Prediction of flow-induced sound and vibration: On different methods for introducing the TBL excitation in the vibroacoustic model

Marion Berton, Laurent Maxit, Daniel Juvé, Christian Audoly

To cite this version:
Marion Berton, Laurent Maxit, Daniel Juvé, Christian Audoly. Prediction of flow-induced sound and vibration: On different methods for introducing the TBL excitation in the vibroacoustic model. Acoustics 2013, Nov 2013, New-Delhi, India. p. 1281, 2013. <hal-00994543>
PREDICTION OF FLOW-INDUCED SOUND AND VIBRATION: ON DIFFERENT METHODS FOR INTRODUCING THE TBL EXCITATION IN THE VIBROACOUSTIC MODEL

M. Berton\textsuperscript{a,b,c}, L. Maxit\textsuperscript{a}, D. Juve\textsuperscript{b}, C. Audoly\textsuperscript{c}

\textsuperscript{a}. LVA, INSA de Lyon, Bâtiment St. Exupéry, 25 bis av. Jean Capelle, 69621 Villeurbanne Cedex
\textsuperscript{b}. Centre Acoustique, LMFA UMR 5509, Ecole Centrale de Lyon, 36 av. Guy de Collongue, 69134 Ecully Cedex
\textsuperscript{c}. DCNS Research, Centre du Mourillon, Rond-point de l’Artillerie de la Marine, 83000 Toulon
e-mail: marion.berton@insa-lyon.fr

The modeling of the vibroacoustic response of a heavy fluid loaded panel excited by a turbulent boundary layer is investigated. The pressure fluctuations due to the Turbulent Boundary Layer (TBL) at the surface of the structure are characterized by cross-spectra which can either be expressed in the physical space or in the wavenumber space. Various models have been proposed in the literature to obtain closed form expressions of these spectra starting from global TBL characteristics. A difficulty to estimate the vibroacoustic response of the panel excited by TBL relies on the coupling of the deterministic vibroacoustic model with the statistical wall pressure fluctuations model. In the present paper, we study different techniques to achieve this coupling: (a), in the physical space; (b), considering an uncorrelated wall plane waves field; (c), using a reciprocity technique; (d), using realizations of the stochastic uncorrelated plane waves field. These techniques require calculating different transfer functions with a vibroacoustic model. For that, we use the Patch Transfer Function formalism (PTF). This approach allows us to couple the structure problem with the heavy fluid problem. The added mass effect of the fluid on the panel (i.e. fluid reactance) is taken into account in the modal frequencies of the panel whereas the acoustic radiation from the panel into the fluid (i.e. fluid resistance) is modeled using the real part of the patch acoustic impedances of the fluid domain. For validation purpose, the results of these vibroacoustic calculations are compared with a result from the literature for a test case composed of a simply supported rectangular thin plate immersed in water. Afterwards, the response of the panel excited by the turbulent boundary layer is predicted in the low and medium frequency domains using the four techniques described above. The response is characterized here by the velocity spectrum at one point on the plate. Convergence of the four techniques is studied. The results with different values of calculation parameters are
presented and analyzed in order to find the optimal ones. We observe that the four techniques give very similar results for these optimal parameters. Finally, we discuss the interest of each technique.

1. Introduction

In the context of the underwater missions, submarine vehicles should be designed to minimize noise sources and acoustic radiation. One of these noise sources is the turbulent flow induced by the vehicle movement. Numerical modeling can help to improve knowledge of interaction between Turbulent Boundary Layer (TBL) and heavy fluid loaded structures. The classical method consists in characterizing the wall pressure fluctuations by a spectrum depending on the TBL parameters, and then to inject it in a vibroacoustic model. The coupling between the statistical model used to describe the wall pressure fluctuations and the deterministic vibroacoustic model constitute a difficult point in the methodology. Several approaches to achieve this coupling are investigated in this paper.

