# Kinematical analysis of 5-axis corner smoothing for transitions between linear (G1) blocks

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**Abstract:** 5-axis high speed machine tools are widely used in industry. The axis movements are generated by the Computer Numerical Controller (CNC) which has to transform the part program into a sequence of axis setpoints. Most of the time, the tool path is described with linear segments (G1) which lead to tangency discontinuities between blocks. With acceleration and jerk limitations, these discontinuities will induce a zero feedrate and thus surface marks and an increase in machining time. The aim of this paper is to study a 5-axis corner smoothing method required to obtain a $C^2$ continuous tool path geometry. Several methods have been developed in 3-axis but the 5-axis corner smoothing is still a challenge. To smooth the tool tip position and the tool orientation, the 5-axis tool path is represented by two B-Spline curves. The proposed corner smoothing model allows to control the contour and orientation tolerances in the workpiece coordinate system. The main difficulty is to obtain a $C^2$ continuous connection between the initial tool path and the newly inserted smoothing portion. This connection is linked to the parametrization of the bottom and top B-Splines. This algorithm can be integrated to a feedrate interpolator to control a 5-axis milling machine equipped with an Open CNC.

**Keywords**: 5-axis corner smoothing; G1; CNC; feedrate; kinematical constraints.

# 1 Introduction

Major improvements in computer aided manufacturing, cutting process and high speed machine tools have been observed in the recent years but advances in the Computer Numerical Control (CNC) have been limited. Even if polynomial and B-Spline representations of the tool paths have been introduced in the industrial CNCs, most of the milling tool paths are defined by G1 blocks (linear interpolation). However, this description leads to generate tangential discontinuities at the connection between segments. Considering that the machine tool axes have acceleration and jerk limitations, the only solution to avoid a full stop at each transition is to smooth the geometry to obtain a $C^2$ continuous tool path. That means that the CNC has to modify the geometry given in the part program using CNC tolerance parameters.
Depending on the context, two main methods are available to smooth the geometry. If the program is composed of several short segments, it is possible to approximate the segments by interpolating all the points with a spline (See 'CompCad/CompCurv' in Siemens 2002). If the program is composed of long segments, it is important to follow precisely these segments and thus each transition has to be locally smoothed. This paper focuses on the latter problem which has been studied by several authors for 3-axis tool paths but which is still a challenge in 5-axis.

In the literature, many 3-axis solutions have been proposed to smooth the transition between two linear segments. The feedrate interpolation is closely linked to the geometry of the corner. It is possible to use a decoupled approach which will first design a $C^2$ continuous geometry and then find an admissible motion law along this fixed geometry. The other solution is to treat the problem globally with a combined approach which will compute both the geometry and the motion law all at once. The combined approach is, for example, used by Pessoles et al. (2012) where a $5^{th}$ order polynomial curve is computed to smooth a 3-axis corner. The time parametrization of this curve requires some hypotheses about the feedrate and acceleration which are not always verified.

For 3-axis corner smoothing, the decoupled approach is preferred in many articles. Different methods, which give similar results, are proposed for 3-axis corner smoothing. They all give a satisfying corner smoothing curve in terms of contour error and $C^2$ continuity. Yutkowitz and Chester (2005) registered a patent describing a 3-axis smooth cornering method based on two $4^{th}$ order polynomial curves. Erkorkmaz et al. (2006) use a $5^{th}$ order polynomial curve to smooth the corner. A cubic B-Spline with 8 control points is used by Pateloup et al. (2010). Ernesto and Farouki (2012) employed a Bézier conic and optimized the feedrate along the curve under acceleration bounds. Bi et al. (2011) used two cubic Bézier curves to smooth the corner. Once the geometry is smoothed the feedrate planning can be performed using, for example, the previously developed Velocity Profile Optimization (Beudaert et al. 2012). The reachable feedrate highly depends on the smoothness of the tool path geometry.

Considering 5-axis tool path, different techniques are used to smooth the tool orientation: quaternion (Shoemake 1985, Ho et al. 2003), spherical B-Spline (Fleisig and Spence 2001), 5xNURBS (position and orientation spline) (Langeron et al. 2004), local smoothing of joint movement (Beudaert et al. 2011). They have been applied to general tool path smoothing and not to corner smoothing which presents several particularities. First the contour and the orientation modification have to be controlled locally and precisely. Then, the corner smoothing curve has to be connected with the initial tool path. That is why a specific method has to be developed to smooth the discontinuities generated by 5-axis G1 blocks. Very little information is available about the specific problem of 5-axis corner smoothing. Although it is possible to know the parameters used by Siemens (2002) to smooth the corner in the continuous-path mode, the algorithms used by the industrial CNCs are unknown. To control an Open CNC, the whole corner smoothing process has to be mastered. To the best of our knowledge, no previously published worked has tackled the problem of 5-axis corner smoothing.

