DYNAMIC ANALYSIS OF A COUPLED FLUID STRUCTURE PROBLEM WITH FLUID SLOSHING

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ABSTRACT

The present paper is related to the study of a generic linear coupled fluid/structure problem, in which an elastic beam is coupled with an inviscid fluid, with or without sloshing effects. A previous study [18] focussed on added mass effects; the present study is devoted to the coupling effects between fluid sloshing modes and structure with fluid added mass modes.

The discretization of the coupled linear equations is performed with an axisymmetric fluid pressure formulated element, expanded in terms of a FOURIER series [14]. Various linear fluid model are taken into account (compressible, incompressible, with or without sloshing) with the corresponding coupling matrix operator. The modal analysis is performed with a MATLAB program, using the non-symmetric LANCZOS algorithm [16]. The temporal analysis is performed with classical numerical techniques [10], in order to describe the dynamic response of the coupled problem subjected to a simple sine wave shock.

The coupling effects are studied in various conditions represented by several non-dimensionnal numbers [12] such as the dynamic FROUDE number and the mass number, based on the geometrical and physical characteristics of the coupled problem.

Comparisons are performed on the coupled problem with or without free surface modeling, with a modal and temporal analysis. Coupling effects are exhibited and quantified; the numerical results obtained in the modal analysis here are in good agreement with other previous studies, carried out on different geometry [3,15]. The temporal analysis gives another point of view on the importance of the coupling effects and their importance at low dynamic FROUDE numbers.

The present study gives and will be completed with a non-linear analysis (for both fluid and structure problems) of the coupled problem, using a finite element and finite volume explicit coupling procedure [19].

INTRODUCTION

The numerical simulation of coupled fluid structure problems is a subject of great interest, and many studies were carried out on this topic over the past years [13], among which the influence of a fluid free surface on the dynamic of the coupled problem (see *e.g.* in some recent studies [1,6,17]). In the present paper, we study the following generic coupled problem, represented by Fig. (1), and focus on the coupling effect occurring between the fluid sloshing modes and the bending structure modes.

Figure 1. Generic coupled fluid structure problem with fluid free surface

The problem is characterized by various non-dimensionnal numbers such as the confinement ratio $\alpha = \frac{R'}{R}$, the filling ratio $\lambda = \frac{l}{h}$ and the length ratio $\eta = \frac{l}{R}$; the dynamic FROUDE number is defined as: $F_D = \frac{e}{R} \times \sqrt{\frac{E}{\rho_s g l}}$.

1. GOVERNING EQUATIONS AND VARIATIONNAL FORMULATION FOR THE DYNAMIC PROBLEM

The structure problem is described in terms of displacement; the governing equations are the following ones (the description includes the dynamic equation, the structure boundary conditions and the coupling condition with the fluid problem):

$$\begin{aligned} \rho_s S \frac{\partial^2 u}{\partial t^2} + EI \frac{\partial^4 u}{\partial z^4} &= f - \int_0^{2\pi} p(R, \theta, t) R \cos \theta d\theta \\ u_{(z=0)} &= 0 & u_{(z=l)} &= 0 \\ \frac{\partial u}{\partial z}_{(z=0)} &= 0 & \frac{\partial u}{\partial z}_{(z=l)} &= 0 \end{aligned} \quad (1)$$

The fluid problem is described in terms of pressure [14]; the governing equations are (the description includes the local equation, the fluid boundary conditions and the coupling condition with the structure):

$$\begin{aligned} \Delta p &= 0 \\ \frac{\partial p}{\partial z}_{(z=0)} &= \frac{\partial p}{\partial r}_{(r=R')} = 0 \\ \frac{\partial p}{\partial r}_{(r=R)} &= -\rho_F \frac{\partial^2 u}{\partial t^2} \cos \theta \end{aligned} \quad (2)$$

The free surface condition can be described with or without gravity waves, that is:

$$p_{(z=h)} = 0 \quad (3)$$

in the former case and:

$$\frac{\partial p}{\partial z}_{(z=h)} = -\frac{1}{g} \frac{\partial^2 p}{\partial z^2}_{(z=h)} \quad (4)$$

in the latter case.

