Passive earth pressures in the presence of hydraulic gradients

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The paper describes a variational approach applied to the limit equilibrium method for calculating the effective passive pressures of a cohesionless soil, taking into consideration the seepage flow. It is shown that in the general case of non-homogeneous and non-isotropic hydraulic properties of the soil medium, the shape of the slip surface which verifies the three limiting equilibrium equations of the soil mass at failure is a log-spiral. It is also shown that the passive earth pressure calculation is independent of the normal stress distribution along this surface. The variational limit equilibrium method is equivalent to the upper bound method in limit analysis for a rotational log-spiral mechanism. Numerical results of the coefficients of passive earth pressures in the presence of seepage flow are presented and discussed.

Cet article présente une approche variationnelle appliquée à la méthode du prisme de rupture permettant le calcul de la pression passive effective des terres en présence d'écoulement dans le cas d'un sol purement pulvérulent. On montre que dans le cas général d'un sol aux propriétés hydrauliques non homogènes et non isotropes, la forme de la surface de rupture qui vérifie les trois équations d'équilibre est une spirale logarithmique. Nous montrons aussi que le calcul de la pression passive ne dépend pas de la distribution des contraintes normales agissant le long de cette surface. La méthode du prisme de rupture variationnelle est équivalente à la méthode de la borne supérieure en analyse limite pour un mécanisme rotationnel en spirale logarithmique. Des valeurs numériques du coefficient de butée en présence d'écoulement sont présentées et discutées.

KEYWORDS: earth pressure; filters; limit state design/analysis; pore pressures; seepage; sheet piles and cofferdams.

INTRODUCTION

The design of deep sheeted excavations is often dominated by the flow of water around the sheet piles. The seepage flow influences the stability of the excavation where bulk heave or piping may occur. While the piping takes place at the excavation level, the heaving is more catastrophic and its risk is usually evaluated by considering a rectangular failure mechanism adjacent to the wall (Terzaghi, 1943). The vertical force equilibrium of this soil mass is then considered by neglecting the vertical frictional forces along the vertical faces of this mechanism.

Based on laboratory model tests, Kastner (1982) has shown that the failure of the sheet piling structures in the presence of seepage flow is not only due to the heaving phenomenon but may also occur due to the reduction of the passive earth pressures in front of the wall. Our aim in this paper is to propose an outline for the calculation of the effective passive pressures, taking into account the seepage forces.

Looking for a simple model capable of correctly describing the behaviour of soil in the passive state in the presence of seepage flow, we opted for the limit equilibrium method. This approach is based on *a priori* hypotheses concerning the shape of the slip surface (kinematic function) and the normal stress distribution (static function) along this surface. The variational approach applied to the limit equilibrium method has been employed in order to avoid the restrictions of such *a priori* hypotheses.

OVERVIEW OF PREVIOUS VARIATIONAL ANALYSIS

The variational limit equilibrium method has been used by several authors in geotechnical engineering. Kopacsy (1957) applied this approach to the three dimensional slope stability problem; however, no explicit solution is offered. Several investigators, for example Dorfman (1965), Garber (1973), Revilla & Castillo (1977) and Ly (1979), have used the calculus of variations to avoid introducing an assumption concerning the shape of the slip surface but they made an assumption concerning the normal stress distribution along this surface. A more interesting analysis consists of finding the two unknown functions without any *a priori* assumptions. Thus, Baker & Garber (1977, 1978) applied the variational approach to the two dimensional slope stability problem, and Garber & Baker (1977) and Castillo & Luceno (1978) applied this approach to the problem of the bearing capacity of a strip footing. Then, Garber & Baker (1979) treated the problems of slope stability, bearing capacity and earth pressure distribution in a unified manner as a single problem. Later, this method was used by Leshchinsky *et al.* (1985), Ugai (1985) and Leshchinsky & Baker (1986) to study the three dimensional slope stability problem, and by Leshchinsky & Reinschmidt (1985), who applied it to the reinforced slope stability problem. Finally, Leshchinsky & San (1994) applied the variational limit equilibrium method to the seismic stability of slopes, and presented interesting results in the form of design charts. It is to be noted here that the variational approach has been the subject of some controversy. Particularly, the work by Castillo & Luceno (1982) shows that the functional has no minimum. Notice, however, that experience indicates that the slip surface determined by the variational analysis reasonably duplicates reality (Leshchinsky & San, 1994). Consequently, the solutions given by the variational limit equilibrium method are very interesting. We present in the following the application of this method to the passive earth pressure problem, taking into account the seepage forces.

VARIATIONAL APPROACH OF THE PASSIVE EARTH PRESSURE PROBLEM

Figure 1 shows a double walled cofferdam subjected to a seepage flow where H is the total head loss and $u(x, y)$ represents the distribution of the pore water pressures in the soil medium.

The assumptions made in the analysis can be summarized as follows:

(*a*) The soil is cohesionless. It is homogeneous and isotropic with respect to its angle of internal friction ϕ.
(*b*) The soil medium is non homogeneous and non isotropic with respect to the hydraulic properties. It is composed of n permeable layers overlying impermeable rock. Each layer is characterized by its coefficients of permeability K_{hi} and K_{vi}.
(*c*) The breadth B_0 (Fig. 1) is large enough so that there is no interaction of the two failure mechanisms which develop in front of the two walls of the cofferdam.