2. Different methods for introducing TBL excitation in the vibroacoustic model

Four methods to evaluate the vibro-acoustic response to TBL excitation are briefly described in this section. Let us consider a fluid loaded baffled panel of surface \( \Sigma_p \) excited by a fully developed, stationary and homogeneous TBL of wall pressure field \( p(x, t) \) at point \( x \) as a function of time. The assumptions are made that the wall pressure fluctuations are not affected by the vibrations of the plate, and that the acoustic waves do not interfere with the flow. The space-frequency spectrum of the plate velocity at point \( x \) in response to a discrete spatial distribution of point pressure forces is defined as [1-2]:

\[
S_{vv}(x, \omega) = \sum_{\tilde{x}} \sum_{\tilde{x}} H_v^*(x, \tilde{x}, \omega) S_{pp}(\tilde{x}, \tilde{x}, \omega) H_v(x, \tilde{x}, \omega) \delta \tilde{x} \delta \tilde{x}.
\]

where: - \( \chi \) is the discretization of \( \Sigma_p \) defined with a criteria which will be discussed later;
- \( S_{pp}(\tilde{x}, \tilde{x}, \omega) \) is the space-frequency cross-spectrum of the wall pressure between points \( \tilde{x} \) and \( \tilde{x} \);
- \( H_v(x, \tilde{x}, \omega) \) is the transfer function in velocity at point \( x \) driven by a point force at point \( \tilde{x} \).

It can be obtained using a vibro-acoustic model considering a driving point force at \( \tilde{x} \). The method to estimate the panel response with (1) and with \( H_v(x, \tilde{x}, \omega) \) evaluated from a vibro-acoustic model with a driving point force at \( \tilde{x} \) is called the “spatial method”.

The TBL excitation can also be described by the space Fourier transform of \( S_{pp}(\tilde{x}, \tilde{x}, \omega) \) which gives the wavenumber-frequency spectrum \( \Phi_{pp}(k, \omega) \). The discrete space Fourier transform of \( H_v(x, \tilde{x}, \omega) \) is defined by:

\[
\tilde{H}_v(x, k, \omega) = \sum_{\tilde{x}} H_v(x, \tilde{x}, \omega) e^{jk\tilde{x}} \delta \tilde{x}.
\]

The combination of equations (1) and (2) and the use of the wavenumber-frequency spectrum \( \Phi_{pp}(k, \omega) \) lead to:

\[
S_{vv}(x, \omega) = \frac{1}{4\pi^2} \sum_{k} \left| \tilde{H}_v(x, k, \omega) \right|^2 \Phi_{pp}(k, \omega) \delta k.
\]
\( H_p(x, k, \omega) \) can be interpreted as the velocity response at point \( x \) of the plate, due to a generalized plane wave of wavevector \( k = (k_x, k_y) \), exciting the plate at points \( \tilde{x} \) (see the definition of the generalized plane waves in [3]). The method to estimate the panel response with (3) and with \( H_p(x, k, \omega) \) evaluated from a vibro-acoustic model considering a generalized wall plane wave excitation is called the “uncorrelated plane waves method”.

Another method [2] is based on the reciprocity principle, which indicates that the ratio of the velocity at point \( x \) over the normal force at point \( \tilde{x} \) is equivalent to the ratio of the velocity at point \( \tilde{x} \) over the normal force at point \( x \):

\[
H_p(x, \tilde{x}, \omega) = H_p(\tilde{x}, x, \omega).
\] (4)

This expression can be introduced in (2), and \( H_p(x, k, \omega) \) can then be interpreted as the space Fourier transform of the response for a point force at \( x \). It results that only one single force should be considered in the vibro-acoustic model to deduce \( \Phi_\omega \). The method to estimate the panel response with (3) and using the reciprocity principle to evaluate \( \Phi_\omega \) is called the “wave-point reciprocity method”.

The last method consists in considering several realizations of the stochastic uncorrelated plane waves. For each sample (i.e. realization), a random phase is assigned to each wall plane wave of amplitude \( \sqrt{\Phi_{pp}(k, \omega)/2\pi} \). The deterministic wall pressure of the sample is then:

\[
p(x) = \sum_{k \in K} e^{-i k \cdot x} e^{i \phi_k} \sqrt{\Phi_{pp}(k, \omega)/2\pi \delta k},
\] (5)

where \( \phi_k \) is the random phase associated to the wall plane wave of wavevector \( k \). The panel response to the TBL excitation is estimated from the average of the panel response over the samples of plane waves. This method is called the “sampling of uncorrelated plane waves method”.

