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Abstract

The interaction of guided waves with non-axisymmetric damages in cylinders is studied. A three dimensional hybrid method involving the classical Finite Element Method (FEM) and the Semi-Analytical Finite Element (SAFE) technique is developed. The damage and its near field are analysed with the standard FEM. Then, eigenmode expansions of the solutions at both inlet and outlet cross-sections of the FEM region are performed. The far field is investigated by using eigenmode expansions based on SAFE method, which is used to determine the elastic guided modes for both inlet and outlet cross-sections of the volume. The amplitudes of the incident modes are enforced. The amplitudes of the scattered modes are determined by solving the global system of the hybrid FEM-SAFE model. The average power flow is directly derived from SAFE matrices. A comparison with results found in the literature for the free-end cylinder is performed with success. Then, some results of the fundamental longitudinal Pochhammer-Chree mode interaction with non-axisymmetric cracks are obtained and discussed. The power balance is shown to be satisfied with a good accuracy.

Résumé

L'interaction des ondes guidées dans un cylindre avec des endommagements non-axisymétriques est étudiée. Une méthode hybride en trois dimensions est développée combinant la méthode classique des éléments finis (FEM) et la méthode semi-analytique éléments finis (SAFE). L'endommagement et son champ proche sont analysés avec la FEM. Ensuite, une décomposition modale des solutions est effectuée en amont et en aval de la région FEM. La décomposition modale est basée sur la méthode SAFE, qui permet de déterminer les modes guidées élastiques des deux sections du volume. Les amplitudes des modes incidents sont imposées. Les amplitudes des modes diffractés sont déterminées à partir de la résolution du système global de la méthode hybride FEM-SAFE. Enfin, le flux de puissance moyen est déterminé directement à partir des matrices SAFE. Une comparaison avec des résultats trouvés dans la littérature pour un bord libre d'un cylindre est effectuée avec succès. Ensuite, quelques résultats de l'interaction du mode de compression L(0,1) avec des fissures non-axisymétriques sont obtenus et discutés. Le bilan de puissance est satisfait avec une bonne précision.

Keywords

Guided waves, Cylinders, FEM-SAFE, Ultrasonics, Hybrid method.

1. Introduction

The motivation of this work is highlighted by the non-destructive testing need of cables, which are widely used in civil engineering. Environmental and operational conditions are likely to cause defects such as corrosions and cracks. Guided waves might be an appropriate technique since they can carry-out energy over long distances. Cables are generally constituted of a central cylinder surrounded by six helical wires. Given the complexity of such a geometry, only a cylindrical structure is studied in this work [1]. The interaction of elastic guided waves with damages is complex since they are multi-modal and dispersive. Models are often required for a better understanding of these interaction phenomena.

A wide range of work has been reported on the interaction of guided waves with defects in plates [2], cylinders [3] and tubes [4]. However, researchers generally use two dimensional models based on a given symmetry to avoid a heavy and complex three dimensional analysis. Hence, these methods can not deal with non-axisymmetric damages. The special case of the free-end cylinder has been studied first by Gregory and Gladwell [5] then by Rattanawangcharoen et al. [6] and Taweel et al. [7].

The aim of the paper focusses on the interaction of guided waves with non-axisymmetric cracks in a cylinder, which require a fully three dimensional approach. In section 2, a hybrid method involving the classical finite element method (FEM) and the semi-analytical finite element (SAFE) technique is developed. The average power flow is directly derived from SAFE matrices. Hence, the reflection and transmission power flows of guided modes can be readily determined. Section 3 deals with the interaction of the first fundamental compressional Pochhammer-Chree mode with non-axisymmetric cracks. Results for cracks with different opening angles are obtained and discussed. A comparison with results found in the literature for the free-end cylinder is performed with success.

2. Analytical development of the hybrid method

2.1. General description of the problem

In this section, a three-dimensional hybrid FEM-SAFE method is proposed. Hybrid methods have successfully been applied to two-dimensional problems (see Ref. [8] for instance). For conciseness, this method is only presented for the special case of a free-end cylinder. It can readily be generalized to an infinite waveguide by adding a right section. Figure 1 depicts a damage located in an arbitrary volume V at the free-end of an arbitrary waveguide section. The arbitrary section have a uniform geometry along the propagation direction. An incident wave is launched in the positive direction (in). The reflected waves (re), from both the damage and the free-end, travel in the negative direction. The arbitrary section is situated in the left while the damage and the free-end are situated in the interior region I of the volume. The two regions are connected each other with the left surface S_L where L designates left. The whole volume V is bounded by the traction free surface S_T and the left section S_L .

