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ABSTRACT

Bouchette, F., Manna, M., Montalvo, P., Nutz, A., Schuster, M.,Ghienne, J.-F., 2014. Growth of cuspate spits. *In:* Green, A.N. and Cooper, J.A.G. (eds.), *Proceedings 13th International Coastal Symposium* (Durban, South Africa), *Journal of Coastal Research*, Special Issue No. 70 pp. 047-052, ISSN 0749-0208.

The present work concerns cuspate spits: slightly symmetrical geomorphic features growing along the shoreline in shallow waters. We develop a new formulation for the dynamics of cuspate spits. Our approach relies on classical paradigms such as a conservation law to the shoreface scale and an explicit formula for alongshore sediment transport. We derive a non-linear diffusion equation and a fully explicit solution for the growth of cuspate spits. From this general expression, we found interesting applications to quantify shoreline dynamics in the presence of cuspate spits. In particular, we point out a simple method for the datation of a cuspate spit given a limited number of input parameters. Furthermore, we develop a method to quantify the mean alongshore diffusivity along a shoreline perturbed by well-defined cuspate spits of known sizes. Finally, we introduce a formal relationship between the geometric characteristics (amplitude, length) of cuspate spits, which reproduce the self-similarity of these geomorphic features.

ADDITIONAL INDEX WORDS: *nearshore, sand spit, Pelnard-Considère, non-linear diffusion equation*

INTRODUCTION

A wide range of large-scale long-standing geomorphic features occur in shallow water environments, from tens of metres of water depth to the shoreline, either in the open sea or on continental settings. Ripples, megaripples, dunes and sandwaves develop in rhythmic or isolated patterns at metre to kilometre scale (Bruun, 1954; Bakker, 1968; Lonsdale and Malfait, 1974; McBride and Moslow, 1991; Reynaud *et al*., 1999; Lykousis, 2001; Todd, 2005; Raynal *et al*., 2009; Bouchette *et al*., 2010; Raynal *et al*., 2010). Sandbanks are a part of this family of bedforms and include features such as mega-dunes, bars and ridges (Dyer and Huntley, 1999). Some sandbanks, termed shoreface-connected ridges and headland-associated banks, correspond to features that develop seaward from high points connected to the coast (McBride and Moslow, 1991; Dronkers, 2005). They are progading down-drift and they usually extend down to deep waters. Obviously these local shoreline perturbations are associated with an accumulation of sand.

Zenkovitch (1959) first described cuspate spits (Figure 1) as a limited category of shore-connected features that result from symmetrical wind/wave forcings and/or peculiar initial shore configuration (Bird, 1994; Coco and Murray, 2007). Asthon *et al.* (2001) and Asthon and Murray (2006) proposed that cuspate spits, flying spits and other shoreline features are derived from instabilities inherent in the relationship between alongshore sediment transport and local shoreline orientation. They presented a comprehensive weakly non-linear theory for cuspate and spit dynamics, and gave a striking numerical solution to the problem.

The present work focuses on cuspate spits, also termed foreland spits, cuspate foreland or v-notches (Gilbert, 1885; Gulliver, 1896; Fisher, 1955; Zenkovitch, 1959), which are slightly symmetrical shoreline-connected features that grow along the shoreline of shallow water environments. Cuspate spits belong to the class of self-similar pattern. That is to say, as the time proceeds, the shoreline varies whilst remaining geometrically similar. From this point, we develop a new formulation for the dynamics of cuspate forelands. We derive a non-linear diffusion equation and an explicit solution for the dynamics of foreland spits. The final objective of this paper is to use the model developed to quantify mean growth velocity of cuspate spits, to contribute to the determination of their age or to the mean longshore diffusivity at their origin. The paper also aims to provide additional ideas on the underlying physics of cuspate spits.

First, we recall the mechanical context driving the edification of cuspate spits, specifying what has been discussed in the literature and what we propose here. Then we present the main steps for the development and the proof of our mathematical model. Finally, we adapt our cuspate model to various simple applied circumstances and initiate a discussion on the physics and origin of cuspate spits.

