetric problem, a steel cylinder is forced downwards (via displacement control of its top surface) into a rigid fixture. The steel is modeled by an elastoplastic law with linear isotropic hardening and has the material properties shown in table 6. Spatial discretization of the cylinder is achieved using 177 bilinear four-nodes elements (3×59) with constant pressure. As the calculation proceeds, the cylinder initially moves down into the die, but when a critical axial load is reached, it begins a series of buckles, as shown in Figure 5. Importantly, this buckling occurs without any initial geometric imperfections. The contact is considered frictionless and is solved using the Penalty method. Information about the initial geometry is given in table 7.

Table 6. Material properties for Steel

Young Modulus	$E = 210000 \text{ N/mm}^2$
Poisson Ratio	$\nu = 0.3$
Density	$\rho = 7850 \text{ kg/m}^3$
Yield Stress	$\sigma_v = 700 \text{ N/mm}^2$
Hardening parameter	$h = 808 \text{ N/mm}^2$

Table 7. Geometrical properties of the cylinder

Internal Diameter	$D_{int} = 27.00 \text{ mm}$
External Diameter	$D_{ext} = 31.75 \text{ mm}$
Thickness	$e = 4.75 \text{ mm}$
Height	$h = 180.00 \text{ mm}$

Table 8. Numerical parameters for time marching algorithms

Explicit	Safety parameter	$\gamma_s = 0.2$
Implicit (CH)	Inertia parameter	$\alpha_M = -0.97$
	Force parameter	$\alpha_F = 0.01$
	Newmark parameters	$\gamma = 0.5$ and $\beta = 0.25$

Figure 5: Buckling of a cylinder, configurations from 0 to 11 milliseconds

To compare the explicit and the implicit schemes, this problem had to be solved dynamically. So it was decided to crash it down 110 mm in 11ms (10m/s at constant velocity). The problem was solved using a large set of Penalty parameters ranging from 10^6 N/mm (which gives a maximum penetration encountered lower than 100 μ m) to as large as possible, with the explicit scheme and with the Chung-Hulbert (CH) implicit scheme (see tables 9 and 10). The low value of the safety parameter ($\gamma_s = 0.2$) in the explicit scheme is necessary to obtain the physical solution of the problem: using a larger one results in a solution with the third and last buckle generated on the top of the cylinder as shown in figure 6. The correct solution – see figure 5 - is only found with a γ_s lower than 0.2 for the explicit scheme and in any case with the implicit scheme.

Table 9. Buckling of a cylinder (Explicit scheme)

Penalty coefficient <i>N/mm</i>	CPU	Steps	Penetration <i>mm</i>
10^6	2 h 38 min 26 sec	367189	7.e-2
5.10^6	2 h 40 min 42 sec	369095	1.e-2
10^7	-----	Failed	-----

Table 10. Buckling of a cylinder (Implicit scheme CH)

Penalty coefficient N/mm	CPU	Steps	Iterations	Penetration mm
10^6	33 sec	178	423	4.e-2
$5 \cdot 10^6$	40 sec	214	494	1.e-2
10^7	41 sec	129	533	6.e-3
10^8	48 sec	178	624	6.e-4
10^9	1 min 00 sec	148	771	6.e-5
10^{10}	1 min 19 sec	176	1038	6.e-6
10^{11}	4 min 32 sec	1647	3503	6.e-7
10^{12}	6 min 38 sec	2275	5261	7.e-8
10^{13}	17 min 18 sec	5814	14188	7.e-9

The CPU-costs and the numbers of time steps and iterations required to solve the problem are given in Table 9 and 10. An idea of the precision obtained is given by the maximum penetration encountered during the process.

Figure 6: Explicit scheme with a safety factor greater than 0.2:
Final configuration and external contact forces.
A spurious buckle appears on the top

Looking at these results, it is obvious that the analysts strongly influenced the precision of modelling of the contact situation by changing the penalty coefficient: the larger the penalty coefficient and the smaller the penetration. The best choice would then be to use infinite or very large penalty coefficient to best fit the impenetrability condition. The problem is that the explicit and

implicit integration methods encounter different numerical difficulties when this coefficient becomes too large.

Explicit scheme does not accept too large Penalty coefficients because of the too important numerical perturbations induced by a new contact configuration: using a discrete step by step process to solve the problem, the explicit scheme suddenly encounters, at a new contact node (unconstrained at the latest time step), a large contact force. As the explicit scheme computes positions at a given time from the forces at the preceding step, and remembering that the contact force only appears at the present step, we understand that the penetration is nothing of a physical equilibrated value but it is the distance that the node would have covered if there were no contact. However the time steps in explicit are generally rather small, preventing too large “first penetrations”, but if there is a large Penalty Coefficient, the contact force induced could be so large that the node in contact is bounced away from the contacting body in the following step. This phenomenon can produce strong numerical oscillations of the contact nodes or even failure of the solving process if the penalty coefficient is too large.