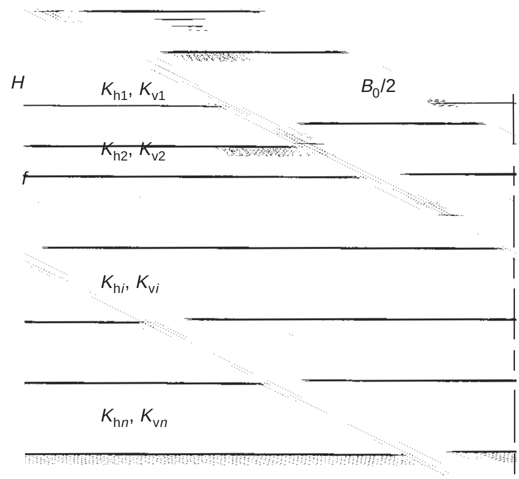


Fig. 1. Double-walled cofferdam in a multi-layered soil medium

(*d*) The resultant P_P of the effective passive pressures is assumed to act at the bottom third of the penetration depth (Fig. 2(a)). This force can be expressed as follows:

$$P_P = K_P \frac{\gamma' f^2}{2} \tag{1}$$

where K_p is the coefficient of passive earth pressure in the presence of seepage flow, γ' is the submerged unit weight of the soil, and f is the penetration depth.

The variational approach is briefly presented in this paper. For more details, refer to Soubra (1989). Fig. 2(a) illustrates the formulated problem and shows the notation used. A potential slip surface $y(x)$ is subjected to a total normal stress $\sigma(x)$. Both functions $y(x)$ and $\sigma(x)$ are assumed to be continuous. Using Coulomb's failure criterion $\tau(x) = [\sigma(x) \quad u(x)]\tan\phi = \sigma'(x)\tan\phi$, the global limiting equilibrium equations for the soil mass (Fig. 2(b)) can be written as

$$P_p \cos\delta + U_1 = \int_{x_0}^{x_1} [\sigma'(\tan\phi + \dot{y}) + u\dot{y}]\,\mathrm{d}x \tag{2a}$$

$$P_p \sin\delta = \int_{x_0}^{x_1} [\sigma'(1 \quad \tan\phi.\dot{y}) + u \quad \gamma_{sat}(f \quad y)]\,\mathrm{d}x \tag{2b}$$

$$P_p \cos\delta X_1 + U_1 X_2 = \int_{x_0}^{x_1} [\sigma'(1 \quad \tan\phi.\dot{y})x + \sigma'(\tan\phi + \dot{y})y + ux + uy\dot{y} \quad \gamma_{sat}(f \quad y)x]\mathrm{d}x \tag{2c}$$

Notice that one of the endpoints x_0 is null

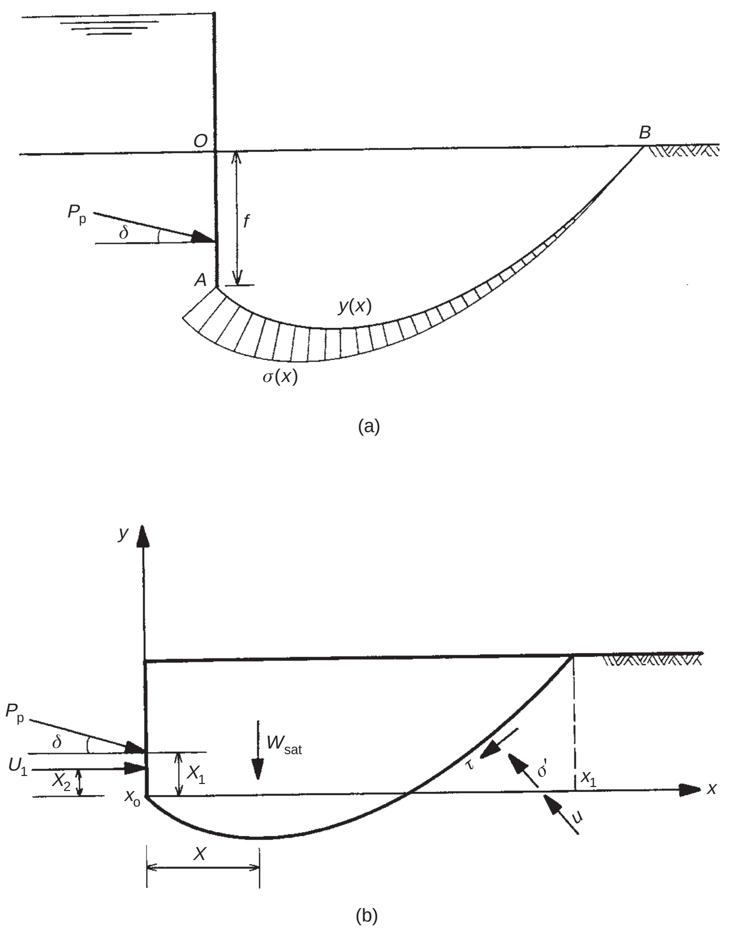


Fig. 2. (a) Slip surface for passive earth pressure analysis. (b) Free body diagram

(Fig. 2(b)), and the other (i.e. x_1), is variable. Observing equations (2a), (2b) and (2c), one realizes that the effective passive force P_p is a functional of two functions, $y(x)$ and $\sigma'(x)$. The mathematical problem of the passive earth pressure is to find those functions which give the minimum value of the passive force functional and simultaneously satisfy all three equations of limiting equilibrium (equations (2a), (2b) and (2c)).

Using equation (2b) to define P_p, while considering the other two equilibrium equations (equations (2a) and (2c)) as constraints (i.e. equations that must be satisfied), the passive earth pressure problem is a variational isoparametric one with a variable end point. This variational problem is equivalent to the minimization of an auxiliary functional G:

$$G = L_0 + \lambda_1 L_1 + \lambda_2 L_2 \tag{3}$$

where L_0, L_1 and L_2 are given as follows:

$$\begin{aligned} L_0 &= \sigma'(1 - \tan\phi\,\dot{y}) + u - \gamma_{sat}(f - y) \\ L_1 &= \sigma'(\tan\phi + \dot{y}) + u\dot{y} \\ L_2 &= \sigma'(1 - \tan\phi\,\dot{y})x + \sigma'(\tan\phi + \dot{y})y \\ &\quad + ux + uy\dot{y} - \gamma_{sat}(f - y)x \end{aligned} \tag{4}$$

λ_1 and λ_2 are the Lagrange undetermined multipliers. Finally, the two extremal functions $y(x)$ and $\sigma'(x)$ must satisfy the following conditions:

(*a*) The system of Euler's differential equations for the functional G:

$$\frac{\partial G}{\partial \sigma'} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial G}{\partial \dot{\sigma}'} \tag{5a}$$

$$\frac{\partial G}{\partial y} = \frac{\mathrm{d}}{\mathrm{d}x}\frac{\partial G}{\partial \dot{y}} \tag{5b}$$

(*b*) The constraint equations.