The four presented approaches are based on equations (1) or (3). Criteria should be considered to define the discretization spaces \( \chi \) and \( K \). For \( \chi \) (related to (1)), spatial discretization has to be fine enough to well describe the correlation decrease between wall pressure signals at points \( \tilde{x} \) and \( \tilde{x} \). For \( K \) (related to (3)), the maximal wavenumber \( k_{max} \) has to be chosen to include the parts of the wall pressure spectrum that impact the response of the panel. Moreover the wavenumber resolution \( \delta k \) should be small enough to take properly into account the variations of \( \Phi_\omega \) and \( \Phi_{pp}(k, \omega) \) over \( k \). These criteria will be studied and discussed on numerical examples in Sec. 4.

3. Patch Transfer Functions (PTF) formalism validation

In this paper, the different transfer functions of the fluid loaded panel will be evaluated with the PTF (Patch Transfer Functions) approach ([4]). It is a substructuring approach which allows us to decompose our problem into two parts (i.e. the panel and the semi-infinite fluid). The coupling interface is divided in patches which size depends on the considered wavelengths. Each part is characterized separately by Patch Transfer Functions (i.e. patch mobilities for the panel, patch impedances for the fluid). Writing the continuity conditions at the coupling interface enables to assemble the two parts. The particularity of the present model compared to [4-5] is that the fluid added mass effect is taken into account through the “wet modal frequencies”, (which are estimated by assuming the fluid incompressible) instead of using the imaginary part of the acoustic impedance of the fluid domain. This allows to overcome the convergence issue evoked in [5] concerning the patch size criterion. Here, a patch size lower than half the wavelength gives converged results.

The test case used here to validate the PTF code corresponds to the case studied by Berry [6]. A simply supported plate immersed in water is excited by one point force at its center. Its
dimensions are 455 mm x 375 mm x 1 mm and its structural damping is taken to be $\eta = 0.01$. The critical frequency is around 220 kHz.

![Figure 1](image.png)

**Figure 1**: Mean quadratic velocity of a thin plate driven by a point force at its centre.

The obtained mean quadratic velocities over the plate surface are compared in Figure 1 between 10 Hz and 500 Hz, with a comparison to the PTF calculation without taking the added mass effect into account. The results show that the modal frequencies calculated with the PTF method with added mass effect are satisfactory in comparison to the case without added mass effect. The frequency shift between the corresponding curves is well visible: for example the resonant peak at 121 Hz in the case without added mass effect is shifted to 43 Hz for both Berry’s results and the PTF results.

### 4. Panel response calculations to TBL excitation

This section investigates the structural response of the plate described in Sec. 3 under a water flow with a convective velocity $U_c = 5\, m/s$. The TBL wall pressure fluctuations which excite the panel are described here by the Corcos cross-spectrum model [7], either in the physical space or the wavenumber space. The observation point $M$ of the plate velocity response is arbitrarily fixed at $x = 5.4 \, cm$ and $y = 18.6 \, cm$. The transfer functions $H_p(x, \bar{x}, \omega)$ and $\bar{H}_p(x, \bar{k}, \omega)$ are calculated with PTF. We propose here to study the results of the different methods described in Sec. 2 in function of different calculation parameters. We indicate that the considered frequency band [10 Hz-500 Hz] is above the hydrodynamic coincidence frequency (which is around 2 Hz).