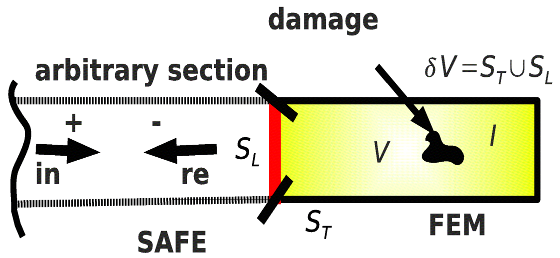


Figure 1. *Representation of a damage in an arbitrary volume at the free-end of an arbitrary waveguide section.*

2.2. The Global system of the hybrid method FEM-SAFE

The volume V is studied with the standard FEM. It can be shown that the variational formulation for the FEM can be expressed as:

$$\langle \delta U_L \ \delta U_I \rangle \begin{bmatrix} D_{LL} & D_{LI} \\ D_{IL} & D_{II} \end{bmatrix} \begin{Bmatrix} U_L \\ U_I \end{Bmatrix} = \langle \delta U_L \ \delta U_I \rangle \begin{Bmatrix} F_L \\ 0 \end{Bmatrix} \quad , \tag{1}$$

where $\{U_L\}$ and $\{U_I\}$ are the left and interior displacements of the degrees of freedom (dofs), $[D_{LL}]$, $[D_{LI}]$, $[D_{IL}]$ and $[D_{II}]$ are the selected dofs of the global dynamic stiffness matrix $[D]=[K]-\omega^2[M]$ (ω is the angular frequency).

The far field is investigated by using eigenmode expansions based on the SAFE method. The SAFE method is first used in order to determine the elastic guided modes for the inlet cross-section of the FEM region. This technique reduces three dimensional waveguide analyses to two dimensions by using a spatial Fourier transform along the propagation direction. Therefore, only the cross section of the waveguide is analysed, which allows a fast and accurate computation of eigenmodes (wavenumbers and modeshapes). These modes correspond to either ingoing (+) or outgoing (-) waves. The variational formulation of the left section can be written as (see Ref. [9] for instance):

$$\langle \delta U_L \rangle \left([K_1] - \omega^2 [M_S] - j k \left([K_2] - [K_2]^{t*} \right) + k^2 [K_3] \right) \{U_L\} = 0 \quad , \tag{2}$$

where the superscript t designates the transpose and $(*)$ is the complex conjugate.

The next step consists in performing an eigenmode expansions of the FEM solutions at the inlet cross-section. These solutions can then be naturally decomposed as incident and reflected normal modes. The application of the eigenmode expansions on the displacements and forces leads to:

$$\{U_L\} = [B_L^+]\{\alpha_L^+\} + [B_L^-]\{\alpha_L^-\} , \ \{\delta U_L\} = [B_L^-]\{\delta \alpha_L^-\} , \ \{F_L\} = [T_L^+]\{\alpha_L^+\} + [T_L^-]\{\alpha_L^-\} \quad , \tag{3}$$

with $[B_L^{\pm}] = \left[\{U_{L1}^{\pm}\} \{U_{L2}^{\pm}\} \ldots \{U_{LN_L^{\pm}}^{\pm}\} \right]$, $[T_L^{\pm}] = \left[\{F_{L1}^{\pm}\} \{F_{L2}^{\pm}\} \ldots \{F_{LN_L^{\pm}}^{\pm}\} \right]$ and $N_L^{\pm}$ is the maximum number of modes used in the computation. $\{\alpha_L^{\pm}\}$ is the vector of the ingoing (+) and the outgoing (-) coefficients of the mode shapes displacements $\{U_L^{\pm}\}$, respectively. Based on Ref. [10], it can be shown that $\{F_L\}$ for each mode can be computed directly from the SAFE matrices:

$$\{F_{Ln}^{\pm}\} = -\left([K_2] + j k_n^{\pm} [K_3] \right) \{U_{Ln}^{\pm}\} \quad , \tag{4}$$

where n is the mode number. The substitution of the above quantities (3) in the equation (1) gives the global system of the hybrid method:

$$\begin{bmatrix} B_L^{-t} & 0 \\ 0 & I_d \end{bmatrix} \left(\begin{bmatrix} D_{LL} & D_{LI} \\ D_{IL} & D_{II} \end{bmatrix} \begin{bmatrix} B_L^- & 0 \\ 0 & I_d \end{bmatrix} - \begin{bmatrix} T_L^- & 0 \\ 0 & 0 \end{bmatrix} \right) \begin{Bmatrix} \alpha_L^- \\ U_I \end{Bmatrix} = - \begin{bmatrix} B_L^{-t} & 0 \\ 0 & I_d \end{bmatrix} \left(\begin{bmatrix} D_{LL} & D_{LI} \\ D_{IL} & D_{II} \end{bmatrix} \begin{bmatrix} B_L^+ \\ 0 \end{bmatrix} - \begin{bmatrix} T_L^+ \\ 0 \end{bmatrix} \right) \{\alpha_L^+\} \quad , \tag{5}$$

where I_d is the identity matrix. The enforced coefficients vector $\{\alpha_L^+\}$ are known while $\{\alpha_L^-\}$ and $\{U_I\}$ are variables to be computed by solving the global system (5).

2.3.The power balance

The power reflection and transmission coefficients are computed by dividing the reflected and the transmitted power by the incident power, respectively ($R_{nm} = P_{n(re)} / P_{m(inc)}$ and $T_{nm} = P_{n(tr)} / P_{m(inc)}$ where: m is the incident mode and n is the reflected or the transmitted mode). The power incident, reflected and transmitted coefficients are determined by using the average power flow [4]. This average power flow can be derived directly from SAFE matrices [10] as, for example, for the m[th] incident single eigenmode:

$$P_{m(inc)} = \frac{\omega}{2} \Im \left(\alpha_{Lm}^{+*} \{U_{Lm}^+\}^{t*} \{F_{Lm}^+\} \alpha_{Lm}^+ \right) \quad , \tag{4}$$

where $\Im$ designates the imaginary part.

The power balance is satisfied when the sum of the power reflected and transmitted coefficients equals unity.

3. Numerical results

3.1. General description

A set of computations is carried out to analyse non-axisymmetric cracks and the free-end of an elastic cylinder. Cracks with different opening angles are studied. The opening angle θ takes values: $n\pi/2$, where $n=1,2,3$ and 4. For the special case of $n=4$, the damage is a free-end cylinder. For conciseness, the range of dimensionless frequency ($\Omega=\omega r/c_L$, r is the cylinder radius and c_L is the longitudinal celerity) is restricted to vary between 1.8 and 2.4. This range is chosen to enable comparisons with results found in the literature (see Section 3.2). The launched wave is the compressional L(0,1) Pochhammer-Chree mode. This mode is applied at the left section of the cylinder with a unit amplitude.

3.2. Non-axisymmetric cracks' analysis

Figures 2a–d illustrate the power reflection coefficients of the reflected compressional modes L(0,1), L(0,2) and L(0,3) with respect to dimensionless frequency. It is shown that the power reflection coefficients are sensitive to the crack opening angle. For all computations, the sum of the power reflected and transmitted coefficients at each frequency equals unity with an error less than 0.1%.

Figure 2d depicts the power reflection coefficients when the L(0,1) mode encounters a free-end cylinder. As expected and due to the axisymmetric geometry, no mode conversions to the flexural modes are observed. These results are compared to those of Gregory and Gladwell [4], which are represented with black points. A very good agreement is found, which validates the hybrid FEM-SAFE method.

Figures 3a and b show the power transmission coefficients of the compressional modes when the L(0,1) interacts with a crack of opening angle $\theta=n\pi/2$ ($n=1,2$) respectively. Unlike power reflection coefficients, the power transmission coefficients of the converted modes (L(0,2) and L(0,3)) are insignificant (less than 0.1, see Fig. 3). Figures 4a and b depict the power reflection and transmission coefficients when the L(0,1) mode encounters a crack with $\theta=\pi$. It is shown that the power reflection and transmission coefficients of the converted flexural modes (F(1,1), F(1,2), F(1,3), F(2,2) and F(3,2)) have the same order of magnitude.

4. Conclusions

A three-dimensional hybrid method combining the FEM and the SAFE techniques is developed. The interaction of the fundamental compressional guided wave with non-axisymmetric cracks and a free-end cylinder is performed. The power balance is satisfied and a comparison with other results is carried out with success. For the considered frequency range, the interaction of the L(0,1) mode with the free-end cylinder and three cracks show significant mode conversions to compressional modes (L(0,2) and L(0,3)). Relatively low mode conversions to flexural modes are observed (F(1,1), F(1,2), F(1,3), F(2,2) and F(3,2)). This work shows that the proposed three-dimensional hybrid FEM-SAFE method is a powerful predicting tool for analysing arbitrary damaged waveguides. This method is relatively easy to implement in an existing FE code and could help optimising inspection strategies for non-destructive evaluation. Experimental measurements are in progress and will be presented in future works.

