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A multiaxial criterion for crack nucleation in rubber

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The present paper is a first step towards the definition of a new multiaxial fracture criterion for rubber-like materials. Assuming that elastomers are subjected to a uniform distribution of intrinsic flaws, the framework of Eshelbian mechanics is considered. More precisely, the properties of the energy-momentum tensor are thoroughly studied to derive the criterion. A basic numerical example is presented and the qualitative discrepancy between the results obtained with this criterion and those relative to more classical approaches is highlighted.

Keywords: Rubber; Crack nucleation; Multiaxiality; Eshelbian mechanics; Energy–momentum tensor

1. Introduction

Rubber parts are widely used in many applications such as tires, engine mounts, bumpers or shoes. For the development of new industrial products, both short- and long-term durabilities should be modelled and estimated. Depending on applications, crack nucleation or crack growth should be considered. As shown by Mars and Fatemi (2002), only few criteria are used for crack nucleation and their efficiency is questionable whereas crack growth approaches are well-established since the pioneering work of Rivlin and Thomas (1953).

The present paper focuses on the prediction of crack initiation in rubber. The most widely used criteria are the maximum principal stretch and the strain energy density. Nevertheless, none of them is applicable to multiaxial loading conditions (see Mars and Fatemi (2002) and the references herein). More recently, some authors consider that rubber contains intrinsic flaws which can be seen as small cracks that grow under loading. It leads to the definition of the cracking energy density by Mars (2001). This energy represents

the portion of the total energy which contributes to the growth of these small cracks. For a given surface, it is defined as “the work performed by the tractions on the surface in deforming the surface”. A similar study was recently proposed by Raoult et al. (2003). The aim of the present paper is to rationalize this approach using the theoretical framework of the Eshelbian mechanics.

In the next Section, principal works on the Eshelbian mechanics are recalled. Section 3 is devoted to the derivation of the new criterion. Then, only a basic numerical example is presented in Section 4 in order to highlight the discrepancy between the present prediction and classical results. Finally, concluding remarks are given in Section 5.

2. A brief literature survey on the Eshelbian mechanics

As mentioned in the introduction, crack nucleation in rubber involves small intrinsic flaws which grow under mechanical loading. Nevertheless, crack nucleation criteria should be defined in terms of continuum mechanics parameters in order to be combined with finite element analysis in engineering applications. In fact, the influence of microstructural defects on damage distribution has to be estimated considering continuum mechanical variables. In order to study a real part which contains defects using a homogeneous model without defect, the Eshelbian mechanics (or the mechanics in material space) is revealed to be an appropriate tool.

This theory is based on the seminal work of Eshelby (1951) who introduced the energy-momentum tensor (also called the Eshelby tensor) to study forces on elastic singularities and defects. This tensor was extended to large strain by Eshelby (1975) and Chadwick (1975). More recently, the general theory of the mechanics in material space was derived by Maugin (1993), Kienzler and Herrmann (2000), and Gurtin (2000).

In most of the works, the Eshelby tensor is used as the integrand of path-independent integrals (after contraction with the normal of the contour) to determine configurational forces (also called material forces) that apply on singularities such as crack tip, inclusion or interfaces (Steinmann, 2000; Steinmann et al., 2001; Gross et al., 2003) and also to improve meshes in finite element analysis (Mueller and Maugin, 2002). In fact, configurational forces generalize the well-known concept of energy release rates in fracture mechanics as defined by the J - (Rice, 1968), M - and L - (Budiansky and Rice, 1973) integrals. Nevertheless, even if the duality between the Cauchy stress tensor in the physical space and the Eshelby tensor in the material space is well-recognized (see for example Chadwick (1975)), only few works deal with local properties of the energy-momentum tensor. The physical significance of its components have been discussed only recently by Kienzler and Herrmann (1997), its invariants have been used to predict crack direction and length (Atkinson and Aparicio, 1999) and the possibility to develop local fracture criteria using its components was investigated only a few years ago (Kienzler and Herrmann, 2002).

