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Test of the Universality of the Three-Body Efimov Parameter at Narrow Feshbach Resonances

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We measure the critical scattering length for the appearance of the first three-body bound state, or Efimov three-body parameter, at seven different Feshbach resonances in ultracold 39K atoms. We study both intermediate and narrow resonances, where the three-body spectrum is expected to be determined by the nonuniversal coupling of two scattering channels. Instead, our observed ratio of the three-body parameter with the van der Waals radius is approximately the same universal ratio as for broader resonances. This unexpected observation suggests the presence of a new regime for three-body scattering at narrow resonances.

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The Efimov effect [1] was first described in the context of nuclear physics but is now explored also in atomic, molecular, and condensed-matter systems [2–5]. Recent experiments on ultracold atoms with Feshbach resonances [6–18] have opened up a new path to study the universal spectrum of three-body Efimov states. The resonant interaction is expected to give rise to a three-body potential scaling as 1/R², where R is the hyperradius that parameterizes the moment of inertia of the system. This leads to an infinite series of trimer states with an universal geometrical scaling for the binding energies. For a finite, negative two-body scattering length a, the three-body potential has a long-range cutoff at R ≃ |a|, and only a finite number of bound states exist. The critical scattering length a for the appearance of the first Efimov state at the three-body threshold, often called the three-body parameter, was expected to be the only parameter to be influenced by nonuniversal physics, i.e., by the microscopic details of two- or even three-body forces [1,3]. While clear evidence of the universal scaling of the Efimov spectrum is still missing, recent experiments on identical bosons suggested that also a might be universal [19]. This surprising result has been interpreted in a recent series of theoretical studies [20–22]. The underlying idea is that the sharp drop in the two-body interaction potential at a distance of the order of the van der Waals radius RvdW results in an effective barrier in the three-body potential at a comparable distance [22]. This prevents the three particles from coming sufficiently close to explore nonuniversal features of the interactions at short distances and leads to a three-body parameter set by RvdW alone, a ≃ −9.5RvdW [19,21,22].

However, this scenario is realized only for the broad Feshbach resonances studied so far in most experiments, which can be described in terms of a single scattering channel, the so-called open channel. For narrow resonances, one must instead take into account the coupling of the open and a second closed channel [23]. It has been shown that in this case a new length scale that depends on the details of the specific Feshbach resonance, the so-called intrinsic length R*, must be introduced to parameterize the two-body scattering. The three-body potentials are also modified, with an expected deviation from the Efimovian dependence into 1/(R²R) for distances R < R* [24]. This tends to reduce the depth of the three-body potential and leads to the nonuniversal result a = −12.90R* [24,25], which is much larger than that obtained for broad resonances. This prediction is valid only close to resonance, where |a| ≫ R*. It is still unclear how a scales in the intermediate regime of |a| ≈ R* or generally for resonances of intermediate widths. Various general models have been proposed [26–31], but they either are not fully predictive or give contradicting results.

In this Letter, we address this problem by performing an experimental study of three-body collisions in ultracold bosonic 39K atoms, where we determine the three-body parameter a at several Feshbach resonances of intermediate or narrow width. In particular, our measurements probe for the first time the regime of very small resonance strengths, sres = 0.956RvdW/R* ≃ 0.1, where R* might be expected to be the relevant length scale that determines a. Surprisingly, we find values of a that are around the same −9.5RvdW measured for broad resonances, suggesting the existence of a novel intermediate regime of three-body scattering.

The investigation of closed-channel-dominated Feshbach resonances is particularly favored in 39K, which has several resonances with moderate magnetic width Δ and relatively small background scattering length −a bg ≃ (20–30)a0 [32]. These parameters, together with the difference of the magnetic moments of the closed and open channels, δμ,
determine the intrinsic length \( R^* = \hbar^2/(m a_0 \Delta \delta a) \) [23], which can be related also to the on-resonance effective range (see the Supplemental Material [33]). In particular, we investigated seven different resonances with \( s_{res} \) in the range 0.1–2.8 in the three magnetic sublevels of the hyperfine ground state \( F = 1 \) [32].