(*c*) The boundary conditions: at the fixed endpoint

A, we have $x_A = y_A = 0$. At the variable end point B, a variational condition must be satisfied. This condition is called the 'transversality condition' and it can be written as follows:

$$\left(G \quad \dot{y}\frac{\partial G}{\partial \dot{y}} \quad \dot{\sigma}'\frac{\partial G}{\partial \dot{\sigma}'}\right)_{x \quad x_1} \delta x_1 + \left(\frac{\partial G}{\partial \dot{\sigma}'}\right)_{x \quad x_1} \delta\sigma'_1 + \left(\frac{\partial G}{\partial \dot{y}}\right)_{x \quad x_1} \delta y_1 = 0 \quad (6)$$

where δ is a variational operator.

The first Euler equation

Combining equations (3) and (5a) yields a differential equation. The solution of this equation in a polar coordinate system is a log spiral (Fig. 3) whose equation is given as follows:

$$r = r_0 \exp(\theta \quad \theta_0)\tan\phi \quad (7)$$

Note that the log spiral function has a particular property, that the resultant of the forces ($\sigma'\,\mathrm{d}l$) and ($\tan\phi\sigma'\,\mathrm{d}l$) passes through the pole of the spiral. Hence, the moment equation about the pole is independent of the stress distribution $\sigma'(x)$, and may be used for the determination of the effective passive force. The two remaining equilibrium equations may be satisfied by every $\sigma'(x)$ distribution that has two degrees of freedom. Thus, one has to find the critical θ_0 and θ_1 angles which satisfy the moment equilibrium equation and give the minimum value of the effective passive force P_p. This is done by a two dimensional minimization procedure of P_p with respect to θ_0 and θ_1.

The independence of the effective passive force from the normal stress distribution can also be shown, due to the special property of the present functional. This functional can be written as follows:

$$G = \sigma' f(x, y, \dot{y}) + g(x, y, \dot{y}) \quad (8)$$

G is linear in σ' and is independent of $\dot{\sigma}'$. The first Euler equation implies that $f(x, y, \dot{y}) = 0$. Substituting this equation into equation (8), one can see that this functional becomes independent of σ' as follows: $G = g(x, y, \dot{y})$. This result is a direct consequence of the shape of the slip surface. Thus, the first Euler equation transforms the passive functional from a functional of two unknown functions to a functional of a single function. Therefore, it is possible to solve the passive earth pressure problem by simply minimizing the new functional G without specifying the normal stress distribution. Finally, it is easy to see that the moment equation of the rotational log spiral mechanism around the centre is identical to the work equation for the same mechanism in the upper bound method in limit analysis. Thus, solving the passive earth pressure problem by writing the moment equation around the centre of the log spiral will give an upper bound solution of the exact solution for an associated flow rule Coulomb material.

The second Euler equation and the transversality condition

Since the aim of this study is the determination of the critical effective passive force, the results obtained so far are enough to solve the problem. Indeed, it has been shown by Baker & Garber (1977) that the second Euler equation and the transversality condition give the normal stress distribution. In this work, this is also the case, but since the $\sigma'(x)$ distribution is not necessary to assess the effective passive force, we will not express these equations.

Existence of a minimum of the functional

The second variation of the passive earth functional shows that this functional is degenerated.

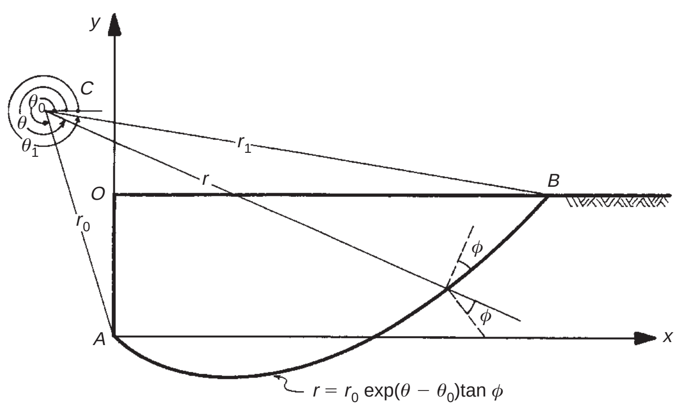


Fig. 3. Log-spiral slip surface for passive earth pressure analysis

Thus, we cannot say mathematically that there is a minimum (Castillo & Luceno, 1982). This difficulty concerning the existence of a minimum can be overcome due to the equivalence between the variational limit equilibrium method and the upper bound method in limit analysis for a rotational mechanism as has been shown before. This equivalence has been established in a more general way by Castillo & Luceno (1983) and Leshchinsky *et al*. (1985).