In this paper, a 5-axis corner smoothing method is proposed and analyzed. This method is based on two B-Spline curves which define the tool tip position and the tool orientation. In 5-axis, the challenge is to smooth both the position and the orientation of the tool. Indeed, a $C^2$ continuous variation of the geometry of the tool path is needed to respect the acceleration and jerk limitations of the linear and rotary axes of the machine. The effect of the parametrization of the curves is especially studied.

The rest of the paper is organized as follows. A 3-axis corner smoothing method is presented in Section 2. Then, in Section 3, different possible approaches to smooth 5-axis corners are evaluated and the solution proposed for 3-axis corners is extended to the 5-axis problem. Section 4 details the specific parametrization problems linked to the orientation smoothing. Finally, the paper is concluded in Section 5.

## 2 3-axis corner smoothing

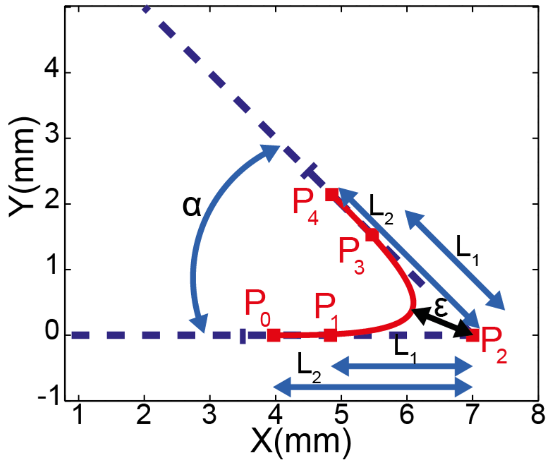
The aim of the corner smoothing algorithm is to obtain a $C^2$ continuous geometry along which the jerk limited motion will further be computed. The modified geometry has to satisfy the specified contour tolerance $\varepsilon$. The proposed model for 3-axis corner smoothing is simpler than the solutions proposed in the literature and is suitable for the extension to 5-axis corners.

To have a $C^2$ continuous transition, a cubic B-Spline with five control points is used. The mathematical formula is given in Eq.1 with $P_k$ the control points and $B_{k3}$ the basis functions. The nodal sequence used for the parametrization of the B-Spline is [0 0 0 0 ½ 1 1 1 1].

$$\vec{C}(u) = \sum_{i=0}^{4} B_{i3}(u)\,\vec{P_i} \qquad (1)$$

The transition model is illustrated in Fig. 1. The B-Spline curve is supposed to be symmetrical so $P_2$ is in the corner; $P_2P_1 = P_2P_3 = L_1$ and $P_2P_0 = P_2P_4 = L_2$. The junction between the segment and the curve is $C^2$ because the curvature at the beginning of the curve is given by the first three points which are aligned with the segment. In order to make each transition independent from the other, the curve should not laid on more than half of the segments so $L_2 < \min\left(\frac{L_{segment_1}}{2};\frac{L_{segment_2}}{2}\right)$.

**Figure 1** Transition model in 3-axis.




With this construction of the B-Spline, the maximum contour error is exactly in the middle of the curve at u=0.5. Taking Eq. 1 and the definition of the basis function $B_{k3}$, it is possible to obtain Eq. 2 which gives the position of the middle point of the curve as a function of the control points $P_1$, $P_2$ and $P_3$.

$$\vec{C}(u = 0.5) = \frac{1}{4}\,\overrightarrow{P_1} + \frac{1}{2}\,\overrightarrow{P_2} + \frac{1}{4}\,\overrightarrow{P_3} \tag{2}$$

Without loss of generality, the point $P_2$ can be the origin, $P_2$ = (0,0,0). Finally, the contour error is defined as follows, with the units vector $\vec{u}$ and $\vec{v}$ in the direction of $\overrightarrow{P_2P_1}$ and $\overrightarrow{P_2P_3}$, respectively.

$$\varepsilon = \left\| \frac{1}{4}\, L_1(\vec{u} + \vec{v}) \right\| \;\Rightarrow\; \varepsilon = \frac{1}{2}\, L_1 \cos\left(\frac{\alpha}{2}\right) \tag{3}$$

Typically, the contour tolerance ε is set to 0.01 mm. The advantage of this definition of the curve is that the contour error depends only on the length $L_1$ as it is given by Eq.3, with α the angle between the segments.