The Non-Linear Pelnard-Considère Equation

In this work, we make the assumption that seabed and shoreline changes driven by strict cross-shore dynamics smooth and counterbalance over time. We consider that the consequence for the net change in the shoreline position over years is weak (Ruessink and Terwindt, 2000; Marino-Tapia *et al*., 2007). Indeed, at a long time scale, mean cross-shore profile is assumed to be at equilibrium (Hanson and Kraus, 1989; Dean, 1991), i.e. net cross-shore transport equals zero. The significant contribution to the long term shoreline change is thus from longshore dynamics

DOI: 10.2112/SI70-009.1 received 6 December 2013; accepted 21 February 2014.

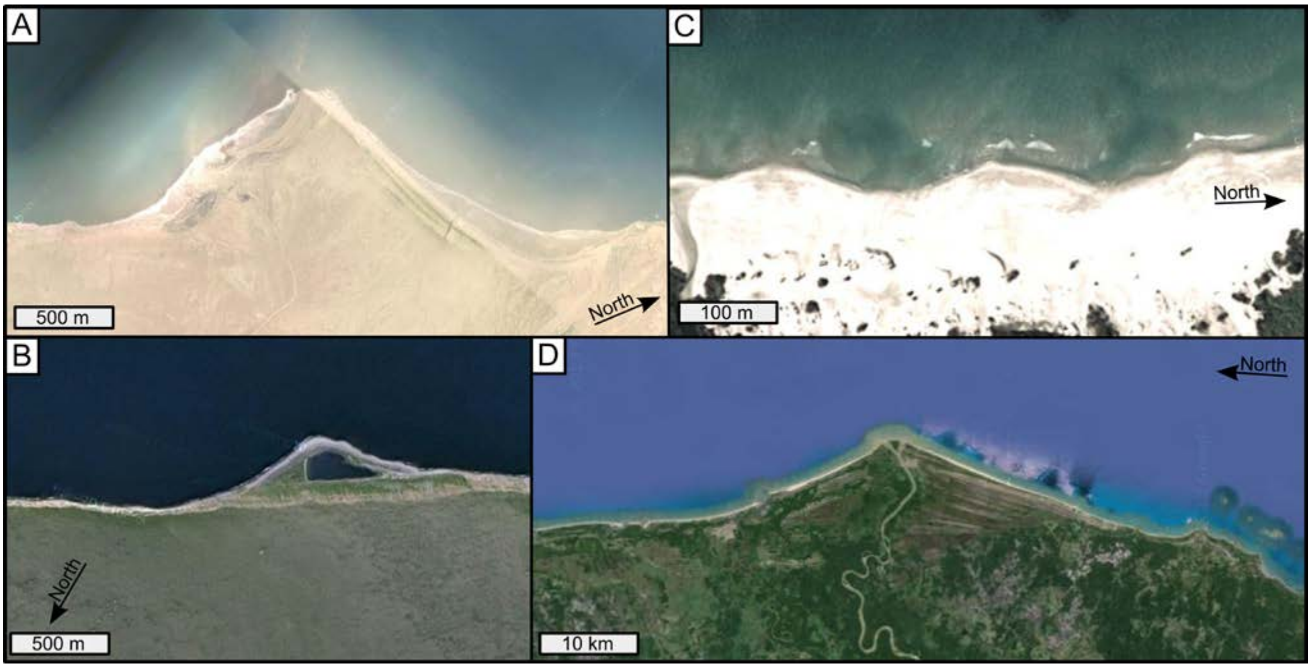

Figure 1. Examples of cuspate spits along shorelines. (A) Cuspate along the eastern basin of the Caspian Sea, the Garabogazköl Aylagy (Lat: 41.7813301; Lon: 54.3315601), (B) Cuspate in the north-eastern part of the Langandensbyggö peninsula in Island (Lat: 66.3351803; Lon: -14.7930432), (C) String of cuspates along the Lebanon shoreline (Lat: 32.8597276; Lon: 35.0629139), (D) Cuspate morphology combining with the Belmonte river mouth, Brazil (Lat: -15.8595914; Lon: -38.8902677).