The variationnal formulation of the dynamic problem is the following one; find $u(t)$ and $p(t)$ such as for all admissible virtual fields of displacement and pressure $(\delta u, \delta p)$:

$$\begin{aligned} \frac{d^2 m_S(u(t), \delta u)}{dt^2} + k_S(u(t), \delta u) &= \\ < f(t), \delta u > - r^*(p(t), \delta u) \\ \frac{d^2 m_F(p(t), \delta p)}{dt^2} + k_F(p(t), \delta p) &= \\ \rho_F \frac{d^2 r(u(t), \delta p)}{dt^2} \end{aligned} \quad (5)$$

with the initial conditions for the pressure and displacement field :

$$\begin{aligned} u(0) &= u_0 & p(0) &= p_0 \\ \frac{\partial u}{\partial t}(0) &= \dot{u}_0 & \frac{\partial p}{\partial t}(0) &= \dot{p}_0 \end{aligned} \quad (6)$$

The various linear and bilinear forms used in the above formulation are the following ones.

- $m_s(u, \delta u) = \int_0^h \rho_s S u(z) \delta u(z) dz$ is the structure mass bilinear form, $k_s(u, \delta u) = \int_0^h EI \frac{d^2 u(z)}{dz^2} \frac{d^2 \delta u(z)}{dz^2} dz$ is the structure stiffness bilinear form ;
- $m'_F(p, \delta p) = \int_0^{R'} \int_0^{2\pi} \frac{p(r, \theta, h) \delta p(r, \theta, h)}{g} r dr \cos \theta d\theta$ and $k_F(p, \delta p) = \int_0^{R'} \int_0^{2\pi} \int_0^h \nabla p(r, \theta, z) \nabla \delta p(r, \theta, z) r dr \cos \theta d\theta dz$ are the fluid mass and stiffness bilinear forms ;
- $r(p, u) = - \int_0^h \int_0^{2\pi} p(R, \theta, z) R \cos \theta d\theta u(z) dz$ is the fluid/structure interaction operator, $r^*(.,.)$ is the adjoint operator.

2. FINITE ELEMENT DISCRETIZATION

The discretization of the variationnal formulation is performed with a finite element approach. Since the coupled problem is geometrically axi-symmetric but the dynamic loading is non axi-symmetric, the pressure is expanded into a FOURIER series as¹:

$$p(r, \theta, z) = p_0(r, z) + \sum_{n \geq 1} p_n(r, z) \cos(n\theta) + \sum_{m \geq 1} p_m(r, z) \sin(m\theta) \quad (7)$$

The 3D fluid problem (r, θ, z) is then turned into a 2D problem (r, z) . The two-dimensionnal fluid domain is meshed with four nodes linear finite element. Figure (2) gives a representation of the fluid reference element.

Figure 2. Linear 2D fluid finite element

¹ In practical application with the beam coupling, only the first cosine term ($n = 1$) is to be taken into account.

The fluid shape functions are linear with respect to both local variables ξ, η :

$$N_i^F(\xi, \eta) = \frac{(1 + \xi_i \xi)(1 + \eta_i \eta)}{4} \quad (8)$$

The discretization of the fluid bilinear forms gives the mass and stiffness matrices M_F , $K_F = \bar{K}_F + n^2 \tilde{K}_F$ for the fluid problem. The corresponding elementary matrices are defined by the following analytical expressions.

$$\bar{k}_{ij}^F = abr_0 \left[\frac{\xi_i \xi_j}{4a^2} (1 + \frac{\eta_i \eta_j}{3}) + \frac{\eta_i \eta_j}{4a^2} (1 + \frac{\xi_i \xi_j}{3}) \right] + \frac{a^2 \eta_i \eta_j}{12b} (\xi_i + \xi_j) \quad (9)$$

$$\tilde{k}_{ij}^F = \frac{b}{8} \left[1 + \frac{\eta_i \eta_j}{3} \right] \times \left[2(\xi_i + \xi_j) - \frac{2r_0 \xi_i \xi_j}{a} + \left(1 - \frac{r_0(\xi_i + \xi_j)}{a} + \frac{r_0^2 \xi_i \xi_j}{a^2} \right) \times \ln \left(\frac{r_0 + a}{r_0 - a} \right) \right] \quad (10)$$

$$m_{ij}^F = \frac{a}{g} \left[r_0 + \frac{r_0 \xi_i \xi_j}{3} + \frac{a(\xi_i + \xi_j)}{3} \right] \times [(1 + \eta_i)(1 + \eta_j)] \quad (11)$$

If compressibility effects are taken into account in the fluid model, the dynamic equation of the fluid problem is $\frac{1}{c^2} \cdot \frac{\partial^2 p}{\partial t^2} - \Delta p = 0$.