CALCULATION SCHEME OF THE PASSIVE EARTH PRESSURES

As mentioned before, the solution of the passive earth pressure problem by a variational limit equilibrium method is equivalent to saying that the equation of moment equilibrium must be satisfied for the soil mass bounded by the log spiral and the ground surface. As shown in Fig. (2b), the forces acting on the collapse mechanism are (*a*) the saturated weight of the soil mass between the log spiral surface and the ground surface, (*b*) the effective passive force P_p which is inclined at δ to the normal of the sheet pile, (*c*) the pore water pressures along the penetration depth and the log spiral surface, and (*d*) the effective normal and tangential stress distributions along $y(x)$. The moment equilibrium equation can be written as follows:

$$W_{sat}(r_0 \cos\theta_0 + X) + P_p \sin\delta(r_0 \cos\theta_0) = P_p \cos\delta\left(\quad r_0 \sin\theta_0 \quad \frac{f}{3}\right) + M_1 + M_2 \quad (9)$$

where M_1 and M_2 represent the moments of the force U_1 and of the pore water pressures along $y(x)$ respectively. They are given as follows:

$$M_1 = \int_0^f u(\quad r_0 \sin\theta_0 \quad y)\,\mathrm{d}y = \quad r_0 \sin\theta_0 \int_0^f u\,\mathrm{d}y \quad \int_0^f uy\,\mathrm{d}y \quad (10)$$

$$M_2 = \int_{\theta_0}^{\theta_1} (u \sin\phi)\frac{r\,\mathrm{d}\theta}{\cos\phi} r = {r_0}^2 \tan\phi \exp(\quad 2\theta_0 \tan\phi) \times \int_{\theta_0}^{\theta_1} u \exp(2\theta \tan\phi)\mathrm{d}\theta \quad (11)$$

From equation (9), one can easily see that P_p is a function of the two parameters θ_0 and θ_1 which completely describe the failure mechanism. The most critical K_p value is obtained by a minimization procedure of the K_p coefficient given by equation (1), that is $[K_P = (2P_P)/(\gamma' f^2)]$ with respect to the two parameters mentioned above. A computer program has been developed for assessing the minimal K_p values and the corresponding critical slip surfaces.

NUMERICAL RESULTS OF THE PASSIVE EARTH PRESSURES

Case of no seepage flow

Table 1 compares the passive earth pressure coefficient K_p for $\phi = 40°$ and $\delta/\phi = 1/2$ obtained from the present analysis with that of other authors in the case of no seepage flow. The comparison of the present upper bound solution with the upper and lower bound solutions given respectively by Chen & Rosenfarb (1973) and Lysmer (1970) shows that the difference with the lower bound solution is smaller than 3%, which means that the present solution is very close to the exact solution for an associated flow rule Coulomb material. On the other hand, the currently accepted values given by Sokolovski (1960) and Caquot & Kérisel (1948) lie in the range between the best upper and lower bound solutions given by the limit analysis method. Finally, the comparison between the present result and the ones given by the limit equilibrium methods (Coulomb, 1776; Shields & Tolunay, 1973) shows that the traditional limit equilibrium method may greatly overestimate or underestimate the passive earth pressure coefficients due to the *a priori* assumptions concerning

Table 1. Passive earth pressure coefficient K_p as given by different authors for ϕ 40°, δ/ϕ 1/2

	Authors	K_p
Limit equilibrium methods	Coulomb (1776)	11·77
	Shields & Tolunay (1973)	8·30
Slip line methods	Caquot & Kérisel (1948)	9·60
	Sokolovski (1960)	9·68
Upper bound methods in limit analysis	Chen & Rosenfarb (1973)	10·10
	Present solution	9·81
Lower bound methods in limit analysis	Lysmer (1970)	9·54

the shape of the slip surface and the normal stress distribution along this surface.

Case of seepage flow

From equation (9), the determination of the effective passive force P_P requires the determination of the terms M_1 and M_2. The hydraulic head distribution $\varphi(x, y, z)$ in the soil medium is governed by the following equation:

$$\frac{\partial}{\partial x}\left(K_x\frac{\partial\varphi}{\partial x}\right)+\frac{\partial}{\partial y}\left(K_y\frac{\partial\varphi}{\partial y}\right)+\frac{\partial}{\partial z}\left(K_z\frac{\partial\varphi}{\partial z}\right)=0 \tag{12}$$

In some simple cases, such as the case of the single sheet pile driven into a semi infinite homogeneous soil medium, the pore water pressure distribution is given analytically (Soubra & Kastner, 1992). For more complex geometry or for a multi layered soil medium with different coefficients of permeability, the pore water pressure distribution cannot be known analytically and it requires a numerical resolution of equation (12).

For the double walled cofferdam considered in this paper, the numerical method used for the determination of the potential field in the soil medium is the well known finite difference method where the differential equation (equation 12) is approximated by a finite difference equation. The boundary conditions used are shown in Fig. 4. The medium is discretized by a rectangular mesh (Fig. 5). The finite difference equations written at the different nodes form a system of linear equations whose unknowns are the values of the hydraulic head at the nodes. This system is solved by the Gauss Seidel method, using over relaxation in order to accelerate the rate of convergence.

The determination of the moments M_1 and M_2 has been made by numerical integration using the Gaussian quadrature method where the pore water pressures along both the sheet pile and the log spiral surface are determined by numerical interpolation. Let us pass now to the presentation of some numerical results obtained from the computer program. Notice that all subsequent results concern the case when $\gamma_{sat}/\gamma_w = 2$ and $B_0/2 = 10$ m (Fig. 1).

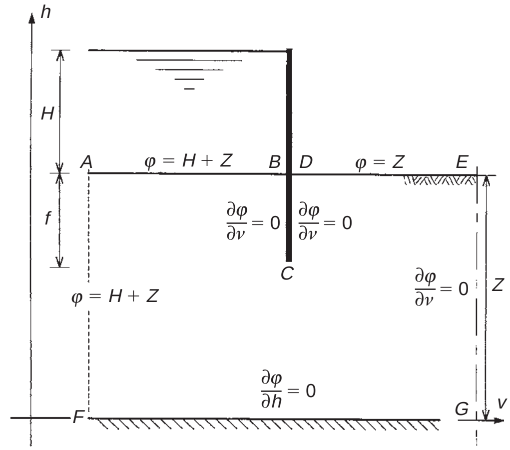


Fig. 4. Boundary conditions for the seepage flow of the double-walled cofferdam

Case of a homogeneous and isotropic soil medium

Case of a single layer of infinite depth. Soubra & Kastner (1992) published the results of the passive earth pressure coefficients in the presence of seepage flow in the case of a single sheet pile wall driven into a homogeneous and isotropic semi infinite soil medium where the hydraulic head can be known analytically. Note that the same results have also been obtained by the present analysis using the finite difference method for the determination of the hydraulic heads.