A detailed description of the experimental setup is given elsewhere [34]. The three-body parameter was determined by finding the maximum of the three-body loss coefficient \( K_3 \) in the region of negative \( a \) at each Feshbach resonance, as in previous experiments [6–18]. In the presence of three-body losses, both the atom number \( N \) and temperature \( T \) evolve according to \( dN/dt = -K_3(n^2)N \) and \( dT/dt = (K_3/3)(n^2)T \), where \( \langle n^2 \rangle = (1/N) \int n(x)^2 dx \) is the mean square density [35]. The temperature increase is due to the preferential removal of atoms in the high-density region around the trap center. The typical starting condition was a noncondensed sample with 3–80 \times 10^4 \) atoms in a temperature range of 20–400 nK, depending on the spin channel and Feshbach resonance (see the Supplemental Material [33]). The atoms were held in a purely optical trap (or with an additional magnetic confinement, depending on the specific resonance) at sufficiently low density to have a negligible mean-field interaction energy. Care was taken to have a trap depth sufficiently large to avoid evaporation associated to the heating. The samples were initially prepared at small negative \( a \) in proximity of the Feshbach resonances; the measurements started 10 ms after \( a \) was ramped to the final value in about 2 ms.

Figure 1 shows a typical evolution of \( N \) and \( T \), as measured by absorption imaging after a free expansion. They were simultaneously fitted with

\[
N(t) = N_0 \left(1 + \frac{3 \beta^2 N_0^2}{27 T_0} K_3 t \right)^{1/3},
\]

(1)

\[
T(t) = T_0 \left(1 + \frac{3 \beta^2 N_0^2}{27 T_0} K_3 t \right)^{1/9}.
\]

(2)

Here \( N_0 \) and \( T_0 \) are the initial atom number and temperature, respectively, and \( \beta = (m \bar{a}^2 / 2 \pi k_B)^{3/2} \), with \( \bar{a} \) the mean trap frequency. In such a fit, one-body losses were neglected, since they occur on a much longer time scale.

Crucial ingredients for a reliable measurement of the \( K_3 \) dependence on the scattering length were an accurate calibration of the magnetic field \( B \) and the use of a high-quality coupled-channel (CC) model for \( a(B) \), based on a large number of experimental observations for the positions and widths of the Feshbach resonances [32,33]. The centers and widths of the Feshbach resonances were redetermined in the present work, finding a good agreement with the theoretical ones. An additional confirmation of the CC model was derived from a direct measurement of the dimer binding energy at the two narrowest resonances by radio-frequency spectroscopy.

We observed for all Feshbach resonances a clear peak in \( K_3 \) in the region of \( |a| = (600–1000)a_0 \), as shown in Figs. 2 and 3. We compared the observations to the known

![FIG. 1 (color online). Example of the time evolution of the atom number (circles) and temperature (triangles), fitted to Eq. (1) (solid line) and Eq. (2) (dashed line) to determine the three-body loss coefficient \( K_3 \).](image1)

![FIG. 2 (color online). Three-body loss rate measured in the proximity of five Feshbach resonances of intermediate strength (see Table I for the assignment of the spin state). The experimental data (squares) are fitted to Eq. (3) (solid line).](image2)
relation for identical bosons at zero collision energy and in the zero-range approximation, for $a < 0$:

$$ K_3(a) = \frac{3 ha^4}{m} \frac{\sinh(2 \eta_-)}{\sin^2(s_0 \ln(a/a_-)) + \sin^2 \eta_-}. \quad (3) $$

Here $s_0 \approx 1.00624$ is an universal constant, and $\eta_-$ is the decay parameter which sets the width of the Efimov resonance and incorporates short-range inelastic transitions to deeply bound molecular states [3]. At the finite temperature of the experiment, there is a limitation in the maximum observable $K_3$ set by unitarity at $K_3^{\text{max}} = 36\sqrt{3} \pi^2 \hbar^5/(k_B T)^2 m^3$ [36,37]. Therefore, we fitted the data with an effective rate of the form $[1/K_3(a) + 1/K_3^{\text{max}}]^{-1}$ [7,13,38], with $a_-, \eta_-$, and $K_3^{\text{max}}$ as fitting parameters. The experimental $K_3(a)$ for the five broadest resonances, shown in Fig. 2, is in good agreement with Eq. (3), besides a multiplicative factor of the order of 3 that can be justified with the experimental uncertainty in the determination of the density (see the Supplemental Material [33]).