The variational formulation for the fluid problem has to take into account the mass bilinear form

$$m_F(p, \delta p) = \int_0^{R/2} \int_0^{2\pi} \int_0^h \frac{p(r, \theta, z) \delta p(r, \theta, z)}{c^2} r dr \cos \theta d\theta dz, \quad \text{which is}$$

discretized into the mass matrix M_F . The corresponding elementary matrix is :

$$m_{ij}^F = \frac{ab}{4} \left[r_0 + \frac{r_0 \xi_i \xi_j}{3} + \frac{a(\xi_i + \xi_j)}{3} \right] \times \left[1 + \frac{\eta_i \eta_j}{3} \right] \quad (12)$$

The discretization of the structure problem is performed with two node finite element with two degrees of freedom $u, \partial u / \partial z$ and cubic shape functions [5].

The coupled dynamic fluid/structure problem is then written in the non-symmetric form:

$$\begin{bmatrix} M_S & 0 \\ -\rho_F R^T & M'_F \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} K_S & R \\ 0 & K_F \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} \phi_S \\ \phi_F \end{Bmatrix} \quad (13)$$

where R is the coupling matrix, which discretizes $r(\cdot, \cdot)$.

When compressibility effects are taken into account in the fluid problem, the coupled problem is:

$$\begin{bmatrix} M_S & 0 \\ -\rho_F R^T & M_F + M'_F \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} K_S & R \\ 0 & K_F \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} \phi_S \\ \phi_F \end{Bmatrix} \quad (14)$$

A MATLAB program is developed in order to perform the numerical analysis. This program is validated with the comparison between calculated eigenvalues of a fluid problem with free surface and analytical eigenvalues.

Frequency	f ₁	f ₂	f ₃	F ₄
Analytical	0.9963	2.8519	3.9734	4.8513
Numerical	0.9965	2.8641	4.0398	5.0367

Table 1. Elementary validation of the fluid finite element for sloshing modes in axi-symmetric problems

Table (1) gives the analytical and numerical frequencies (in Hz) for the elementary problem².

3. MODAL ANALYSIS OF THE COUPLED PROBLEM

The modal analysis of the coupled problem cannot be performed with the mass and stiffness matrix given in Eq. (13) because the matrix M'_F is singular. One has to split the degrees of freedom of

the fluid problem according to $\{P\} = \begin{Bmatrix} P_o \\ \tilde{P} \end{Bmatrix}$ where P_o are the

degrees of freedom for the fluid free surface and $\tilde{P}$ are the degrees of freedom of the rest of the fluid. Equation (13) becomes:

$$\begin{bmatrix} M_S & 0 & 0 \\ -\rho_F R_o^T & M_F^o & 0 \\ -\rho_F R^T & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{P}_o \\ \ddot{\tilde{P}} \end{Bmatrix} + \begin{bmatrix} K_S & R_o & \tilde{R} \\ 0 & K_F^o & \tilde{K}_F^o \\ 0 & \tilde{K}_F^{oT} & \tilde{K}_F \end{bmatrix} \begin{Bmatrix} U \\ P_o \\ \tilde{P} \end{Bmatrix} = \begin{Bmatrix} \phi_S \\ \phi_o \\ \tilde{\phi} \end{Bmatrix} \quad (15)$$