(Allen, 1981; Aagaard and Greenwood, 1995). This assumption is at the origin of the formulation proposed here for the development of cuspate spits.

Having this in mind, a basic mass balance equation states that the volume of sand required to move a profile cross shore is the shift of shoreline times the height of the active profile. Let be $y = S(x, t)$ in the equation of shoreline position in a fixed (x, y) coordinate system with the x-axis oriented alongshore, the y-axis oriented offshore and t the time (Figure 2A). $S(x, t)$ satisfies:

$$\frac{dS}{dt} + \frac{1}{h_0 + B}\frac{dQ_L}{dx} = 0 \quad (1)$$

where h_0 is the closure water depth (seaward of which no significant transport occurs), B is the active berm height, $h_0 + B$ is the height of the active profile (Figure 2B). The total amount of sediment transported alongshore Q_L is related to the alongshore flux of energy available for the nearshore per unit length along the shoreline (Inman and Bagnold, 1963):

$$Q_L(x) = \frac{KF_L(x)}{(\rho_s - \rho)g(1-p)} \quad (2)$$

where ρ_s and ρ are densities of sediment and water respectively, p is the porosity, g is the acceleration of gravity. The dimensionless parametre K is an empiric constant. The energy flux to the beach F_L is defined by :

$$F_L = C_g.\varepsilon_0\cos(\delta_0 - \theta)\sin(\delta_0 - \theta) \quad (3)$$

where $\cos(\delta_0 - \theta)$ is the ratio of incoming energy that flows from the closure water depth through the nearshore to the shoreline. In other words, it is the ratio of energy between two infinitely (dL) close wave rays that acts on an infinitely small dx shoreline segment (Figure 2A). Hence, $\sin(\delta_0 - \theta)$ is the longshore contribution of the total incoming energy. At the closure water depth h_0, the incoming energy is classically defined with the expression derived from linear wave theory:

$$\varepsilon_0 = \frac{1}{8}\rho g H_0$$

with H_0 representing the wave height. This energy propagates at the group velocity C_g . This velocity must be calculated at the point where the energy flows into the active domain, that is at the closure water depth h_0. In this case, the linear wave theory provides the simple formulation:

$$C_g = C_{g0} = \frac{g}{4\pi}T_0$$

The longshore transport rate Q_L is thus:

$$Q_L = 2C_L\cos(\delta_0 - \theta)\sin(\delta_0 - \theta) \quad (4)$$

with:

$$C_L = \frac{K\rho\rho g\,{}_0^2T_0}{64\pi(\rho_s - \rho)(1-p)} \quad (5)$$

Several formulations for the alongshore transport rate were successively derived from Eq. (3) (e.g. Komar and Inman, 1970; Komar, 1971; Bailard, 1984). Reviews and compared analyses of alongshore transport formulae were also performed (Bayram *et al.*, 2001). Here the sediment transport is strictly controlled by F_L, the