3. Formulation of the criterion

In the material space, rubber can be considered as a material with a uniform distribution of defects. From a microscopic scale, these defects can be voids (pre-existing porosities and cavities), micro-cracks or rigid inclusions (filler agglomerates). Crack nucleation is assumed to be a consequence of the growth of these pre-existing small flaws as proposed by Mars (2001). Depending on the geometry of the considered part and loading conditions, not all of the stored elastic energy is available for the growth of defects (see for example the experiments conducted by Roberts and Benzies (1977) under fatigue loading conditions). Thus, for a given rubber part the energy release rate of small flaws has to be determined in each particle to exhibit the most propitious zones for crack occurrence.

Consider a rubber body and a current particle M in the reference configuration of the physical space as shown in Fig. 1(a). In the material space, the body is subjected to a smoothly distributed defects density. Zooming in near M leads to Fig. 1(b) in which the defect in M is represented by a grey disk.

In M , the second-order Eshelby tensor is defined by

$$\Sigma = W\mathbf{I} - \mathbf{C}\mathbf{S}, \quad (1)$$

where W is the strain energy density per unit volume of the reference configuration, \mathbf{I} is the identity tensor, \mathbf{C} is the right Cauchy–Green strain tensor and \mathbf{S} is the second Piola–Kirchhoff stress tensor. In the general case, W depends on both the deformation gradient \mathbf{F} and the reference position \mathbf{X}

$$W = W(\mathbf{F}, \mathbf{X}) \quad (2)$$

and Σ is symmetric with respect to \mathbf{C}

$$\Sigma\mathbf{C} = \mathbf{C}\Sigma^T. \quad (3)$$

Considering that the material is macroscopically homogeneous, the dependence of W on \mathbf{X} in Eq. (2) is irrelevant. Moreover, assuming that rubber is isotropic and elastic, tensors \mathbf{C} and \mathbf{S} are coaxial and commute, and the Eshelby tensor is symmetric, i.e. Eq. (3) reduces to

$$\Sigma = \Sigma^T. \quad (4)$$

In the present study, rubber is assumed hyperelastic and incompressible (its more complex properties such as viscoelasticity, hysteresis and Mullins effect would be considered in further works). The stress–strain relationship can be written as (see for example the book of Holzapfel (2000) for details):

$$\mathbf{S} = -p\mathbf{C}^{-1} + 2\frac{\partial W}{\partial \mathbf{C}} \quad (5)$$

where p is an arbitrary hydrostatic pressure which has to be determined using equilibrium equations. Recalling the isotropy assumption, the strain energy density only depends on the two first principal invariants of \mathbf{C}

$$I_1 = \text{tr}\mathbf{C} \quad \text{and} \quad I_2 = \frac{1}{2}[(\text{tr}\mathbf{C})^2 - \text{tr}(\mathbf{C}^2)] \quad (6)$$

and Eq. (5) becomes

$$\mathbf{S} = -p\mathbf{C}^{-1} + 2\left(\frac{\partial W}{\partial I_1} + I_1\frac{\partial W}{\partial I_2}\right)\mathbf{I} - 2\frac{\partial W}{\partial I_2}\mathbf{C} \quad (7)$$

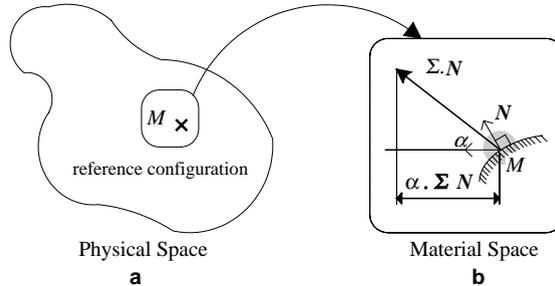


Fig. 1. (a) A rubber body in its undeformed configuration and (b) zoom in near the particle M in the material space.

in which $\partial W/\partial I_1$ and $\partial W/\partial I_2$ are the material functions. Finally, the Eshelby tensor of a homogeneous, isotropic, incompressible, hyperelastic material is given by

$$\boldsymbol{\Sigma} = (W + p)\mathbf{I} - 2\left(\frac{\partial W}{\partial I_1} + I_1 \frac{\partial W}{\partial I_2}\right)\mathbf{C} + 2\frac{\partial W}{\partial I_2}\mathbf{C}^2. \quad (8)$$