After static condensation, the system is reformulated in terms of U and P_o . The corresponding modal problem is then:

$$\begin{bmatrix} K_S & R_o - \tilde{R} \tilde{K}_F^{-1} \tilde{K}_F^{oT} \\ 0 & K_F^o - \tilde{K}_F^{oT} \tilde{K}_F^{-1} \tilde{K}_F^o \end{bmatrix} \begin{Bmatrix} U(\omega) \\ P_o(\omega) \end{Bmatrix} = \begin{Bmatrix} \phi_S \\ \phi_o \end{Bmatrix} \quad (16)$$

$$\omega^2 \begin{bmatrix} M_S + \rho_F \tilde{R} \tilde{K}_F^{-1} \tilde{R}^T & 0 \\ -\rho_F (R_o^T - \tilde{K}_F^{oT} \tilde{K}_F^{-1} \tilde{R}^T) & M_F^o \end{bmatrix} \begin{Bmatrix} U(\omega) \\ P_o(\omega) \end{Bmatrix}$$

The coupled problem has the same structure as the initial problem given by Eq. (13): the non-symmetry is typical of a displacement/pressure formulation [14,16]. Equation (16) also exhibits the coupling matrix between the fluid and structure problem $R_o - \tilde{R} \tilde{K}_F^{-1} \tilde{K}_F^{oT}$, which takes into account the free surface terms. On the other hand, the structure part of the coupled problems exhibits the hydrodynamic added mass term $\rho_F \tilde{R} \tilde{K}_F^{-1} \tilde{R}^T$, which correspond to the structure problem coupled with the incompressible fluid with boundary condition $p=0$ for the free surface. Thus, restricting Eq. (16) to the structure or fluid separate terms leads to the determination of the eigenmodes of:

✓ the fluid sloshing without structure coupling effects the problem to solve is:

$$(K_F^o - \tilde{K}_F^{oT} \tilde{K}_F^{-1} \tilde{K}_F^o) \times P_o(\omega) = \omega^2 M_F^o \times P_o(\omega) \quad (17)$$

² The validation case is that of a cylindrical tank for which the analytical eigenvalues are given by [8], p. 288. Table (1) gives the comparison of numerical and analytical model with $R=0.1$ m, $\eta=1$, $\alpha=2$ and $g=10$ m/s². The mode order is $n=1$.

- ✓ the structure problem coupled with the fluid without fluid sloshing effects; the problem to solve is:

$$K_s \times U(\omega) = \omega^2 (M_s + \rho_f \tilde{R} \tilde{K}_f^{-1} \tilde{R}^T) \times U(\omega) \quad (18)$$

The non-symmetric problem given by Eq. (16) is solved using the LANCZOS algorithm adapted to non-symmetric problems [16]. Table (2) compares the computed eigenvalues with the **lanzcos** function developed in the MATLAB code and the theoretical eigenvalues for the non-symmetric example proposed in [16]; this comparison validates the numerical calculation of eigenvalues for generalized non-symmetric problem with our numerical code.

Theoretical eigenvalues	MATLAB lanzcos function
$0.10000 + 0.00000 i$	$0.1000 + 0.0000 i$
$4.00000 \pm 1.00000 i$	$4.0000 \pm 1.0000 i$
$2.40000 \pm 7.32000 i$	$2.39974 \pm 7.31897 i$
$7.00000 + 0.00000 i$	$6.99999 + 0.00000 i$
$7.00000 + 0.00000 i$	$7.00088 + 0.00000 i$
$8.00000 \pm 127.00000 i$	$7.99997 \pm 127.00001 i$
$11.00000 + 0.00000 i$	$11.00005 + 0.00000 i$
$130.000000 + 0.00000 i$	$129.99998 + 0.00000 i$
$-2000.00000 + 0.00000 i$	$-1999.99995 + 0.00000 i$

Table 2. Elementary validation of the lanzcos function implemented in the Matlab code

A modal analysis of decoupled and coupled fluid/structure problem is performed on the generic case represented by Fig. (1). The geometrical and physical data of the problem are characterized by the non-dimensionnal numbers $\alpha = 2$, $\eta = 5$, λ , F_D and M_A . Table (3) gives the eigenfrequencies of the structure without fluid coupling.

Frequency (Hz)	f_1	f_2	f_3	f_4
Structure mode ($F_D \sim 1$)	3.4705	21.7493	60.90	119.34
Structure mode ($F_D \sim 10$)	34.71	217.49	608.98	1193.4

Table 3. Structure mode for $F_D \sim 100$ and $F_D \sim 10$

Tables (4) to (7) show the eigenfrequencies of the fluid problem with and without structure coupling, the structure problem coupled with fluid with or without fluid sloshing, with $F_D = 100$ and $M_A = 8$, for different values of λ .