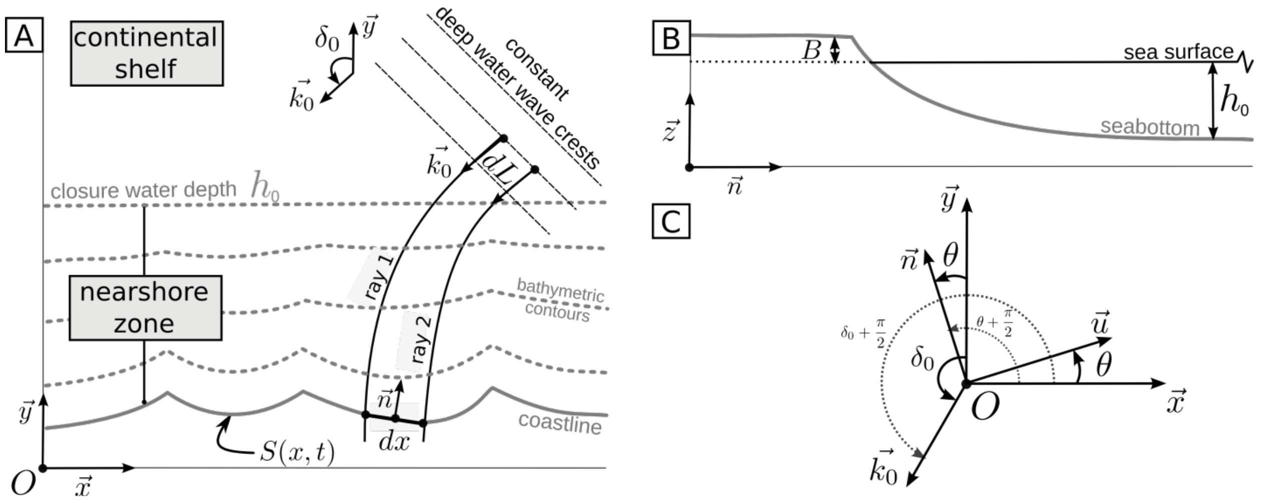

Figure 2. Sketches for the design of the mathematical model (A) a plan view of the nearshore and shoreline Wherein the *x*-axis is oriented longshore and the *y*-axis is oriented seaward. From deep water, waves (example of Ray 1 and Ray 2) propagate from the top to the bottom of the figure and refract depending upon bathymetric contours (B) Definition of the normal *n* to the shoreline, the berm height *B*, and of the closure water depth h_0. (C) Relative orientation of vectors and angles used in the paper.

longshore portion of flux of energy ε_0 per shoreline unit length. No matter what type of wave transformation occurs in the nearshore, the only significant information is the fact that $\delta_0-\theta$ varies along the shoreline and that the energy that flows in the nearshore up dip of the shoreline depends upon this. This point of view is quite different from that chosen by other authors (for more explanations, see Ashton *et al*., 2001).

The combination of Eqs (4) and (1) is a model of long-term shoreline changes *S*(*x, t*) under mean wave forcings and mean sediment texture conditions. To date, several strategies have been tested to find solutions for this kind of problem. First, Pelnard-Consideré (1956) (and a significant amount of subsequent literature) linearized the problem so that a single linear diffusion equation describes the planform evolution of *S*(*x, y*). More recently, Asthon *et al*. (2001) and Asthon and Murray (2006) solved the problem numerically with an expression of Q similar (but not equal) to that of Dean and Dalrymple (2002) and introduced a diffusion coefficient that depends on θ (and may be thus negative). Asthon *et al*. (2001) focussed on rythmic foreland spits. Another striking idea was to introduce some non-linearity with an "expansion of the flow and the bottom perturbations in a truncated series of eigenfunctions of the linear problem" (Calvete *et al*., 2002; Falquès *et al*., 2008) which is not discussed here. The latter works concerned more specifically rythmic shoreline patterns like beach cusps. These works argued that shoreline-connected features including foreland spits originate in instabilities. We provide a new solution to the problem.

The angle θ between the local normal to the shoreline and the *y*-axis (Figure 2C) satisfies:

$$\sin\theta = \frac{dS/dx}{\sqrt{1+(dS/dx)^2}}$$
$$\cos\theta = \frac{1}{\sqrt{1+(dS/dx)^2}} \quad \text{(6) and (7)}$$

Eq. (4) can be rewritten in the following Eq (8):

$$Q_L = C_L\left[\sin 2\delta_0\left(\cos^2\theta - \sin^2\theta\right) - 2\cos 2\delta_0 \sin\theta\cos\theta\right]$$

Developing Eqs (6), (7) in Taylor series until order two in ∂S/∂x , and combining Eqs (1) and (8) results in

$$\frac{dS}{dt} = G_0 cos2\delta_0 \frac{d^2S}{dx^2} + 2G_0 sin2\delta_0 \frac{dS}{dx}\frac{d^2S}{dx^2} \quad (9)$$