![Figure 2](image.png)

**Figure 2**: Velocity spectrum at point $M$ for different spatial resolutions $\delta x$.

For the “spatial method”, the spatial resolution is set dependant to a characteristic wavelength $\lambda_c = U_c/f$ in order to adapt the spatial discretization to the decrease of correlation of the wall pressure fluctuations. Results with different spatial resolutions are shown on Figure 2. A spatial
The criterion related to the “uncorrelated plane waves method” concerns the wavenumber range defined with $k_{max}$ and $\delta k$: at the first sight, the analysis of the wavenumber-frequency spectrum $\Phi_{pp}(k, \omega)$ leads us to consider a maximum wavenumber $k_{max}$ greater than the convective wavenumber $k_c = \omega/U_c$. However, the dynamic behaviour of the plate can filter the excitation and decrease the impact of the convective region of the wall pressure spectrum on the plate response. If it is the case, $k_{max}$ could be defined from the natural bending wavenumber of the structure $k_f$ (which is well lower than $k_c$ for the present case: at 500 Hz $k_c \sim 630 \text{ m}^{-1}$ and $k_f \sim 63 \text{ m}^{-1}$). Figure 3 shows the results for the two cases: $k_{max} = 2k_f$ and $k_{max} = k_c + Dk$ with $Dk = 100 \text{ m}^{-1}$. The results show that the convective region of the TBL wall pressure spectrum does not influence the plate response in the considered frequency range. It is thus unnecessary to take into account the convective part of $\Phi_{pp}(k, \omega)$. The criterion $k_{max} = 2k_f$ can be applied for this case, which saves computing time. We underline that this criterion can be questioned in regards to the TBL model. For example, as the Chase model presents a lower pressure level than the Corcos model in the low wavenumber region, the convective part could have a higher contribution on the plate response.

Figure 3: Velocity spectrum at point M for different cut-off wavenumbers, and comparison with the spatial method.

The converged results for both “spatial” and “uncorrelated plane waves” methods are also compared in Figure 3, and show a good agreement.

The result of the “wave-point reciprocity method” is proposed in Figure 4 a. A good agreement with the result of the “uncorrelated plane waves method” is observed. Although one single point force excitation should be considered in the vibro-acoustic model, the time cost (due to the space Fourier transform) appears to be higher than for the “uncorrelated plane waves method” for this case (650 seconds for the reciprocity method against 460 seconds for the plane waves method).

Finally, the results obtained from the last method, the “sampling of uncorrelated plane waves”, are presented in Figure 4 b. The results for each realization are plotted in grey lines whereas the mean values of the 10 realizations are plotted in black line. The figure shows a good agreement with the “uncorrelated plane waves method” for a nearly equivalent computation time. One advantage of this approach is that few load case (here 10) should be considered in the vibro-acoustic model. If a commercial vibro-acoustic code is used to evaluate the transfer functions, the small number of load case allows reducing the amount of input/output data compared to the “uncorrelated plane waves method”.

Figure 4: (a) Velocity spectrum at point M for different cut-off wavenumbers, and comparison with the spatial method. (b) Mean and standard deviation of the results for the “sampling of uncorrelated plane waves” method.
Figure 4: Velocity spectrum at point M. a.: Reciprocity method. b.: Sampling method.

5. Conclusion

Four methods have been tested and compared with good agreement in order to introduce the TBL excitation in the calculation of the response of an elastic plate submitted to a water flow. The “spatial method” is the most time consuming. In order to increase its efficiency, investigations could be carried out to limit the excitation to an area around a reference point. The process based on the Cholesky decomposition of the space-frequency cross-spectrum of the wall pressure described in [8] could be an alternative to reduce the computing time. The “uncorrelated plane waves method” has shown its efficiency when the cut-off wavenumber $k_{max}$ can be fixed in regards to the panel characteristics (i.e. natural bending wavenumber) and not of the excitation characteristics (i.e. convective wavenumber). Additional investigations should be done to study the influence of the hydrodynamic part of other TBL models. The “wave-point reciprocity method” has also been validated. For the studied case, it has not allowed to reduce the computing time. This approach is more appropriate for stiffened structure ([2]) for which the point response in the wavenumber domain (i.e. $\tilde{H}(x, k, \omega)$) can easily be estimated. Finally the technique using samples of plane waves with random phase gives also a good accuracy with few realizations. It constitutes another way to introduce the TBL excitation with few load case in the vibro-acoustic model.

Acknowledgements: This work was performed within the framework of the LabEx CeLyA of Université de Lyon, operated by the French National Research Agency (ANR-10-LABX-0060/ ANR-11-IDEX-0007). This work was performed with the financial support of DCNS.

REFERENCES