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.0996	2.8757	4.0789	5.1344
Structure w/o sloshing	34.6724	212.768	555.541	1046.20
Fluid with structure	1.0996	2.8757	4.0789	5.1344
Structure with sloshing	34.6729	212.770	555.543	1046.20

Table 4. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 10$, $M_A = 8$ and $\lambda = 25\%$

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2546	2.8761	4.0789	5.1344
Structure w/o sloshing	33.6284	164.728	461.918	944.62
Fluid with structure	1.2545	2.8760	4.0788	5.1344
Structure with sloshing	33.6349	164.7326	461.918	944.620

Table 5. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 10$, $M_A = 8$ and $\lambda = 50\%$

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2905	2.8761	4.0789	5.1344
Structure w/o sloshing	27.2966	136.706	404.423	869.101
Fluid with structure	1.2899	2.8756	4.0787	5.1343
Structure with sloshing	27.3250	136.707	404.423	869.110

Table 6. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 10$, $M_A = 8$ and $\lambda = 75\%$

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2962	2.8761	4.0789	5.1344
Structure w/o sloshing	19.3432	122.546	377.832	804.374
Fluid with structure	1.2943	2.8750	4.0783	5.1341
Structure with sloshing	19.4094	122.552	377.834	804.374

Table 7. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 10$, $M_A = 8$ and $\lambda = 100\%$

For a high dynamic FROUDE number, the fluid and structure coupled modes are practically uncoupled. The fluid eigenfrequencies are slightly increased as well as the structure eigenfrequencies with sloshing compared to the eigenfrequencies without sloshing. These observations are made for all fluid filling ratios. In this case, a separate modal analysis for fluid and structure problem can be performed.

Tables (8) to (11) show the eigenfrequencies of the fluid problem with and without structure coupling, the structure problem coupled with fluid with or without fluid sloshing, with $F_D = 10$ and $M_A = 8$, for different values of λ .

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.0996	2.8757	4.0789	5.1344
Structure w/o sloshing	3.4672	21.2768	55.5542	104.618
Fluid with structure	1.0989	2.8728	4.0798	5.1345
Structure with sloshing	3.4698	21.2952	55.5690	104.623

Table 8. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 1$, $M_A = 8$ and $\lambda = 25\%$

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2546	2.8761	4.0789	5.1344
Structure w/o sloshing	3.3628	16.4728	46.1918	94.4619
Fluid with structure	1.2413	2.8402	4.0904	5.1369
Structure with sloshing	3.4199	16.5525	46.1949	94.4664

Table 9. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 1$, $M_A = 8$ and $\lambda = 50\%$

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2905	2.8761	4.0789	5.1344
Structure w/o sloshing	2.7297	13.6706	40.4423	86.9110
Fluid with structure	1.2126	2.6420	4.1006	5.1410
Structure with sloshing	3.1352	13.6925	40.4485	86.9158

Table 10. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 1$, $M_A = 8$ and $\lambda = 75\%$

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2962	2.8761	4.0789	5.1344
Structure w/o sloshing	1.9343	12.2546	37.7832	80.4374
Fluid with structure	1.0813	2.9832	4.0931	5.1385
Structure with sloshing	2.2147	12.3579	37.8024	80.4451

Table 11. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 1$, $M_A = 8$ and $\lambda = 100\%$

For a low dynamic FROUDE number, some fluid and structure modes are likely to be coupled. When an eigenfrequency of the structure coupled with a fluid without sloshing is near an

eigenfrequency of fluid sloshing, a coupling effect can be observed.³ This coupling effect is characterized by the following facts:

- ✓ the sloshing mode eigenfrequency is decreased when coupled with the structure;
- ✓ the structure mode eigenfrequency is raised because of the fluid added mass is lower due to sloshing effect;
- ✓ the sloshing modes above the coupled one are increased whereas the sloshing modes under the coupled one are decreased.

In this case, an uncoupled analysis is not valid. The previous observations are more significant for a low mass number, as shown by Tab. (12).

