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To cite this version:

HAL Id: hal-00967830
https://hal.archives-ouvertes.fr/hal-00967830
Submitted on 31 Mar 2014

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Non-Local Non-Negative Spherical Deconvolution for Single and Multiple Shell Diffusion MRI

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I. INTRODUCTION

In diffusion MRI (dMRI), Spherical Deconvolution (SD) is a category of methods which estimate the fiber Orientation Distribution Function (fODF). Existing SD methods, including the widely used Constrained SD [1], normally have two common limitations: 1) the non-negativity constraint of the fODFs is not satisfied in the continuous sphere; 2) many spurious peaks are detected, especially in the regions with low anisotropy. In [2], we proposed a novel SD method, called Non-Negative SD (NNSD), to avoid these two limitations. NNSD guarantees the non-negativity constraint of fODFs in the continuous sphere $S^2$, and it is robust to the false positive peaks. In this abstract, we propose Non-Local NNSD (NLNNSD) which considers the non-local spatial information because of the Laplace-Beltrami regularization. The constraint

$$\sum_{x=0}^{2\pi} \sum_{y=0}^{\pi} c_{x,y} e^{-J(x,y)} = 0,$$

where for any fixed vector $q = (x,y)$, $K(q,u)$ is a square matrix with the elements $K_{mm}(q,u) = \sum_{l=0}^L \sum_{p,m} \int \frac{4\pi}{8} c_{x,y} c_{l,m} Q_{l,m}^p h_l(q) Y^p_{m}(u)$.

Then the elements $\Lambda_{nm}$ can be calculated from the Wigner 3-j symbol. Then based on the closed form of spherical convolution using SH basis, for a given axisymmetric fiber response function along $z$-axis $H(q)(0,0,1) = \sum_{\lambda=0}^L h_\lambda(q)\Omega_\lambda$, the convoled diffusion signal is

$$E(q,u) = \sum_{a,b} \sum_{p,m} \sum_{x=0}^{2\pi} \sum_{y=0}^{\pi} \sqrt{\frac{4\pi}{8}} c_{x,y} c_{l,m} Q_{l,m}^p h_l(q) Y^p_{m}(u) = e^c K(q,u) e,$$

where $\Lambda = \sum_{i=1}^N \Lambda_{ini} = \Lambda_{NNSD} \Lambda_{ii} (l+1)^2$ for the Laplace-Beltrami regularization. The constraint $\|e\| = 1$ is because of $\int_{S^2} K(q,u)du = 1$. In this abstract, we propose Non-local NNSD (NLNNSD) which considers the non-local spatial information and Rician noise. Non-local mean has been used in image denoise [3], [4] and [4]. The cost function in NLNNSD is

$$J(e^c) = \frac{1}{2} \sum_{i=1}^N \left[ (e^c)^T \Lambda_{ini} (e^c) - \Lambda_{ini} \right] + (e^c)^T \Omega (e^c) + \frac{1}{2} L_{SLLM} \|e^c - \Lambda_{NNSD} e^c\|^2,$$

where $\lambda^c$ and $E^c$ are the coefficient vector and diffusion signal at voxel $x$, $V$ is the number of voxels, $L_{SLLM}(\lambda^c) = \arg\min E^c$, $\sum_{x \in V} w_i \lambda^c (x, e^c)^2$ is the non-local Riemannian mean of $e^c$ [6], $\Lambda_{NNSD} = \sqrt{\sum_{x \in V} p_x (E^c)^2 - 2\lambda^c}$ is the non-local mean of $E^c$ considering Rician noise with standard deviation of $\sigma$, $w_i$ is the non-local weights determined by the distance of coefficient vectors, i.e.

$$w_i = \frac{1}{2\pi} \exp\left( \frac{-\sum_{y \neq x} G_a |e^c(x) - e^c(y)|^2}{28} \right),$$

where $e^c$ and $e^c$ are the coefficient vectors respectively in the neighborhood $N_x$ of $x$ and the neighborhood $N_y$ of $y$, $G_a$ is the Gaussian weighting with standard deviation of $a$, and $Z_0$ is the normalization factor. $p_n$ is the non-local weight determined by the distance of $|E^c|$ with another set of $[a,b]$. To minimize Eq. (3) with the constraint $\|e\| = 1$, we first set $\lambda_{NNSD} = 0$, and perform a Riemannian gradient descent on the sphere $\|e\| = 1$ [6] to minimize $J(c^c)$ individually for each voxel.