Eq.3 allows to determine the parameter $L_1$ so the last parameter $L_2$ has to be selected. The parameter $L_2$ allows us to control the level of continuity of the transition between the segment and the B-Spline. If $L_2$ is too small, the abrupt transition will cause a slowdown. Indeed, this parameter is closely linked with the curvature in the corner and thus with the allowable feedrate. Pateloup et al. 2010 propose for example to chose $L_2$ which minimizes the sum of the squares of the curvature.

## 3 5-axis corner smoothing

In 5-axis, the orientation of the tool has also to be smoothed in order to have a smooth movement of the drives after the inverse kinematical transformation. First, the different possibilities available for orientation smoothing are evaluated considering the advantages and the drawbacks for the 5-axis corner smoothing. Then, the proposed method is developed.

*3.1 Analysis of the different solutions available*

To smooth the orientation, it is possible to work in the workpiece coordinate system or in the machine coordinate system. Working in the machine coordinate system presents several drawbacks. First, the solution will be dedicated to a specific machine tool kinematic. Then, it is not possible to use only an angular tolerance for the rotary drives because the effect on the geometrical deviation in the workpiece coordinate system will depend on the position of the workpiece in the machine. That is why the workpiece coordinate system is preferred to realize the corner smoothing. To have a precise movement in the machine coordinate system, a 5-axis tool path have to be discretized accurately in the workpiece coordinate system in order to have a small effect of the linear interpolation of the rotary axes.

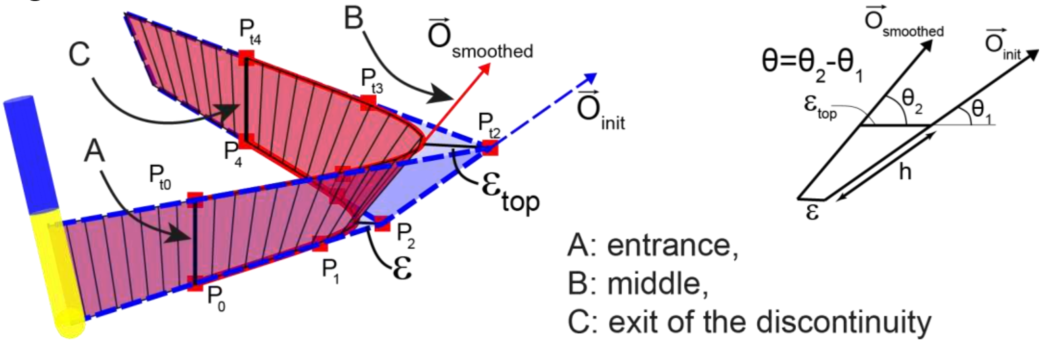
Even if the kinematical transformation is non-linear, it preserves the continuity of the tool path; whatever the machine tool kinematical structure. So if a $C^2$ continuous position and orientation is obtained in the workpiece coordinate system, this continuity level is kept after the inverse kinematical transformation (except for the singularities which are not considered here).

*3.2 Geometrical model for 5-axis corner smoothing*

The proposed solution to smooth a 5-axis corner uses the method presented for 3-axis tool paths. As shown in Fig. 2, the 5-axis tool path can be described by a bottom curve which gives the tool tip position and by a top curve which gives the orientation. The control points of the top curve ($P_{t1}$ .. $P_{t4}$) are computed using the same technique as for the bottom control points (see Section 2). The nodal sequence of the top curve is [0 0 0 0 ½ 1 1 1 1]. Eq. 4 gives the orientation vector $\vec{O}$ in the corner which depends on the values of $u_{top}$ and $u_{bottom}$ chosen to parameterize the corner.

$$\vec{O} = \overrightarrow{C_{top}}(u_{top}) - \overrightarrow{C_{bottom}}(u_{bottom}) \tag{4}$$

**Figure 2** Transition model in 5-axis.




The orientation tolerance is defined as shown in Fig. 2. This tolerance $\varepsilon_{top}$ corresponds to the geometrical deviation seen on the part in flank milling; the parameter h would correspond to the height of the part which is machined. A typical value for the orientation error parameter could be $\varepsilon_{top}$ = 0.05mm. This parameter is more explicit than an orientation tolerance in degree even if the angle θ can be used instead of $\varepsilon_{top}$ (Fig.2 right).