This is a nonlinear diffusion equation.When waves are directed along the *x*-axis (alongshore wind/wave forcings) sin $2\delta_0$ is zero and Eq. (9) reduces to a classical diffusion equation (Pelnard-Considère, 1956) with G_0 the longshore diffusivity. Another way to obtain Pelnard-Considère is to linearize Eq. (9). For this reason, we could name Eq. (9) the *"non-linear Pelnard-Considère equation"*. In such a formulation, G_0 is given by:

$$G_0 = \frac{C_L}{h_0 + B} \quad (10)$$

and one will notice that

$$G_0 = G\left(H_0, T_0, \delta_0, \rho, \rho_s, p, h_0 + B\right) \quad (11)$$

which means that G_0 is a function of wave properties, sediment properties and basic geometrical informations.

Derivation of the Cuspate Equation

From Eq. (11) we know that G_0 depends upon most of the 'environmental' variables of the problem, i.e., those relative to the geometrical context and the forcings. As long term dynamics are mostly driven by mean values averaged to the historical/ geological time scale, we can consider that H_0, T_0, ρ, ρ_s, *p* are constant through time or that they vary very slowly. In the same manner, h_0 + *B* may not change as sea bottom is always in equilibrium (Short, 1999, p. 45, Fig 3).

Let us consider the following particular scenario in the frame (0, *x*, *y*) (Figure 2A). At t = 0, we have a non perturbed shoreline for x in [−∞, +∞]. At time t_0, an *x*-symmetric and positively defined perturbation appears that develops on both sides of the origin O and extends in [$-x_f$, $+x_f$], being zero beyond. The building of such a cuspate spit supposes that the alongshore sediment transport results from two main dominant forcings varying close enough to $\delta_0 = \pm\pi/4$. Under these conditions Eq. (11) splits in two equations with solutions S_R (for $\delta_0 = +\pi/4$) and S_L (for $\delta_0 = -\pi/4$) satisfying :

$$\frac{dS_{R/L}}{dt} = 2G_0\sin(\pm\pi/2)\frac{dS_{R/L}}{dx}\frac{d^2S_{R/L}}{dx^2} \qquad (12)$$

As one can consider that the two forcings compete through time, we can substitute the real system represented by the two Eqs. (12) by a model based on a distributed solution satisfying :

$$\frac{dS}{dt} = \begin{cases} 0 & \text{for } x \le -x_f \\ 2G_0 \dfrac{dS_R}{dx}\dfrac{d^2S_R}{dx^2} & \text{for } -x_f < x \le 0 \\ -2G_0 \dfrac{dS_L}{dx}\dfrac{d^2S_L}{dx^2} & \text{for } 0 \le x \le x_f \\ 0 & \text{for } x \ge x_f \end{cases} \qquad (13)$$