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid w/o structure	1.2905	2.8761	4.0789	5.1344
Structure w/o sloshing	1.4055	8.0652	22.1989	45.7151
Fluid with structure	0.8375	3.0122	4.0972	5.1359
Structure with sloshing	2.0310	8.4110	22.2076	45.7264

Table 12. Uncoupled and coupled fluid structure mode with or without fluid sloshing for $F_D \sim 1$, $M_A = 1$ and $\lambda = 75\%$

Figure 3. Evolution of the frequency ratio ε of structure with or without fluid sloshing for different dynamic FROUDE numbers

Figure (3) gives the evolution of the ratio $\varepsilon = \frac{\omega_{w/o} - \omega_w}{\omega_{w/o}}$ between the first structure eigenfrequency coupled with the fluid with (ω_w) or without ($\omega_{w/o}$) fluid sloshing the two dynamic FROUDE numbers toward the filling ratio λ , and shows the importance of the fluid sloshing for low FROUDE numbers and high filling ratio.

Figures (4) and (5) illustrates the coupling process for the case $F_D = 10$, $\lambda = 75\%$.

α	Structure	Fluid
2	+14.86%	-8.14%
5	+6.26%	-2.40%

Table 13. Influence of confinement on the coupling effect

The influence of confinement on the coupling effect is given by Tab. (13), which compares the ratio ε for the fluid and structure eigenfrequencies with or without sloshing (for the structure) and

³ These effects of the coupling between fluid mode and structure mode in the case of a free surface have been observed by many authors for other fluid-structure coupled problems (see e.g. [3], and [15]).

with or without structure coupling (for the fluid), for two different values of α . The coupling effect is amplified by confinement (i.e. for small values of α).

Figure 4. Structure mode without fluid (3.4705 Hz) and fluid sloshing mode without structure (2.8761 Hz)

Figure 5. Structure mode coupled with fluid without fluid sloshing (2.7297 Hz) and with fluid sloshing (3.1352 Hz)

When the fluid is supposed compressible, the coupled problem is represented by Eq. (14); due to the presence of the fluid mass matrix M_F , the global fluid mass matrix $M_F + M'_F$ is non-singular, and a modal analysis can be performed on system given by Eq. (14), with the LANCZOS algorithm. As the study is related to low frequency phenomena, the matrix M_F (e.g. the influence of the compressibility effects) has a slight influence on the numerical results, as shown by Tab. (14). The problem represented by Eq. (14) involves large scaled sparse matrices, whereas the problem represented by Eq. (16) deals with small scale filled matrices. Both systems are unsymmetric.

Frequency (Hz)	f_1	f_2	f_3	f_4
Fluid modes - Eq. (14)	1.2905	2.8761	4.0789	5.1344
Fluid modes - Eq. (17)	1.2905	2.8761	4.0789	5.1344
Fluid coupled mode - Eq. (14)	1.2126	2.6420	3.1352	4.1006
Fluid coupled mode - Eq. (16)	1.2126	2.6420	3.1352	4.1006
Structure coupled mode - Eq. (14)	2.7297	13.671	40.442	86.911
Structure coupled mode - Eq. (16)	2.7297	13.670	40.441	86.905

Table 14. Modal analysis with Eq. (14) or with Eq. (16)

The effects of coupling between fluid modes and structure modes can be represented in the following simple model with a 2D

acoustic fluid and a single degree of freedom structure problem, as represented by Fig. (6).

Figure 6. 2D fluid problem coupled with a single degree of freedom structure problem

$F(\omega)$ and $G(\omega)$ are defined by $G(\omega) = \frac{M_A}{\lambda(\alpha)} \times [(\omega_0 / \omega)^2 - 1]$ and

$$F(\omega) = \frac{1}{\omega R / c} \times \frac{J(\omega R / c) Y'(\alpha \omega R / c) - J'(\omega R / c) Y(\alpha \omega R / c)}{J'(\omega R / c) Y'(\alpha \omega R / c) - J(\omega R / c) Y(\alpha \omega R / c)}$$

where J and Y are the BESSEL functions of first kind and order one. One can show that the acoustic eigenpulsations are the roots of $\frac{1}{F(\omega)} = 0$ and the structure eigenpulsation is the root of $G(\omega) = 0$. The eigenpulsations of the coupled problem are roots of the equation

$$F(\omega) = G(\omega) \quad (19)$$

When the fluid is supposed incompressible, $F(\omega) = 1$. Equation (19) can be solved with Fig. (7), which plots F and G in the following cases.