Figure 6 shows the variation of the passive earth pressure coefficient with H/f for $\phi = 30°$ and for four values of δ/ϕ ($\delta/\phi = 0$, 1/3, 1/2, 2/3). For a zero K_p value, the corresponding H/f value is the same for different δ/ϕ values ($H/f = 2{\cdot}78$). This means that the angle of friction at the soil structure interface has no effect on the H/f value causing failure by heaving. This fact can

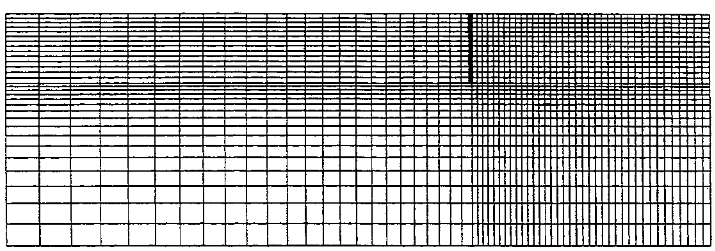


Fig. 5. Finite difference mesh

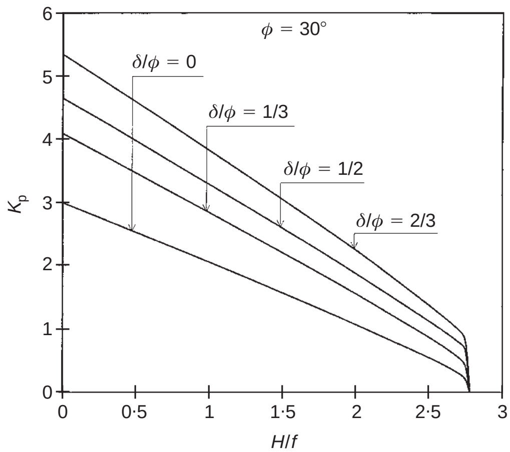


Fig. 6. K_p versus H/f for ϕ 30° and δ/ϕ 0, 1/3, 1/2 and 2/3 in the case of a homogeneous isotropic semi-infinite medium

be explained as follows: when the effective passive force vanishes, there is no interaction at the soil structure interface and we have the traditional heaving phenomenon. For the same case, Terzaghi's approach gives a value of H/f at failure equal to 2·82. Furthermore, the piping phenomenon which appears for the critical hydraulic gradient at the point D (Fig. 4) occurs for a value of H/f equal to 3·14. It should be mentioned that the numerical results have shown that in the case of a homogeneous and isotropic semi infinite soil medium, the failure by heaving will occur before the piping phenomenon as long as ϕ is smaller than 45°.

Figure 7 shows some charts of the variation of the passive earth pressure coefficient as a function of H/f for different values of ϕ ($\phi = 20°$, 25°, 30°, 35°, 40°) and δ/ϕ ($\delta/\phi = 0$, 1/3, 2/3). From these figures, the reduction of this coefficient is quasi linear for the H/f values varying from 0 to 2·5.

Finally, it should be mentioned that the K_p value increases with γ_{sat}. This increase is to be expected, since the soil weight has the favourable effect of increasing the stability of the soil mass in front of the sheet pile. For example, when $\phi = 35°$, $\delta/\phi = 2/3$ and $H/f = 2$, the increase of the passive earth pressure coefficient attains 22% when γ_{sat}/γ_w increases from 2 to 2·2.

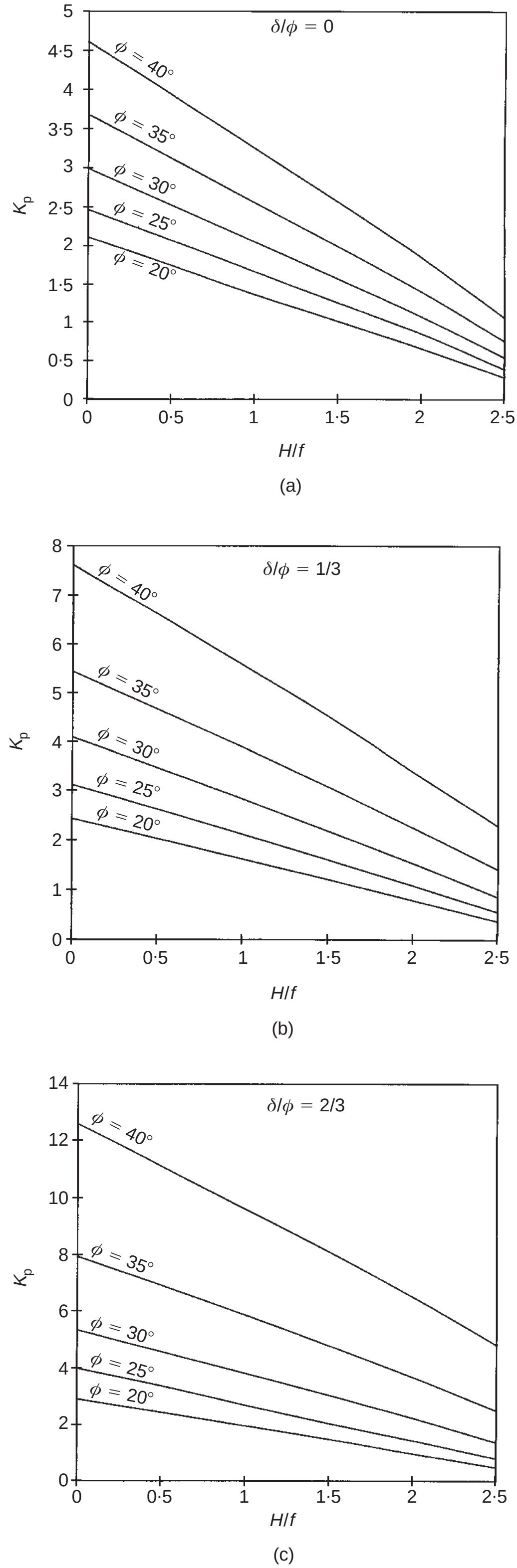


Fig. 7. K_p versus H/f for different values of ϕ and δ in the case of a homogeneous isotropic semi-infinite medium

Case of a single layer of finite depth D. The numerical results obtained from the present pro gram have shown that the passive earth pressure coefficient increases with the D/f decrease. The minimum relative depth D/f necessary to obtain the results of the semi infinite case must be greater than or equal to 6.

