Improvement in solar declination computation

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INTRODUCTION

A simple formula for solar declination angle computation, as a function of the day of the year, is available. It is generally reliable for most of the applications. Nevertheless, this formula introduces some systematic errors, sometimes larger than one degree. This fact is not acceptable in some solar radiation data processing. Another formula has been investigated, which needs, as input data, the concerned year, and leads to errors smaller than 0.02°, with quite acceptable computations.

USUAL FORMULA

The following formula, given in /1/ and reproduced in many reference books (e.g. /2/) and papers gives the solar declination angle, d (degrees), of the n-th day of the year

$$d = 23.45 \sin (360/365 . (284 + n)) \quad (1)$$

Values computed from this formula are compared with values of reference tables (Ephemerides Nautiques /3/) (Table 1). The error is within 1°, being the largest at the equinoxes. Thus, in march or September, the error on the number of hours between sunrise and sunset can be close to 10 minutes, at mid-latitudes.
This difference of 1° is not much larger than the sun diameter (32') and this is generally of small importance for most of the applications.
Nevertheless, when attention is focused on solar radiation phenomena at low solar altitude or on solar irradiance received by non-south oriented planes, precise knowledge of the sun-position is required. In such cases, errors introduced by formula (1) are not quite acceptable.

SOLAR DECLINATION VARIATIONS DURING THE YEAR

Solar declination is equal to zero at the equinoxes. It is at its maximum at the summer solstice (d = 23.442°) and at a minimum at the winter solstice (d = -23.442°).
As the earth's orbit is not exactly circular, the seasons (intervals between equinox and solstice) do not have the same duration, as shown in Table 2.

A NEW FORMULA

A new formula has been investigated, using the form suggested by DOGNIAUX (/4/):

$$d = a_0 + a_1.\sin wt + a_2.\sin 2wt + a_3.\sin 3wt + a_4.\cos wt + a_5.\cos 2wt + a_6.\cos 3wt \quad (2)$$

where

$$w = 360 / 365.2422 \quad (3)$$

and t is the time (in days, from the spring solstice).
Following times are defined

$t_0 = 0$		(Spring equinox)
$t_1 = 92.7763$		(Summer solstice)
$t_2 = 186.4226$		(Fall equinox)
$t_3 = 276.2489$		(Winter solstice)

Equation (2) has to reconstitute the declination values for these dates, so

$$d (t_0) = 0° \quad (4.1)$$

$$d (t_1) = 23.442° \quad (4.2)$$

$$d (t_2) = 0° \quad (4.3)$$

$$d (t_3) = -23.442° \quad (4.4)$$

Besides, as there are some extrema at t_1 and t_3, derivatives are equal to zero

$$dd / dt_{(t_1)} = 0 \quad (4.5)$$

$$dd / dt_{(t_3)} = 0 \quad (4.6)$$

Using these 6 linear relationships for the a_0, ..., a_6 parameters, the last coefficient is found by a least-squares procedure, using real declination values for a particular year (1967). It yields to the final formula

$$d = 0.3723 + 23.2567 \sin wt + 0.1149 \sin 2wt - 0.1712 \sin 3wt - 0.7580 \cos wt + 0.3656 \cos 2wt + 0.0201 \cos 3wt \quad (5)$$

That can be summarized into the additionnal linear constraint

$$dd / dt_{(t_0)} = 0.3952 \text{ degree/day} \quad (4.7)$$

The spring-equinox time, n_0, expressed in days from the beginning of the year is given by

$$n_0 = 78.801 + 0.2422.(YEAR-1969) - INT(0.25.(YEAR-1969)) \quad (6)$$

where INT( ) is the INTEGER PART function.
Therefore, to compute the solar declination for the n-th day of the year at noon (Universal time), it comes

$$t = n - 0.5 - n_0 \qquad (7)$$

For long period averages, a mean value, $\bar{n}_0$, of the spring equinox time may be adopted. For example, the value

$$\bar{n}_0 = 79.164 \text{ day}$$

may be adopted for the period 1961-1980. This leads to additional errors within 0.2°.

COMPARISON WITH REFERENCE VALUES

Solar declination values, computed from eqns (3), (5), (6) and (7), are compared with values given by the "Ephemerides Nautiques" for a particular year. Results are presented in Table 3.
Errors are generally within 0.01° (The mean error is +0.008°); the maximal error occurs in February and August and is equal to 0.02° (about 1′).
This is a very satisfactory accuracy and a significant improvement over usual formulae, with an acceptable increase of computation, negligible when a computer is used.
Secular variations of solar declination have not been considered here, so the present formulae can be used (with the accuracy above mentioned) for the secund half of this century; they have to be slightly corrected for other periods.

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Table 1: Comparison of the solar declination angle real values at noon (Universal Time) and values computed from eqn (1).

Table 2: Yearly variations of solar declination angle and duration of seasons (in days) (Year: 1984).

Table 3: Comparison of real and computed solar declination angle values for a particular year (1967). Computed values are given by eqns (3), (5), (6) and (7).

Table 2

	Date	Time (UT)	Dec.	Season duration
Spring Equinox	20/03	10h 25m 15s	0°	
				92.77632
Summer Solstice	21/06	5h 3m 9s	23.44°	
				93.64630
Fall Equinox	22/09	20h 33m 49s	0°	
				89.82633
Winter Solstice	21/12	16h 23m 44s	-23.44°	

Table 1

Solar Declination Angle (degree)

Date	n	Real Value 1969	Real Value 1979	Computed
01/01	1	-23.0	-23.03	-23.01
22/03	81	+ 0.68	+ 0.50	0.0
22/09	265	+ 0.28	+ 0.44	- 0.61
01/11	305	-14.47	-14.33	-15.36

Table 3

DATE	n	REAL DEC.	COMP. DEC.	DIFF.
5/ 1	5	-22.660	-22.665	.005
15/ 1	15	-21.198	-21.213	.015
25/ 1	25	-19.062	-19.084	.022
5/ 2	36	-16.047	-16.069	.022
15/ 2	46	-12.815	-12.836	.021
25/ 2	56	- 9.242	- 9.254	.012
5/ 3	64	- 6.212	- 6.216	.004
15/ 3	74	- 2.300	- 2.300	.000
25/ 3	84	1.648	1.650	-.002
5/ 4	95	5.915	5.913	.002
15/ 4	105	9.618	9.612	.006
25/ 4	115	13.055	13.047	.008
5/ 5	125	16.135	16.126	.009
15/ 5	135	18.770	18.761	.009
25/ 5	145	20.877	20.870	.007
5/ 6	156	22.503	22.498	.005
15/ 6	166	23.297	23.292	.005
25/ 6	176	23.405	23.402	.003
5/ 7	186	22.832	22.826	.006
15/ 7	196	21.597	21.588	.009
25/ 7	206	19.752	19.737	.015
5/ 8	217	17.092	17.073	.019
15/ 8	227	14.188	14.167	.021
25/ 8	237	10.910	10.890	.020
5/ 9	248	6.972	6.960	.012
15/ 9	258	3.190	3.187	.003
25/ 9	268	- 0.687	- 0.685	-.002
5/10	278	- 4.572	- 4.566	-.006
15/10	288	- 8.363	- 8.360	-.003
25/10	298	-11.957	-11.961	.004
5/11	309	-15.557	-15.566	.009
15/11	319	-18.382	-18.395	.013
25/11	329	-20.670	-20.680	.010
5/12	339	-22.320	-22.323	.003
15/12	349	-23.250	-23.248	-.002
25/12	359	-23.410	-23.408	-.002