- ✓ Case #1: the eigenfrequency of the structure is near an eigenfrequency of the fluid. Then, the structure eigenfrequency coupled with the fluid is greater in the case of a compressible fluid than in the case of an incompressible fluid. The acoustic coupled modes are increased for modes above the coupled fluid/structure mode and decreased for other modes.
- ✓ Case #2: the eigenfrequency of the structure is lower than all eigenfrequencies of the fluid. Then, the coupled eigenfrequency of the structure with the fluid is identical in the compressible and incompressible cases. All the acoustic coupled eigenfrequencies are slightly increased, but the raise in frequency is significant for the first coupled acoustic modes.
- ✓ Case #3: the coupling effects are increase in the case of a low mass number.

The simple analytical model with acoustic and structure mode coupling can account for the observed coupling phenomena in the case of an elastic structure coupled with an incompressible fluid with sloshing effects.

4. DYNAMIC ANALYSIS

A temporal analysis is performed on the structure problem, the fluid problem (i.e. with inner and outer cylinder considered as rigid walls) and the coupled fluid/structure problem, subjected to a sine wave shock.

Figure 7. Graphical resolution of the elasto-acoustic problem of Fig. (6)

The imposed acceleration is given by:

$$\gamma(t) = \begin{cases} \gamma_M \sin(2\pi f_0 t), & \forall t \leq 1/f \\ 0, & \forall t > 1/f \end{cases} \quad (20)$$

where $\tau = \frac{1}{f}$ is the shock duration. The discretized equation of motion for the structure, fluid or coupled fluid/structure problem are solved with the NEWMARK algorithm [10] with MATLAB.

4.1. Structure problem

Figure 8. Reduced displacement and acceleration for the structure problem

Figure (8) gives a graphical representation of the reduced relative displacement and acceleration of the beam free end for various reduced shock frequencies, by plotting the mappings:

$$f / f_0 \mapsto \delta = \max \left(\frac{\ddot{Y}(h, t, f)}{\gamma_M}, \forall t \geq 0 \right) \quad \text{and}$$

$$f / f_0 \mapsto \gamma = \max \left(\frac{Y(h, t, f)}{X_M}, \forall t \geq 0 \right), \quad \text{where } Z(h, t, f) \quad \text{and}$$

$\ddot{Y}(h, t, f)$ denotes the relative displacement and the acceleration of the beam free end. γ_M and X_M are the maximum imposed acceleration and displacement of shock with frequency f and are linked according to: $X_M = \frac{\gamma_M}{2\pi f^2}$. f_0 is the first structure frequency.

The curves are of course the same for the structure mass and stiffness corresponding to the two cases presented in Tab. (1): for a given problem (geometry and boundary conditions) the reduced displacement and acceleration only depend on the imposed motion [11].

4.2. Fluid problem

The dynamic equations of the fluid problem are discretized with finite element technique, as described in §.3, to obtain the dynamic system:

$$[M_F + M'_F]\{\ddot{P}\} + [K_F]\{P\} = \{\Phi_F(\gamma)\} \quad (21)$$

The fluid mass terms takes into account the sloshing and acoustic modes of the fluid.

The analytical solution of the incompressible fluid problem with DIRICHLET conditions for the free surface is given by:

$$p(r, \theta, z, t) = \frac{2\rho_F \gamma(t)}{h} \times \sum_{n=0}^{n=+\infty} [\alpha_n I(q_n r) + \beta_n K(q_n r)] \cdot \frac{(-1)^n \cos(q_n z)}{q_n^2} \cdot \cos(\theta) \quad (22)$$

and the fluid forces on the inner rigid cylinder is given by:

$$\phi(t) = - \int_{z=0}^{z=h} \int_{\theta=0}^{\theta=2\pi} p(R, \theta, z, t) R d\theta dz = - \left(\frac{2\pi\rho_F R}{h} \cdot \sum_{n=0}^{n=+\infty} \frac{[\alpha_n I(q_n R) + \beta_n K(q_n R)]}{q_n^3} \right) \times \gamma(t) = -M_H \times \gamma(t) \quad (23)$$

The added mass M_H appearing on Eq. (23) characterizes the incompressible behavior of the fluid, and is used to normalize the calculated fluid force.