Figure 8 shows some charts of the variation of the passive earth pressure coefficient as a function of H/f for different values of ϕ ($\phi = 20°$, $25°$, $30°$, $35°$, $40°$) and δ/ϕ ($\delta/\phi = 0$, $1/3$, $2/3$) and for $D/f = 2$.

Case of a homogeneous and non isotropic soil medium. Figure 9 shows the variation of the passive earth pressure coefficient with K_h/K_v when $\phi = 35°$, $H/f = 2$ and $\delta/\phi = 0$, $1/3$, $2/3$ and 1. There is a large decrease of the passive earth pressure coefficient up to $K_h/K_v = 100$. Beyond this limit, the passive earth pressure coefficient tends to an asymptote. This can be explained by the fact that the equipotential lines in front of the sheet pile become quasi horizontal beyond a certain value of K_h/K_v and, thus, the potential field does not change any more in the zone concerned with the failure mechanism.

Figure 10 shows some charts of the variation of the passive earth pressure coefficient as a function of H/f for different values of ϕ ($\phi = 20°$, $25°$,

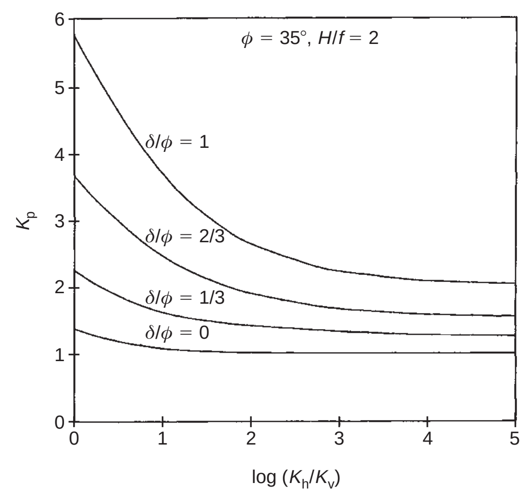


Fig. 9. K_p versus K_h/K_v for ϕ 35° and H/f 2 in the case of a homogeneous and non-isotropic semi-infinite medium

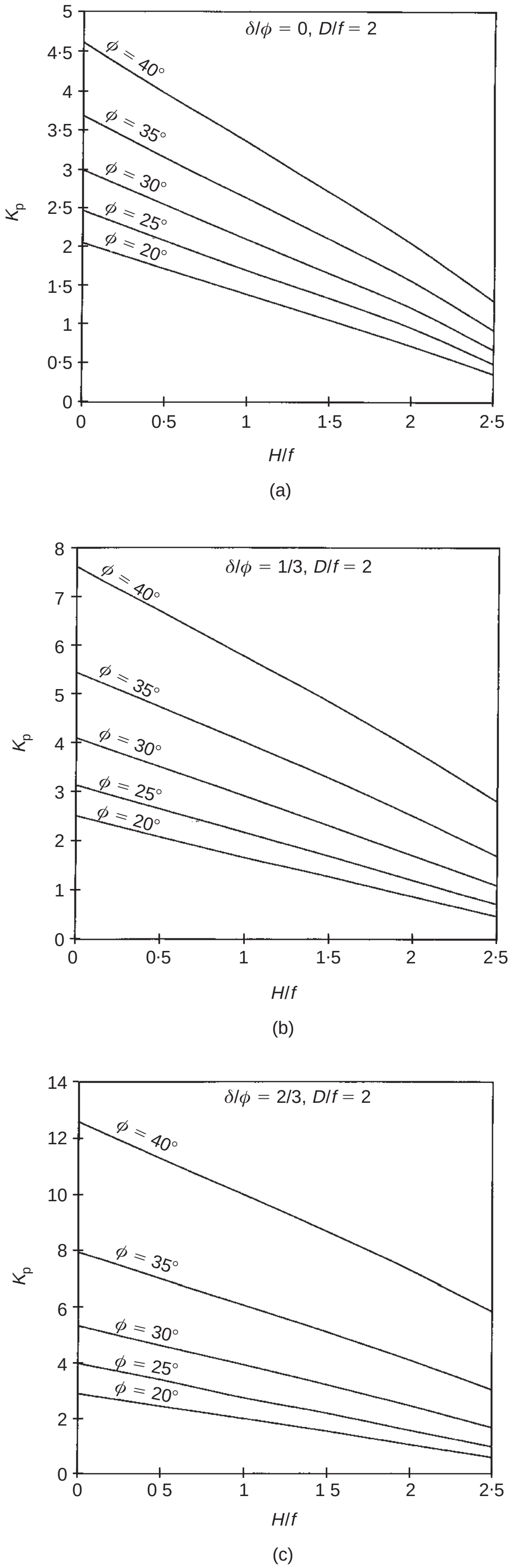
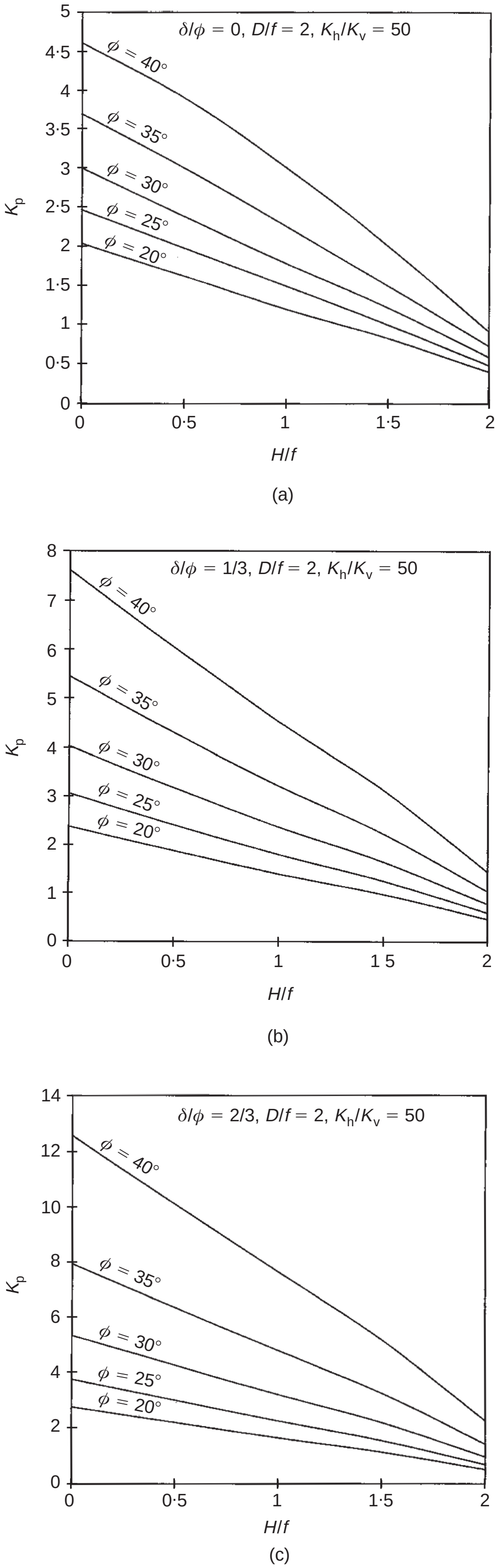