Implicit schemes do not like too large Penalty coefficient too but for quite another reason: a too high penalty coefficient ill-conditions the tangent stiffness matrix of the system; this matrix is necessary to lead the iterative process towards an equilibrated solution (Newton-Raphson process). The tangent stiffness matrix has contact terms directly linked to the Penalty coefficient and if this latter is too large, the corresponding terms can become much greater than the other physical terms. It can then swap out most of the information necessary to converge towards a solution that the method can no more see. In table 10, it can be seen that not only the precision increases as the penalty coefficient grows, but also the CPU-cost. This is due to the crescent ill-conditioning of the stiffness matrix that makes the process more demanding with respect to time steps and iterations to solve the problem (with a better accuracy).

Figure 7: Buckling of a cylinder: variation of the CPU-cost with the precision on a log-log scale

It can also be seen that (Fig. 7) the variation of the CPU-cost as a function of the maximum penetration. In summarising the explicit scheme, CPU-cost is found to be more or less independent of the Penalty coefficient (then, also of the desired precision, as far as this one can be reached) and that for the implicit scheme, it follows, for this problem, the relation $CPU-cost = C_1 g_n^{-1/4} = C_2 \varepsilon_n^{1/4}$ with g_n , the normal penetration and ε_n , the penalty coefficient (C_1 and C_2 are two constants). This graphic shows that even for a very large penalty parameter (that gives a very low penetration) the implicit

scheme is cheaper than the explicit one. At the limit of precision that can be reached by the explicit scheme, the implicit one gives the same solution with a CPU 240 times lower.

CONCLUSIONS

In this paper, two examples show that, for high speed dynamic problems, the explicit scheme can be much more expensive (200 times in the second problem) than implicit schemes. This is in contradiction with what is generally thought in the computational mechanics community. Moreover, if it is agreed to “pay the price”, the implicit schemes can lead to a much more precise solution. This is simply done by increasing the penalty parameters which in turn lead to smaller penetrations. We have shown too that the explicit scheme has to be very carefully used (safety parameter γ_s lower than 0.2 in some case) to get an accurate solution.

REFERENCES

1. T. Belytschko and T-J-R. Hughes, editors. *Computational Methods for Transient Analysis*. North Holland, 1983.
2. T-J-R. Hughes. *The Finite Element Method*. Prentice-Hall, 1987.
3. J-P. Ponthot. *Traîtement unifié de la Mécanique des Milieux Continus solides en grandes transformations par la méthode des éléments finis*. PhD thesis (in French), Université de Liège, Liège, Belgium, 1995.
4. J-P. Ponthot and M. Hogge. *On relative merits of implicit / explicit algorithms for transient problems in metal forming simulation*. In International Conference on Numerical Methods for Metal Forming in Industry, Vol 2, pp 128-148, Baden-Baden, Germany, Sept. 1994.
5. M. Hogge and J-P. Ponthot. *Efficient implicit schemes for transient problems in metal forming simulation*. In NUPHYMAT'96, Numerical and Physical Study of Material Forming Processes, CEMEF - Ecole Nationale supérieure des Mines de Paris Sophia-Antipolis, France, June 5-7, 1996.
6. J. Chung and G-M. Hulbert. *A time integration algorithm for structural dynamics with improved numerical dissipations: the generalized- α method*. In Journal of Applied Mechanics, Vol 60, pp 371-375, 1993.
7. J-P. Ponthot. *Mode d'emploi pour la version pilote de METAFOR, module de calcul en grandes déformations*. Technical Report TF-24 (in French), Université de Liège, LTAS, 1992.
8. Beltrán F. and Goicolea J. *Large strain plastic collapse: a comparison of explicit and implicit solutions*. In Owen R., Hinton H. and Oñate E., editor, International Conference on Computational Plasticity, COMPLAS II, pp 1125-1136, Barcelona, Spain, 1989.
9. C. García Garino. *Un modelo numerico para el analysis de solidos elastoplasticos sometidos a grandes deformaciones*. PhD thesis (in Spanish), E.T.S. Ingenieros de Caminos, UPC, Barcelona, Spain, 1993.
10. Tod-A. Laursen. *Formulation and Treatment of Frictional Contact Problems using Finite Elements*. PhD thesis, Department of Mechanical Engineering, Stanford University, 1992.