Also, the two narrowest Feshbach resonances feature a maximum in $K_3$ around $-1000a_0$, as shown in Fig. 3. There is, however, a slower background variation of $K_3$ with $a$, not reproduced by Eq. (3). It was shown that for narrow resonances one should expect a slower evolution in the regime $|a| < R^*$, with $K_3 \propto |a|^{7/2}$ [24], but also this does not seem to reproduce the data at small $|a|$. In the absence of a better model and in analogy with the broad resonances, we determined the position of the measured maximum in $K_3$ with a Gaussian fit, as shown in Fig. 3, and we interpreted it as the $a_-$ parameter. As uncertainty, we conservatively took the $1/e^2$ half-width of the Gaussian.

For all the resonances in excited spin states, there is in principle also a contribution of two-body losses, which have a slower dependence on $a$ [23]. While it was not possible to distinguish in a reliable way two- from three-body losses in the experiment, we have verified that the calculated two-body losses from the CC models are typically negligible, besides some large-$a$ regions close to the two narrow resonances (see the Supplemental Material [33]).

A summary of our analysis is reported in Table 1. For the calculation of $a(B)$, we used the experimentally determined Feshbach resonance centers $B_0^{\text{exp}}$ and the resonance widths and the background scattering lengths from the CC model. The uncertainties in $B_0^{\text{exp}}$ include those in the calibration of $B$ and in the determination of $B_0$ from the loss resonances. Particular care was put in the determination of $B_0$ for the two narrowest resonances, where we found a rather good agreement between independent measurements of the atom losses and of the binding energy (see the Supplemental Material [33]). The uncertainties in $a_-$ include the statistical uncertainties from the fit of the $K_3$ data and from the determination of $a(B)$. For the two narrowest resonances, the dominant source of uncertainty comes from the determination of $B_0$. These two resonances are coupled, and $a(B)$ can be represented only over an

### Table 1. Theoretical and experimental parameters at Feshbach resonances in the $m_F$ spin channels: measured resonance center $B_0^{\text{exp}}$; intrinsic length $R^*$ and strength $s_{\text{res}}$ of the Feshbach resonances from the CC model; measured three-body parameter $a_-$ and decay parameter $\eta_-$; initial temperature $T$. For $^{39}$K, $R_{\text{eff}} = 64.49a_0$. Figures in parentheses represent one standard deviation.

<table>
<thead>
<tr>
<th>$m_F$</th>
<th>$B_0^{\text{exp}}$ (G)</th>
<th>$R^*$ ($a_0$)</th>
<th>$s_{\text{res}}$</th>
<th>$-a_-$ ($a_0$)</th>
<th>$\eta_-$</th>
<th>$T$ (nK)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>471.0 (4)</td>
<td>22</td>
<td>2.8</td>
<td>640 (100)</td>
<td>0.065 (11)</td>
<td>50 (5)</td>
</tr>
<tr>
<td>+1</td>
<td>402.6 (2)</td>
<td>22</td>
<td>2.8</td>
<td>690 (40)</td>
<td>0.145 (12)</td>
<td>90 (6)</td>
</tr>
<tr>
<td>−1</td>
<td>33.64 (15)</td>
<td>23</td>
<td>2.6</td>
<td>830 (140)</td>
<td>0.204 (10)</td>
<td>120 (10)</td>
</tr>
<tr>
<td>−1</td>
<td>560.72 (20)</td>
<td>24</td>
<td>2.5</td>
<td>640 (90)</td>
<td>0.22 (2)</td>
<td>20 (7)</td>
</tr>
<tr>
<td>−1</td>
<td>162.35 (18)</td>
<td>59</td>
<td>1.1</td>
<td>730 (120)</td>
<td>0.26 (5)</td>
<td>40 (5)</td>
</tr>
<tr>
<td>0</td>
<td>65.67 (5)</td>
<td>456</td>
<td>0.14</td>
<td>950 (250)</td>
<td>0.065 (11)</td>
<td>330 (30)</td>
</tr>
<tr>
<td>0</td>
<td>58.92 (3)</td>
<td>556</td>
<td>0.11</td>
<td>950 (150)</td>
<td>0.065 (11)</td>
<td>400 (80)</td>
</tr>
</tbody>
</table>
extended range of magnetic fields in terms of a two-pole expression containing two widths and a single $d_{bg}$ [32]. The reported values of $R^*$ are determined on resonance, from a comparison of the predictions of our CC calculation to a generalization of the quantum-defect model of Ref. [39] to the case of coupled resonances. The coupling causes a dependence of $R^*$ on $B$, which is, however, limited to about 20% in the experimental range (see the Supplemental Material [33]).