Fig. 8. K_p versus H/f for different values of ϕ and δ in the case of a homogeneous isotropic single layer of finite depth (D/f 2)



30°, 35°, 40°) and δ/ϕ ($\delta/\phi = 0$, 1/3, 2/3) and for $D/f = 2$ when the permeability ratio $K_h/K_v = 50$.

Case of an isotropic two layered soil medium. In this section, we consider the frequent case of a cofferdam driven into a two layered soil medium.

Case where the bottom of the sheet pile lies in the upper layer (case A). Figure 11 shows the case of an isotropic two layered soil medium where the permeability coefficients of the upper and lower layers are respectively K_1 and K_2.

Figure 12 shows the variation of the passive earth pressure coefficient with K_1/K_2 when $\phi = 35°$, $\delta/\phi = 2/3$ and $H/f = 2$.

For the case of a single layer ($K_1/K_2 = 1$), the

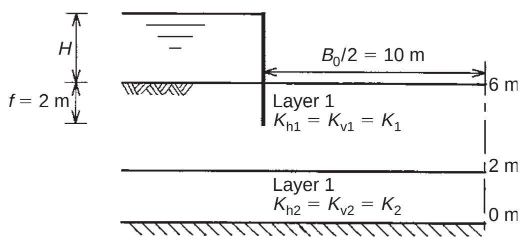


Fig. 11. Case of an isotropic two-layered soil medium: case A

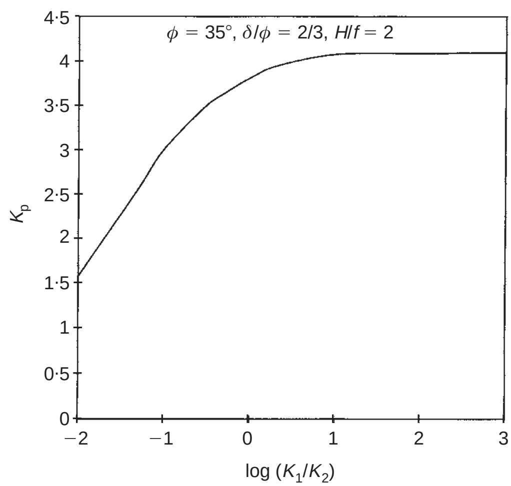


Fig. 12. K_p versus K_1/K_2 for ϕ 35°, δ/ϕ 2/3 and H/f 2 for case A

Fig. 10. K_p versus H/f for different values of ϕ and δ in the case of a homogeneous and non-isotropic single layer (D/f 2)

passive earth pressure coefficient is equal to 3·81. When $K_1/K_2 > 10$, one obtains the passive earth pressure coefficient corresponding to the case of a single layer of limited depth ($\phi = 35°$, $\delta/\phi = 2/3$, $H/f = 2$ and $D/f = 2$), since the lower layer can be considered as an impermeable substratum. Notice, however, that for cases when the lower layer has a greater permeability coefficient than the upper layer ($K_1/K_2 < 1$), most of the head loss is concentrated in the upper layer, resulting in greater pore water pressures. Consequently, the passive earth pressure coefficient decreases with the K_1/K_2 decrease.

Case where the bottom of the sheet pile lies in the lower layer (case B). Figure 13 shows the case of an isotropic two layered soil medium. The unique difference from the previous case is that the bottom of the sheet pile wall lies in the lower layer.

Figure 14 shows the variation of the passive earth pressure coefficient with K_1/K_2 when $\phi = 35°$, $\delta/\phi = 2/3$ and $H/f = 2$.