The Eshelby tensor being now defined in M , its local properties have to be examined. Consider a unit surface with the outward normal vector \mathbf{N} and a given direction $\boldsymbol{\alpha}$ as shown in Fig. 1(b). Following Kienzler and Herrmann (1997), the scalar $\boldsymbol{\alpha} \cdot \boldsymbol{\Sigma} \cdot \mathbf{N}$ is the change in the total energy density at the point M due to a material unit translation in direction $\boldsymbol{\alpha}$ of the unit surface with normal \mathbf{N} . It is to note that this quantity is nearly similar to the definition of the cracking energy density proposed by Mars (2001). The unit surface on which the maximum of energy is released is defined by its outward normal vector $\tilde{\mathbf{N}}$ such as

$$\|\boldsymbol{\Sigma} \cdot \mathbf{N}\| \text{ is maximum for } \mathbf{N} = \tilde{\mathbf{N}}. \quad (9)$$

In other words, the norm of the material traction is maximum on the surface with normal $\tilde{\mathbf{N}}$. Moreover, it can be written as

$$\|\boldsymbol{\Sigma} \cdot \mathbf{N}\| = \sqrt{\mathbf{N} \cdot \boldsymbol{\Sigma}^T \boldsymbol{\Sigma} \cdot \mathbf{N}}. \quad (10)$$

As $\boldsymbol{\Sigma}$ is a symmetric tensor, $\boldsymbol{\Sigma}^T \boldsymbol{\Sigma}$ is a positive semi-definite tensor. Thus, its eigenvalues are real, and positive or null, and its three eigenvectors are perpendicular to each other; these eigenvalues are denoted $(\Sigma_i^2)_{i=1,3}$. In fact, $(\Sigma_i)_{i=1,3}$ are the three real eigenvalues of $\boldsymbol{\Sigma}$. Then, the outward normal vector $\tilde{\mathbf{N}}$ in Eq. (9) is one of the eigenvectors of $\boldsymbol{\Sigma}$ which are equal to those of $\boldsymbol{\Sigma}^2$. Moreover, recalling that the body ever tends to reduce its potential energy and that positive material forces correspond to increases of energy, the material traction should be opposite to the normal vector direction:

$$\tilde{\mathbf{N}} \cdot \boldsymbol{\Sigma} \cdot \tilde{\mathbf{N}} \leq 0. \quad (11)$$

Thus, $\tilde{\mathbf{N}}$ is the eigenvector which corresponds to the minimum eigenvalue of $\boldsymbol{\Sigma}$ if at least one of the eigenvalues is negative. Otherwise, in the case of three positive eigenvalues, no vector $\tilde{\mathbf{N}}$ is defined and this case can be considered as a ‘‘crack closure effect’’ in which no energy is released for the growth of the flaw.

As a conclusion, the new crack nucleation criterion for rubber can be summarized as follows:

$$\text{crit} = |\min[(\Sigma_i)_{i=1,3}, 0]| \quad (\Sigma_i)_{i=1,3} : \text{eigenvalues of the Eshelby tensor}. \quad (12)$$

Moreover, if $\text{crit} \neq 0$ the eigenvector associated with the minimum eigenvalue of $\boldsymbol{\Sigma}$ is normal to the plane of crack growth.

4. A basic example

The present paper being a work-in-progress, only a simple numerical example is presented in order to qualitatively compare damaged areas exhibited by our criterion with those obtained with previous approaches.