At first sight, this method seems to generate a C² continuous transition between the 5-axis G1 blocks. Indeed, the bottom and top curves are C² continuous but the important parameter is the continuous evolution of the tool orientation. Thus the main difficulty comes from the parametrization of the curves which is studied in Section 4.

## 4 Problems of the orientation smoothing technique

This section highlights the parametrization problem which occurs with this bottom and top curve corner smoothing technique. The link between the geometrical derivatives and the kinematical constraints is first presented. Then, different possible parametrizations are studied and their impact on the tool orientation is shown.

### *4.1 Link between the geometry and the feedrate, acceleration and jerk*

The link between the geometry of the tool path and the kinematical parameters is presented in Eq. 5-7. Using the formula for the derivative of the composition of two functions (Eq. 5), it is possible to express the velocity of the drives $\dot{q}$ as a function of the geometry $q_s$ multiplied by a function of the motion $\dot{s}$ (s is the path displacement so $\dot{s}$ is the feedrate and q contains the X,Y,Z,A,C axis movements). Therefore, the motion is decoupled of the geometry. The acceleration $\ddot{q}$ and jerk $\dddot{q}$ of the drives are obtained identically in equations (6) and (7). $q_s$, $q_{ss}$, $q_{sss}$ are the geometrical derivatives with respect to displacement s along the tool path. One can note that Eq. 5-7 are valid for linear and rotary drives.

$$\dot{\boldsymbol{q}}(s) = \frac{d\boldsymbol{q}(s)}{dt} = \frac{d\boldsymbol{q}(s)}{ds}\frac{ds}{dt} = \boldsymbol{q}_s(s)\,\dot{s} \tag{5}$$

$$\ddot{\boldsymbol{q}}(s) = \boldsymbol{q}_{ss}(s)\,\dot{s}^2 + \boldsymbol{q}_s(s)\,\ddot{s} \tag{6}$$

$$\dddot{\boldsymbol{q}}(s) = \boldsymbol{q}_{sss}(s)\,\dot{s}^3 + \boldsymbol{3q}_{ss}(s)\,\dot{s}\,\ddot{s} + \boldsymbol{q}_s(s)\,\dddot{s} \tag{7}$$

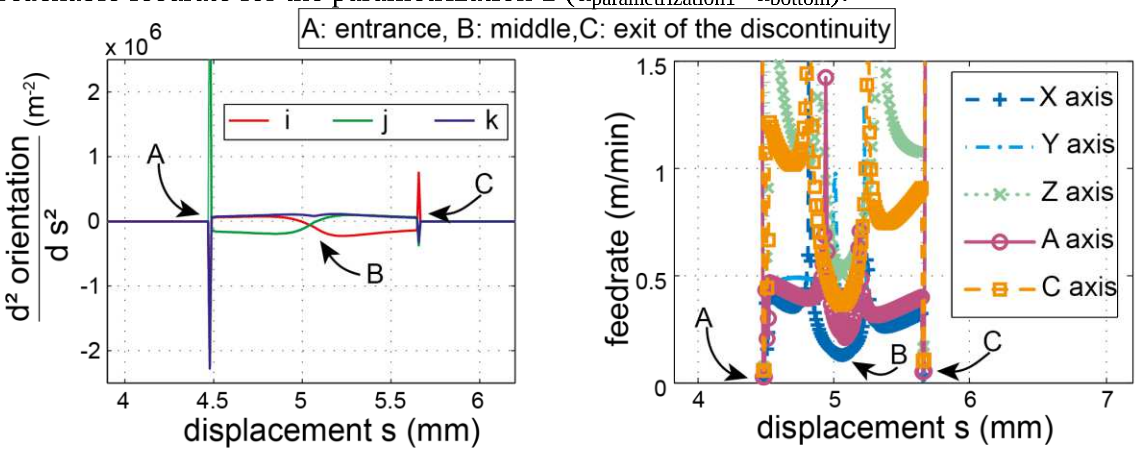
Eq. 7 shows that if $\boldsymbol{q}_{sss}$ (third derivative of the axis movement with respect to the displacement) is infinite, the feedrate has to be equal to zero to respect the jerk limitation. To have the highest feedrate in the corner, it is important to minimize the geometrical derivatives and thus to create a smooth transition in the corner.