Figure (9) gives the representation of the mapping $f/f_o \mapsto \varphi = \max\left(\frac{\Phi(t, f)}{\gamma_M M_H}, \forall t \geq 0\right)$ where f_o is the first fluid sloshing frequency.

Figure 9. Reduced fluid force on the inner cylinder for the fluid problem

The curves exhibit resonance effects in the low (sloshing effects) and high (acoustics effects) reduced frequency range; in the

medium frequency range, the fluid forces are governed by added mass effects.

4.3. Coupled fluid/structure problem

The discretization of the dynamic equations of the coupled problem with imposed motion leads to the following system, in which U denotes the unknown displacement for the structure problem and P represents the fluid pressure.

$$\begin{bmatrix} M_s & 0 \\ -\rho_F R^T & M_F + M'_F \end{bmatrix} \begin{Bmatrix} \ddot{U} \\ \ddot{P} \end{Bmatrix} + \begin{bmatrix} K_s & R \\ 0 & K_F \end{bmatrix} \begin{Bmatrix} U \\ P \end{Bmatrix} = \begin{Bmatrix} -m\ddot{X} - kX \\ \phi_F + \rho_F r\ddot{X} \end{Bmatrix} \quad (24)$$

X and $\ddot{X}$ are the imposed displacement and acceleration; m and k are extracted vectors of the structure global mass and stiffness matrices and r is extracted of the global fluid/structure coupling matrix, taking into account the imposed displacement [5]. The filling ratio is $\lambda = 75\%$.

Figure (10) shows the evolution of the reduced displacement of the structure coupled to the fluid without sloshing effects, i.e. only added mass effects are described.

Figure 10. Reduced displacement for the coupled problem without fluid sloshing and various filling ratios

The reduced displacements are the same for the two FROUDE numbers. Applying a shock on both the structure and the fluid leads to an ARCHIMEDE fluid force that reduces the displacement of the structure when coupled with the fluid; this added mass effect is reduced for low filling ratios ($\lambda \leq 50\%$).

Figure (11) shows the evolution of the reduced displacement of the structure coupled to the fluid with sloshing effects for two dynamic FROUDE numbers. The shock frequency f is reduced with the first frequency f_o of the structure coupled with the fluid.

In the low reduced frequency range, the calculated displacement is higher with $F_D = 10$ than $F_D = 100$, due to coupling fluid sloshing and structure modes. In the medium and high frequency range, the computed displacement for the two dynamic FROUDE numbers are equivalent: pure added mass effects and acoustic effects have the same influence on the two systems.

Figure (12) compares the reduced displacement for the coupled problem with $F_D = 10$, with and without fluid sloshing.

Figure 11. Reduced displacement for coupled problem with fluid sloshing in the case $F_D \sim 10$ and $F_D \sim 1$

Figure 12. Reduced displacement for the coupled problem with and without fluid sloshing

It shows the importance of the coupling effect with sloshing mode for a low dynamic FROUDE number, since the calculated displacement of the coupled problem is greater when taking fluid sloshing into account. In both cases, the added mass (with or without compressibility) effects have the same influence.

CONCLUSION

A generic linear fluid structure coupled problem with fluid free surface is numerically studied by coupling fluid and structure finite element.

A modal and temporal analysis is performed on the coupled problem with or without taking into account the fluid gravity waves, for various dynamic Froude numbers defined with the geometrical and physical characteristics of the coupled problem.

For low dynamic Froude number, coupling effect between fluid sloshing modes and structure bending modes occur: the first frequencies of the coupled problem calculated with or without dynamic free surface effects can then be very different. The influence of other parameters such as the mass number and the confinement ratio is also exhibited on the coupling process.

A temporal analysis is also performed on the system, when subjected to a simple sine wave shock of given amplitude and duration.

Coupling effects are also illustrated by a comparison of the structure displacement under shock in various conditions.

Future study will focus on the non-linear sloshing effects by using a finite element and finite volume numerical coupling procedure.

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