$$e^c = \exp(\lambda_{NNSD}) \left( \begin{array}{c} \frac{\partial J(c^c)}{\partial e^c} \nonumber \end{array} \right).$$

The isotropic fODF with $e = (1,0,0,0)^T$ is chosen as the initialization. Then the non-local Riemannian mean is performed to calculate $\Lambda_{NNSD} e^c$ at each voxel. Then the Riemannian gradient descent is performed again with $\lambda_{NNSD}$ and the estimated non-local mean $\Lambda_{NNSD} e^c$ in Eq. (3). Note that this procedure can be iteratively performed to update $\Lambda_{NNSD} e^c$ and $E^c$, however in practice we found the result with just one iteration is enough.

II. METHOD

We represent the square root of fODF $\Phi(u)$ as a linear combination of real Spherical Harmonic (SH) basis $\sum_{l=0}^L \sum_{p,a} \sum_{m} \int \frac{4\pi}{8} c_{l,m} Q_{l,m}^p h_{\lambda}(q) Y^p_{m}(u)$ with even order, i.e.

$$\Phi(u) = \sum_{l=0}^L \sum_{p,a} \sum_{m} \int \frac{4\pi}{8} c_{l,m} Q_{l,m}^p h_{\lambda}(q) Y^p_{m}(u),$$

where $\Phi(u)$ is the integral constant of three SHs which can be calculated from the Wigner 3-j symbol. Then based on the closed form of spherical convolution using SH basis, for a given axisymmetric fiber response function along $z$-axis $H(q)(0,0,1) = \sum_{\lambda=0}^L h_\lambda(q)\Omega_\lambda$, the convoled diffusion signal is

$$E(q,u) = \sum_{a,b} \sum_{p,m} \sum_{x=0}^{2\pi} \sum_{y=0}^{\pi} \sqrt{\frac{4\pi}{8}} c_{x,y} c_{l,m} Q_{l,m}^p h_l(q) Y^p_{m}(u) = e^c K(q,u) e,$$

where any fixed vector $q = (x,y)$, $K(q,u)$ is a square matrix with the elements $K_{mm}(q,u) = \sum_{l=0}^L \sum_{p,a} \int \frac{4\pi}{8} c_{x,y} c_{l,m} Q_{l,m}^p h_l(q) Y^p_{m}(u)$. Then NNSD [2] is to estimate $e^c$ by minimizing

$$J(e^c) = \frac{1}{2} \sum_{i=1}^N \left[ (e^c)^T K(q,\Omega x - L_{NNSD} e^c) \right] + \frac{1}{2} e^c \Omega e^c, \text{ s.t. } \|e\| = 1$$

III. ISBI HARDI RECONSTRUCTION CHALLENGE

In the ISBI reconstruction challenge, the testing data was generated based on Numerical Fibre Generation toolbox [7]. We test the proposed NLNNSD in the data with three kind of sampling schemes: 1) single shell DTI scheme with 32 directions, $b = 1200\text{s/mm}^2$; 2) single shell HARDI scheme with 64 directions, $b = 3000\text{s/mm}^2$; 3) multiple shell DSII-like scheme with 514 directions, $b = (0,4000)\text{s/mm}^2$. For all night datasets (three schemes with three SNR 10, 20, 30), we fixed $L = 8$, $L_{NNSD} = 0$, $\lambda_{NNSD} = 1$, and used the tensor fiber response function with FA of 0.8, mean diffusivity of 0.8. In the non-local mean of $e^c$ and $E^c$, we uses a $11 \times 11 \times 11$ search window, and a $3 \times 3 \times 3$ patch to define the weights, where the parameters $a$ and $h$ were tuned respectively for $e^c$ and $E^c$ to obtain visually good results for each dataset.

REFERENCES