The velocity, acceleration and jerk of the 5 drives are limited so $|\dot{\mathbf{q}}| \leq \mathbf{V}_{\max}^{\text{axis}}$, $|\ddot{\mathbf{q}}| \leq \mathbf{A}_{\max}^{\text{axis}}$, $|\dddot{\mathbf{q}}| \leq \mathbf{J}_{\max}^{\text{axis}}$. As it is explained in detail in Beudaert et al. (2011), it is possible to have a really good approximation of the maximum reachable feedrate using Eq. 8. This approximation is used to evaluate the quality of the smoothed geometry in the corner.

$$\dot{s} \leq \min\left(\frac{\mathbf{V}_{\max}^{\text{axis}}}{|\mathbf{q}_s|},\ \sqrt{\frac{\mathbf{A}_{\max}^{\text{axis}}}{|\mathbf{q}_{ss}|}},\ \sqrt[3]{\frac{\mathbf{J}_{\max}^{\text{axis}}}{|\mathbf{q}_{sss}|}}\right) \tag{8}$$

Fig. 3 shows that for a given tool path geometry, the derivatives of the orientation with respect to the path displacement are linked to the maximum reachable feedrate along the tool path. Indeed, on the right plot, the curves represent the limitation given by each axis of the machine tool using Eq. 8. It is clear that the huge peaks of the derivative of the orientation at the entrance (A) and exit (C) of the discontinuity (on the left plot) induce a really low feedrate limitation (on the right plot). This large variation of the second derivative will also be present in the machine coordinate system after the inverse kinematical transformation. Moreover at the connection between the corner smoothing curve and the segments the required feedrate is lower than in the middle of the transition which means that the connection is not satisfying. So a sharp variation of orientation at the connection between the curve and the segments will requires a really low feedrate to respect the acceleration and jerk constraints because the geometrical derivatives of the tool path $q_s$, $q_{ss}$, $q_{sss}$ will be large (see Eq. 8).

**Figure 3** Link between the second derivative of the orientation and the maximum reachable feedrate for the parametrization 1 ($u_{parametrization1}=u_{bottom}$).




*4.2 Parametrization of the curves in the corner*

In 3-axis, the parametrization does not modify the geometry of the tool path whereas in 5-axis the parametrization of the curves has an important effect on the orientation of the tool in the corner and thus on the maximum reachable feedrate. Of course, the number of discretized points on the top and bottom curves has to be equal. The bottom curve is discretized in order to obtain a constant arc length between two points; this choice will be kept for the rest of the paper. So a special attention should be paid to the parametrization of the top curve which gives the final orientation of the tool.

There are many different possible parametrizations for the top curve such as:

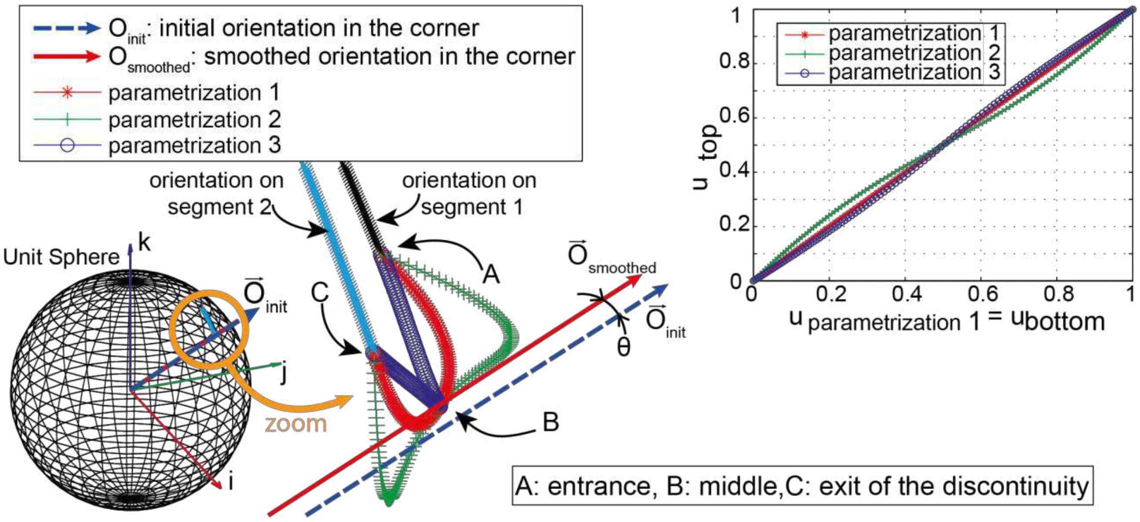
- parametrization 1: same proportion of the parameter u as for the bottom curve $u_{parametrization1}=u_{bottom}$,
- parametrization 2: linearly spaced in u ($\Delta u$=cst),
- parametrization 3: linearly spaced in arc length s ($\Delta s_{top}$=cst),
- …

The parametrizations of the top curve as a function of the parametrization of the bottom curve are presented on the right hand side of Fig. 3. The first parametrization gives a straight line in this plot because $u_{discretization\ 1} = u_{bottom}$.