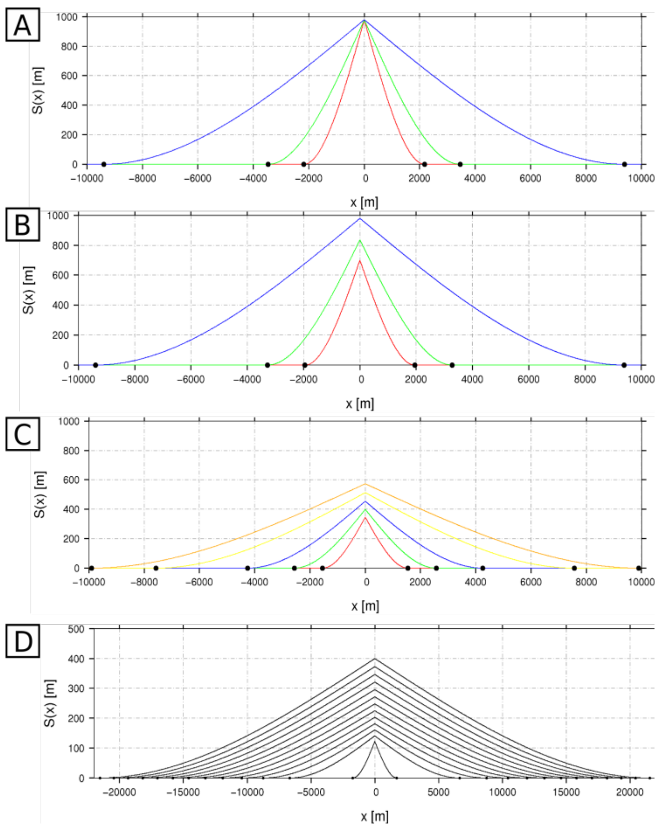

Figure 3. Various cuspate spits calculated from the mathematical model proposed in Equation (16). The different plots correspond to the configurations given below. Parameters to be set are G_0 and *a*. G_0 is given by K = 0.77, ρ = 1025 kg.m^{-1}, ρ_s = 2400 kg.m^{-1}, H_0=1.5 m, T_0 = 10 s, κ = 0.55,h_0 = 10m. The definition of the parameter *a* (arising from the integration) changes in the various plots: (A) parameter *a* is a constant set to 20; (B) parameter *a* ranges from 16 to 20; (C) parameter a ranges from 10 to 14; (D) parameter *a* ranges from 5 to 11; the various curves represent cuspate geometry from 1 year to 450 years.

We already recalled that cuspate spits possess self-similar patterns. Thus it is obvious to take into account the (x,t) dependence of S through a self-similar variable ξ so that:

$$S(x,t) \Leftrightarrow S(\xi,\tau) \text{ with } \xi = \frac{x}{t^{1/3}} \qquad (14)$$

Applying this variable substitution to the operators d/dx and d/dt, we derive a new writing of Eq. (13):

$$\begin{aligned} &2G_0 S_{R,\xi\xi} + \frac{1}{3}\xi = 0, \xi \in [-\xi_0, 0] \\ &2G_0 S_{L,\xi\xi} - \frac{1}{3}\xi = 0, \xi \in [0, \xi_0] \end{aligned} \qquad (15)$$

Integrating twice, we obtain another expression with 4 distinct constants to be determined by the geometrical behavior of the cuspate spit. We impose to *S* to be continuous and positively defined at ξ=0. In addition, we impose discontinuity of the derivative of *S* at ξ=0. And we impose to *S* to be zero at the points ξ_0 where S_ξ=0. We obtain a set of equations with a single unknown parameter *a*. Going back to the original coordinates (*x*,*y*), we get a new equation.

This equation is an exact solution to the problem developed in Eq. (9) adapted to the growth of any cuspate spit. Figure 3 displays some examples of plots of the expression *S*(*x*,*y*) derived here at various arbitrary times and for various values of the control parameters. Each curve could be cuspate spits like those in Figure 1. The expression of the solution in the original coordinates is given by equation (16) given below.

$$\frac{dS}{dt} = \begin{cases} 0 & x \le -\sqrt{6a}t^{1/3} \\ \dfrac{1}{2G_0}\left\{\dfrac{2}{3}a\sqrt{6a} + \dfrac{ax}{t^{1/3}} - \dfrac{x^3}{18t}\right\} & -\sqrt{6at}t^{1/3} \le x \le 0 \\ \dfrac{1}{2G_0}\left\{\dfrac{2}{3}a\sqrt{6a} - \dfrac{ax}{t^{1/3}} + \dfrac{x^3}{18t}\right\} & 0 \le x \le \sqrt{6at}t^{1/3} \\ 0 & x \ge \sqrt{6at}t^{1/3} \end{cases}$$

At this stage the model must be developed further. Indeed, unlike the longshore diffusivity G_0 which affects the ability of the system to transport sediment alongshore, the parameter *a* has no clear physical meaning as it simply results from an integration process. The plots in the Figure 3 are consistent with highly symmetric geomorphic features; but, at this stage, we have no way to use the model for applications.