The study concerns the uniaxial extension of an axisymmetric rubber dumbbell sample with metal plates. The sample geometry is similar to the one proposed by Beatty (1964) and is presented in Fig. 2(a). The sample is stretched under uniaxial quasi-static enforced displacement conditions. Its significant radius of curvature leads to a higher local deformation state in its middle than the enforced nominal deformation. Thus, the predominant crack initiates at the centre of the sample (see for example the work of Le Cam et al., 2004). The global stretching level varies from 1.0 (undeformed configuration) to 3.0. The mechanical problem is solved using the finite element method, and particularly the program Abaqus (Hibbit et al., 1999). The model is presented in Fig. 2(b). It is axisymmetric (Oz is the axis of revolution) and only a semi-sam-

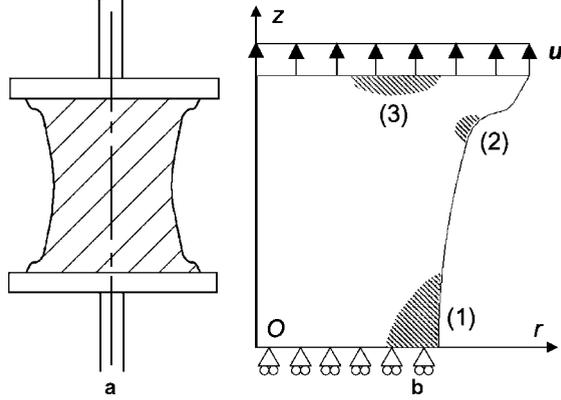


Fig. 2. (a) The dumbbell sample and (b) The model considered for simulation and the three damaged areas.

ple is considered due to the symmetry with respect to the $z = 0$ -axis. Displacements of the top nodes are incrementally enforced until 300% of nominal stretch. The material is assumed to obey the neo-Hookean constitutive equation, i.e. the strain energy function is simply given by:

$$W = C(I_1 - 3), \quad (13)$$

where C is the material parameter; here it is arbitrary set to 1 MPa. Consequently, material functions involved in Eqs. (7) and (8) reduce to

$$\frac{\partial W}{\partial I_1} = C \quad \text{and} \quad \frac{\partial W}{\partial I_2} = 0. \quad (14)$$

The model is discretized with 510 nodes and 464 four-node bilinear hybrid with constant pressure finite elements. All criteria examined in the following are implemented in Abaqus with the help of the UVARM facility.

Four crack nucleation criteria are studied: the maximum principal stretch, the strain energy density, the cracking energy density and the present proposal. The maximum principal stretch is the square root of the maximum eigenvalue of the right Cauchy–Green strain tensor, and in the present case the cracking energy density is simply defined as the maximum eigenvalue of the energy tensor $\int \sigma d\epsilon$ in which the integration is performed on the loading path.

The distribution of criteria in the mesh is examined as a function of the stretch level. In order to simplify the following discussion, the three damaged zones which are highlighted by the criteria are depicted in Fig. 2(b) by hash-marked areas:

- (1) is the area in which the macroscopic crack takes place;
- (2) is the area situated in the curved region near the sample basis;
- (3) is the area situated under the metal plates.

First, results obtained with the three first criteria are examined. In the three cases, two damaged areas are highlighted: areas (1) and (2) are simultaneously predicted for relatively small stretching levels (about 1.1). For the maximum stretch, i.e. 3.0, criteria exhibit that area (1) is the most damaged zone and that area (2) tends to disappear. For the cracking energy density, no more damaged zone in area (2) is observed for the maximum stretching level. Consider now the results obtained with our criterion. They are noticeably different from those obtained with previous criteria. In fact, the three damaged areas exhibited in Fig. 2(b) are predicted by the criterion. For small stretching levels, i.e. about 1.1, damage occurs in the most curved

region of the sample (area (2)), then internal cracks take place under the metal plate (area (3)) for stretching levels between 1.2 and 1.4, and finally for stretches greater than 1.7 area (1) is revealed to be the most damaged zone in which the macroscopic crack appears.

5. Concluding remarks

In this paper, a new multiaxial criterion for crack nucleation in rubber was proposed. This criterion is founded on the local properties of the Eshelby second-order tensor. First results show quantitative differences with other commonly used criteria.

This study being only a first step, it leaves unresolved some issues of fundamental importance. First, the present criterion should be validated with analytical solutions and more complex industrial problems. Moreover, the physical significance of the eigenvector associated with the criterion have to be thoroughly investigated. Second, the relevance of the criterion for the prediction of fatigue life of rubber parts is obviously a critical issue. Finally, the influence of the complex mechanical response of elastomers on the formulation of the criterion should be analyzed.

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