We observe a whole range of values of $\eta_-$ for the different Efimov resonances; this is probably a consequence of the different measurement temperatures but possibly also of the nonuniversal nature of $\eta_-$ [3,40].

A comparison of the results in Table I leads to the striking conclusion that the three-body parameter $a_-$ stays around values of the order of $-10R_{vdW}$ for all the Feshbach resonances explored in $^{39}$K, including the ones with $R^*$ as large as $\sim 600a_0$, hence much larger than $R_{vdW}$. We note that in the earlier measurement at the $m_F = 1$ resonance, we found two $K_3$ resonances at $|a| \approx 700a_0$ and $|a| \approx 1500a_0$, which we tentatively identified as a four- and a three-body resonance, respectively [7]. We now think that the previous resonance around 1500$a_0$, which we no longer observe, was an artifact of the analysis of the limited time-dependent data, and we reassign the one around 700$a_0$ as the three-body resonance (see the Supplemental Material [33]).

Figure 4 shows the measured $|a_-|/R_{vdW}$ as a function of $s_{res}$. We observe just a moderate deviation of our data from the mean value $9.73(3)$ measured for open-channel-dominated resonances [17,19,21,41] and also for other intermediate resonances [9–11,18,19,41]. This observation is far from the already mentioned prediction for narrow resonances [24,25], which indicates that the Efimov resonances should appear at scattering lengths that are multiples of $a_- = -12.9R^*$ by a factor $\exp(\pi/s_0) \approx 22.7$. One might note that this result is expected to be valid only in the limit of a scattering length larger than any other length scale, $|a| \gg R^* \gg |a_{bg}|$, where the three-body potential at large hyperradii $R \gg R^*$ has an Efimovian character [24]. The present experiment does not access this extreme limit but is in an intermediate regime also for the two narrowest resonances, which show indeed $R^* \approx |a_-|$.

Other models for the three-body physics at Feshbach resonances of intermediate strength have been proposed [26–31]. The specific problem of connecting the results for the three-body parameter in the open-channel-dominated regime, where $a_-$ is determined by $R_{vdW}$, and the closed-channel limit, where it is $R^*$ which sets the scale for $a_-$, has been addressed recently [26,31], finding, however, considerably different results. In particular, the model of Ref. [26] predicts that a crossover between the two regimes of broad and narrow resonances would take place around $s_{res} \approx 1$, as shown in Fig. 4. Additionally, the regime of $a_- = -12.9R^*$ should be reached only for excited Efimov states, while the first one has a slightly smaller $a_- = -10.3R^*$. Although an increase of $|a_-|$ with decreasing $s_{res}$ might be present in the experimental data, there is a clear disagreement with such predictions. Experiments on $^7$Li and $^{133}$Cs have also measured similar values for $a_-$. at three intermediate resonances with $s_{res} = 0.5–1$ [10,11,18,19], indicating that this behavior might not be peculiar of $^{39}$K. Also, a system without Feshbach resonances like metastable $^4$He might be consistent with these results [42].

We note that for the two narrowest resonances $|a_-|$ is only a factor of 2 larger than $R^*$. This observation seems to indicate that the three-body potential can support a bound state that resides only in the region with hyperradius $R \approx 2R^*$. This is a regime that was not accessible in previous one-channel models, and a multichannel approach [43], possibly comprising the coupled-resonance aspect, will presumably be necessary to model the observations.

In conclusion, our study showed an apparent universal behavior of the three-body parameter on several different Feshbach resonances of the same atomic species, down to a resonance strength $s_{res} \approx 0.1$. This gives important information on the three-body physics in this narrow-resonance regime, where an interplay of the open and the closed molecular channels is expected.

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