Using the Cuspate Model

From Eq. (27), the length λ(t) of the foreland spit is :

$$\lambda(t) = 2\sqrt{(6at^{1/3})} \qquad (17)$$

For *x*=0, Eq. (16) results in:

$$S(0) = \sqrt{(6)}a^{3/2}/(3G_0) \qquad (18)$$

Making power three Eq. (28), and deleting *a* to the power of 3/2 from equations, we get the expression:

$$S(0) = \frac{\lambda^3}{2G_0.72t} \quad (19)$$

If t_p is the present time, one may say that

$$\Delta t = t_p - t_0 \quad (20)$$

is the time required to nucleate and to develop the cuspate spit from an initial moment t_0. For convenience, let us rename $S(0)$ as A_p. With this formalism, Eq. (19) can be rewritten as:

$$\Delta T = \frac{1}{2G_0} \cdot \frac{\lambda_p^3}{72A_p} \quad (21)$$

where A_p and λ_p are the amplitude and the alongshore length of the cuspate spit at the present day, respectively. These parameters can be easily measured on an aerial photograph or by satellite imagery (Figure 4) following the simple protocol described below.

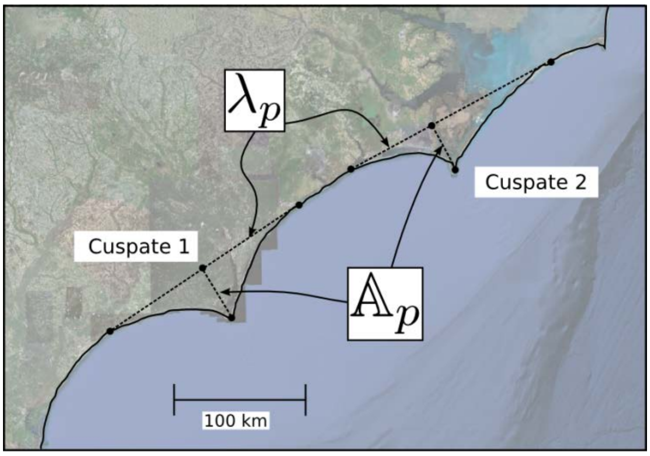

Figure 4. Sketch of two cuspate spits along the Carolina Coast, from Asthon *et al*. (2001). The two cuspates (1 and 2) display well defined wave lengths λ_p and wave amplitudes A_p. Those parameters can be estimated with the methodology described in the text.

To quantify A_p and λ_p for a given cuspate, the following method is applied: a) draw a line at the tangent to the shoreline; it intersects the shoreline in two points. This defines a segment which is the cuspate length λ_p; (b) from the head of the cuspate spit, draw the perpendicular line to the segment defined in (a); the point at their intersection and the point at the head of the cuspate form a new segment whose length is the amplitude A_p of the cuspate spit.

Eq. (21) can be considered as a method for the datation of a cuspate spit knowing the mean alongshore diffusivity and the geometrical features of the cuspate in the present day. Alternatively, the cuspate model can be thought as a tool to quantify the mean alongshore diffusivity G_0 having information on the geometry of the cuspate spit and its age. Indeed, considering that the present day geometry of a cuspate is known, as well as its age Δt (e.g. dated by its occurence in an artificial continental water body damned at a given period; or directlty dated by any well-adapted datation method). This information can be robustly retrieved through field work by geologists. From Eq. (21) it reads directly:

$$2G_0 = \lambda_p^3 / (72.A_p.\Delta T) \quad (22)$$

The calculation of such a mean alongshore diffusivity from parameters expressed on a geological/historical time scale is interesting for the classification of the paleo-system concerned with respect to well-known existing littoral systems (such as described in Dean & Dalrymple (2002)).