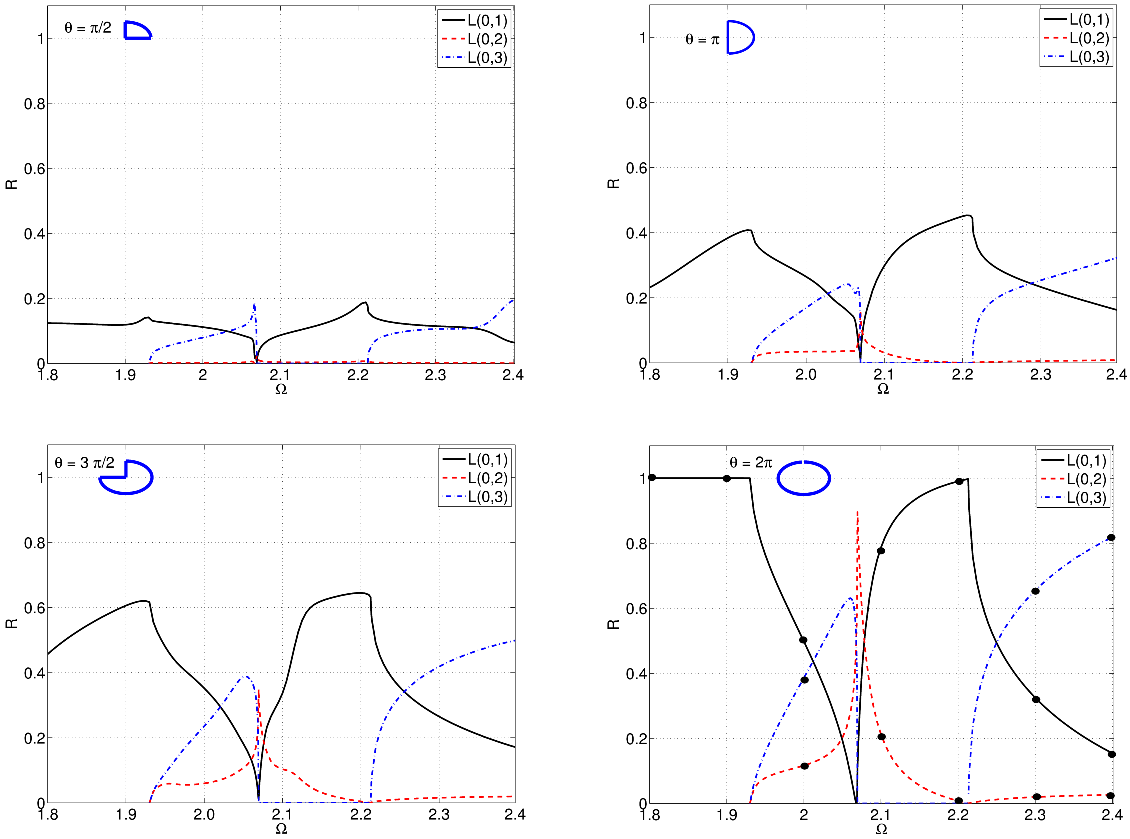


Figure 2. *Power reflection coefficients of the compressional modes when the L(0,1) mode encounters a crack with an opening angle* θ *equal to* $\pi/2$ *(a),* π *(b),* $3\pi/2$ *(c) and* 2π *(d).*