As in the previous section (case A), the passive earth pressure coefficient in the case of an isotropic single layer ($K_1/K_2 = 1$) is equal to 3·81. For $K_1/K_2 > 100$, the head loss takes place solely in the lower layer, and the upper layer can be considered as a filter increasing the global stability of the soil in front of the sheet pile. Consequently, the increase of the passive earth pressure coefficient with the K_1/K_2 increase is a logical phenomenon. On the other hand, for cases when the lower layer has a greater permeability coefficient than the upper layer ($K_1/K_2 < 1$), most of the head loss takes place in the upper layer, resulting in a significant reduction of its resultant body force (buoyant weight + seepage force). Consequently, the passive earth pressure coefficient decreases with the K_1/K_2 decrease. However, it should be noted that this calculation scheme is only valid as long as the vertical hydraulic gradient in the upper layer is smaller than the critical gradient $i_c = \gamma'/\gamma_w$, as otherwise failure by heaving of the upper layer will occur due to the fact that the saturated weight of this layer is equal to the resultant of the pore water pressures on the base of this layer. This limitation is shown in Fig. 14 by a vertical dotted line.

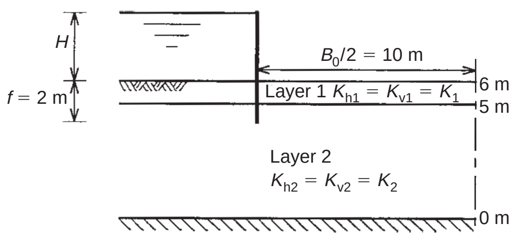


Fig. 13. Case of an isotropic two-layered soil medium: case B

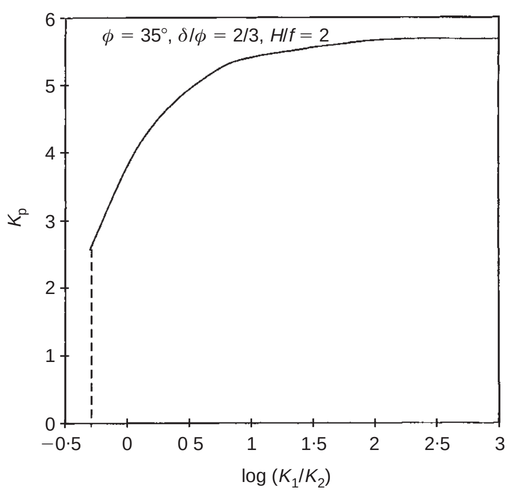


Fig. 14. K_p versus K_1/K_2 for ϕ 35°, δ/ϕ 2/3 and H/f 2 for case B

CONCLUSION

The variational limit equilibrium method was applied to the passive earth pressure problem, taking into account the seepage flow due to dewatering. It showed that the failure mechanism, in the general case of non homogeneous and non isotropic hydraulic properties of the soil medium, is a log spiral. This method is equivalent to the upper bound method in limit analysis for a rotational log spiral mechanism.

In the case of no seepage flow, the present upper bound solution is the smallest one existing in the literature and is very close to the currently accepted results of Caquot & Kérisel (1948).

In the case of seepage flow, the present mechanism allowed us to determine the reduction of the passive earth pressures. For the limiting case corresponding to zero passive pressures, the present mechanism describes the traditional heaving phenomenon in front of the sheet pile. The numerical results have shown that:

(*a*) The heaving of a soil mass in front of the sheet pile occurs before the piping phenomenon in the case of a homogeneous and isotropic semi infinite soil medium as long as ϕ is smaller than 45°.
(*b*) The effective passive pressures increase with the decrease of the layer depth in the case of a single layer problem.

(*c*) The anisotropy of the permeability coefficient of the soil medium can induce a significant reduction of the effective passive pressures.

(*d*) The study of the two layered soil medium has shown that: (*i*) for great values of K_1/K_2, when the bottom of the sheet pile lies in the upper layer, one obtains the case of a single layer of limited depth, and when the bottom of the sheet pile lies in the lower layer, the upper layer can be considered as a filter; and (*ii*) for small values of K_1/K_2, most of the head loss takes place in the upper layer, thus resulting in a significant reduction of the effective passive pressures.

Finally, one can see that the effect of soil aniso tropy and non homogeneity is significant for the reduction of the passive earth pressures. Thus, the assessment of the reduction of the effective passive pressures taking into account these parameters is of great interest in the practice of geotechnical engi neering.

NOTATION

B_0	breadth of the double-walled cofferdam
D	layer depth
$\mathrm{d}l$	elementary surface along the slip surface
f	penetration depth
H	total hydraulic head loss
h, v	coordinate system whose origin is at point F
i_c	critical hydraulic gradient
K_1, K_2	isotropic permeability coefficients of layers 1 and 2
K_h, K_v	horizontal and vertical permeability coefficients
K_p	coefficient of passive earth pressure
K_x, K_y, K_z	permeability coefficients in the principal directions x, y and z
M_1	moment of the force U_1
M_2	moment of the pore water pressures acting on the slip surface
P_p	effective passive force
r_0, r_1	initial and final radius of the log-spiral slip surface
U_1	resultant of pore water pressures at the soil structure interface
u	pore water pressure
W_{sat}	saturated weight of the soil mass OAB
X	distance between the wall and the line of action of the force W
X_1	distance between the bottom of the wall and the normal component of the effective passive force
X_2	distance between the bottom of the wall and the line of action of force U_1
$y(x)$	equation of the slip surface in the (x, y) coordinate system
$\dot{y}$	$\frac{\mathrm{d}y}{\mathrm{d}x}$
δ	friction angle at the soil structure interface
ϕ	angle of internal friction of the soil
φ	hydraulic head
γ'	submerged unit weight of the soil
γ_{sat}	saturated unit weight of the soil
γ_w	unit weight of water
λ	relaxation factor
λ_1, λ_2	Lagrange's undetermined multipliers
θ_0, θ_1	angles defining the log-spiral slip surface
$\sigma(x)$	total normal stress distribution along the slip surface
$\sigma'(x)$	effective normal stress distribution along the slip surface
$\dot{\sigma}'$	$\frac{\mathrm{d}\sigma'}{\mathrm{d}x}$
$\tau(x)$	tangential stress distribution along the slip surface

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