*4.3 Effect of the parametrization on the tool orientation*

These different parametrizations induce a small variation of the position of the discretized points on the top curve but a large variation of the tool orientation as it is shown in Fig. 4. The discretized orientations are plotted on a unit sphere. A zoomed view of the connection between the corner smoothing curve and the segments allows to see that the different parametrizations induce a large variation of orientation compared with the orientation tolerance θ.

**Figure 4** Effect of the parametrization of the top curve on the orientation.

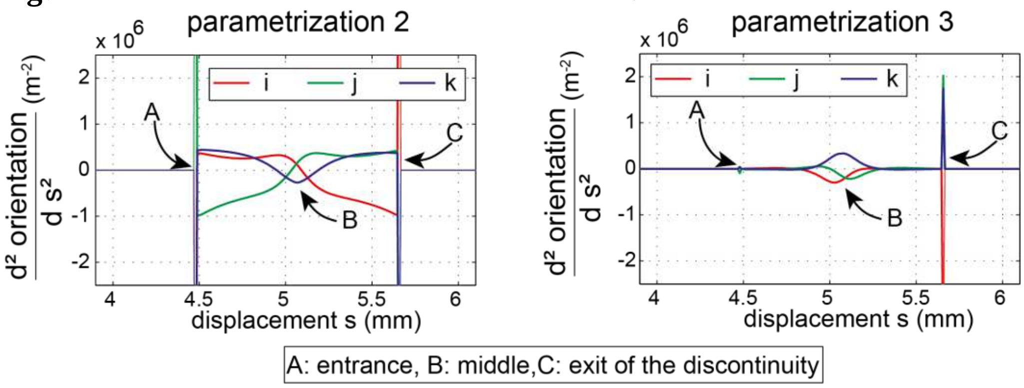



*4.4 Problem at the connection between the segment and the smoothing curve*

In the zoomed view of the orientation in Fig. 4, it is clear that the parametrization of the top curve plays a key role to determine the orientation of the tool in the corner. Moreover, the smoothness of the connection between the corner smoothing curve and the segments highly depends on that parametrization.

As the link between the second derivative of the orientation and the maximum reachable feedrate has been shown in Fig. 3, the rest of the analysis will focus on the orientation derivatives. The Fig. 5 presents the orientation derivatives for the parametrizations 2 and 3. At the connection with the segment 1 (A), the parametrization 3 gives a smoother variation of the orientation than the other parametrizations. At the connection with the segment 2 (C), it is the parametrization 1 which gives a smoother variation of orientation. This analysis shows that it might be possible to find an appropriate parametrization of the top curve to obtain a smooth evolution of the orientation at the connections.

**Figure 5** Evolution of the orientation derivatives.




*4.5 Conclusion about the orientation smoothing technique*

As it has been presented, a special attention should be paid to the connection between the segments and the smoothing curve. Three different parametrizations have been studied to show that they have a huge impact on the orientation in the corner. It has been shown that the connection between the segments and the curve is not necessary smooth even with the construction of the bottom and top curves presented in Fig. 2.
The proposed method which uses a bottom and a top curve to describe the geometry in the corner allows to control precisely the contour and orientation tolerances. However it does not solve the continuity problem between the smoothing curve and the segments.
Indeed, the simple parametrizations tested here do not allow to obtain a C² continuous variation of the tool orientation at the connections. A solution to this problem might be to use another parametrization based on a new parametrization spline to discretize the top curve.

## 5 Conclusion

The linear interpolation (G1) is the main and by far the most used solution to program a tool path. This format creates discontinuities in tangency at the end of each block of the program. Considering the acceleration and jerk limitation, the only solution to avoid a zero feedrate is to smooth the corners in order to obtain a C² continuous geometry. After a presentation of the problem in 3-axis, a 5-axis corner smoothing method is proposed in this paper. A solution based on two B-Spline curves is presented and the problem of parametrization is addressed. The main difficulty for the orientation smoothing is highlighted. Further developments will combine the 5-axis corner smoothing with the VPOp feedrate interpolator previously developed (Beudaert et al. 2012) to control a 5-axis Open CNC.

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