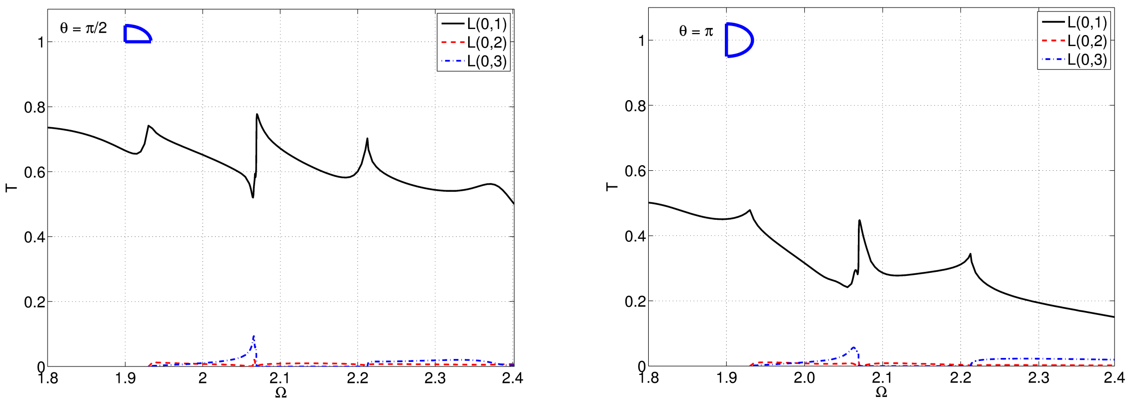
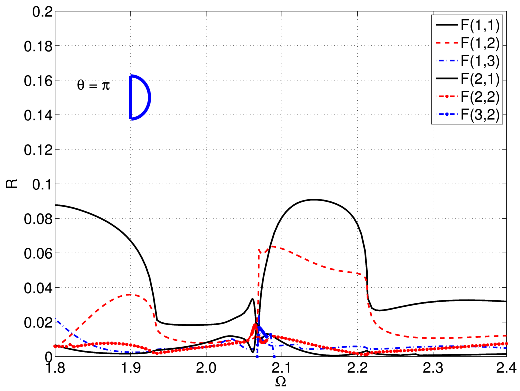


Figure 3. *Power transmission coefficients of the compressional modes when the L(0,1) mode encounters a crack with an opening angle* θ *equal to* $\pi/2$ *(a) and* π *(b).*



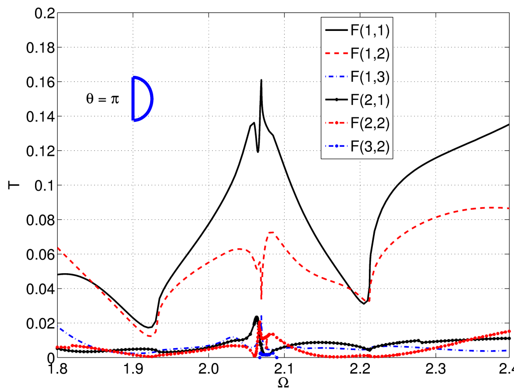


Figure 4. *Power reflection (a) and transmission (b) coefficients of the flexural modes when the L(0,1) mode encounters a crack with an opening angle* $\theta=\pi$.

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References

1. Pochhammer, J. (1876) "Über die fortpflanzungsgeschwindigkeiten kleiner schwingungen in einem unbegrenzten isotropen kreiszylinder". "On the propagation velocities of small vibrations in an infinite isotropic cylinder", *J. Reine Angew. Math*., Vol. 81, pp.324-336.

2. Benmeddour, F, Grondel, S, Assaad, J, Moulin, E. (2008) "Study of the fundamental Lamb modes interaction with asymmetrical discontinuities". *NDT & E Int*., Vol. 41, Issue 5, pp.330-340.

3. Engan, H.E. (1998) "Torsional wave scattering from a diameter step in a rod". *J. Acoust. Soc. Am*., Vol. 104, Issue 4, pp.2012-2024.

4. Demma, A, Cawley, P, Lowe, M, Roosenbrand, A.G, Pavlakovic, B. (2004) "The reflection of guided waves from notches in pipes: a guide for interpreting corrosion measurements". *NDT & E Int*., Vol. 37, Issue 3, pp.167-180.

5. Gregory, R.D, Gladwell, I. (1989) "Axisymmetric waves in a semi-infinite elastic rod", *Q. JI Mech. Appl. Math.,* Vol. 42, Pt. 2, pp.327-337.

6. Rattanawangcharoen, N, Shah, A.H, Datta, S.K. (1994) "Reflection of waves at the free edge of a laminated circular cylinder", *Journal of Applied Mechanics*, Vol. 61, pp.323-329.

7. Taweel, H, Dong, S.B, Kazic, M. (2000) "Wave reflection from the free end of a cylinder with an arbitrary cross-section", *International Journal of Solids and Structures*, Vol. 37, pp.1701-1726.

8. Rattanawangcharoen, N, Zhuang, W, Shah, A.H, Datta, S.K. (1997) "Axisymmetric guided waves in jointed laminated cylinders", *journal of Engineering Mechanics,* Vol. 123, pp.1020-1026.

9. Hayashi, T, Song W-J, Rode, J.L. (2003) "Guided wave dispersion for a bar with an arbitrary cross-section, a rod and rail example", *Ultrasonics,* Vol. 41, pp.175-183.

10. Treyssède, F. (2008) "Elastic waves in helical waveguides", *Wave Motion,* Vol. 45, pp.457-470.