Another obvious application is to calculate the mean active profile (or the mean wave height or period) for the system at a given time knowing the present day geometry and all the other long-term mean forcings. Combining Eqs (5), (10) and (21), we have:

$$h_0 + B = \frac{9K\rho g H_0^2 T_0 A_p \Delta T}{2\pi(\rho_s - \rho)(1-p)\lambda_p^3} \quad (23)$$

The benefit of this last application could be debated for recent cuspate spits. Indeed, one will claim that there exist other ways of calculating an active profile directly from data in the field (e.g. considering the analysis of wave data at a buoy, and the subsequent extraction of a closure depth). Nevertheless, for ancient cuspate spits it seems obvious that the calculation of $H_0^2 T_0$ directly from Eq. (23) would provide significant semi-quantitative restrains for the reconstruction of paleo-wave regimes. Indeed, the parameters ρ_s, ρ and p could be determined by analysis of sand properties within the cuspate deposits. λ_p and A_p could be determined by direct surface observation (or seismic investigation if the cuspate is buried). h_0 could be determined by the identification of the location where wave ripples vanish and B by the identification of the position of the shoreline and the highest location concerned by wave impact.

DISCUSSION

The cuspate model offers the opportunity to calculate useful parameters for coastal engineering and geosciences. But applications under real conditions are beyond the objective of this paper, although they could be presented in future works.

Nevertheless, the cuspate model highlights a more fundamental result. In their numerical analysis of the growth of cuspate spits, Ashton & Murray (2006) suggested that there exists an approximate relationship between the age and the length of a cuspate feature. They wrote (Eq. (10) in their paper):

$$\Delta t \approx \left(\frac{K_1}{D_{sf}} H_0^{12/5} T^{1/5} \right) \Delta x^2 \quad (24)$$

where Δt is our ΔT (the age of the cuspate), Δx^2 is λ_p^2 the cuspate length squared, and D_{sf}, K_1 and other variables can be considered as constant and are ignored in the following. The authors derived this approximate relationship from the equation at the origin of their numerical model and from an analysis of numerical simulations.

In this paper we demonstrate with Eq. (21) that a formal relationship between the age and the length of the cuspate is mathematically correct, and that this relationship is definitively controlled by a power law as Ashton & Murray (2006) suggested. Although the term to the power is different (3 in our case; 2 in the case of Ashton & Murray), these two totally independent proofs give more confidence in the reality of such a relationship. The occurrence of a power law suggests also that more underlying physics remain to be analyzed.

Finally, Ashton & Murray (2006) sustained the idea that, contrary to a traditional point of view, the wave angle in deep water strongly controls shoreline perturbations through their so-called anti-diffusional high wave angle instability. In this paper we demonstrate that deep water wave features H_0 and T_0 drive non-linear shoreline instabilities in a simple way with absolutely no dependance on what occurs to waves in the nearshore. This is also a formal confirmation of what was suggested by Ashton & Murray (2006). Fourth, we demonstrate that there is no need to transform – by quite complex and obscure operations – shallow water variables into deep water ones (Equations (1), (4) and (5) of Ashton & Murray, 2006) to get this result.

CONCLUSION

From what can be called the non-linear Pelnard-Considère equation, we develop an explicit formulation of the growth of a symmetrical cuspate spit through time. This formulation can be applied (a) to the calculation of the age of a cuspate spit, (b) to the determination of a mean alongshore diffusivity in the vicinity of a cuspate spit, or (c) to the calculation of a paleo-active profile or information on paleo-wave regimes after some simple geological field data has been acquired. More substantially, the paper confirms the works of Ashton & Murray (2006) in the sense that it provides an alternative formal proof of what was suggested. In a near future the cuspate model will be more deeply explored, tentatively extended to other geomorphic features and engineering/ geological applications will be engaged to confirm its relevancy.

ACKNOWLEDGEMENT

This work was funded by NUCLEASPIT (CNRS Mathematics and Physics) and by KUNSHEN (ANR international program). The authors thank GLADYS (www.gladys-littoral.org) and SO LTC (www.soltc.org) for the comments on the original work as well as the reviewer of the previous version of the document.

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