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I. Introduction

In diffusion MRI (dMRI), Spherical Deconvolution (SD) is a category of methods which estimate the fiber Orientation Distribution Function (fODF). Existing SD methods, including the widely used Constrained SD [1], normally have two common limitations: 1) the non-negativity constraint of the fODFs is not satisfied in the continuous sphere; 2) many spurious peaks are detected, especially in the regions with low anisotropy. In [2], we proposed a novel SD method, called Non-Negative SD (NNSD), to avoid these two limitations. NNSD guarantees the non-negativity constraint of fODFs in the continuous sphere \( S^2 \), and it is robust to the false positive peaks. In this abstract, we propose Non-Local NNSD (NLNNSD) which considers non-local spatial information and Rican noise in NNSD, and apply it to the testing data in ISBI contest.

II. Method

We represent the square root of fODF \( \Phi(u) \) as a linear combination of real Spherical Harmonic (SH) basis \( Y_l^m(u) \) with even order, i.e., \( \Phi(u) = \sum_{l=0}^{2L} \sum_{m=-l}^{l} c_{lm} Y_l^m(u) \). The cost function in NLNNSD is

\[
NNSD(\Phi(u)) = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=0}^{N} \sum_{l=0}^{L} \sum_{m=-l}^{l} c_{lm} Y_l^m(u) Y_l^m(u)^T c_{lm} Y_l^m(u) Y_l^m(u)^T, \quad \text{s.t. } |c_{lm}| = 1
\]

where \( c_{lm} \) is the non-local Riemannian mean of \( c_{lm} \) and \( c_{lm} \) is the non-local mean of \( c_{lm} \). The constraint \(|c_{lm}| = 1\) is chosen as the initial value. Then the Riemannian gradient descent on the sphere is performed to update \( c_{lm} \).

\[
J(c) = \frac{1}{2} \sum_{l=0}^{L} \sum_{m=-l}^{l} \left| c_l^T \mathbf{K}(\mathbf{c}) c - \mathbf{E}_l \right|^2 + \frac{1}{2} c_l^T \mathbf{L} c, \quad \text{s.t. } |c_l| = 1
\]

where \( \mathbf{K} \) is a diagonal matrix with elements \( \Lambda_{lm} = \lambda_{LNSD} \omega^2(l+1)^2 \) for the Laplace-Beltrami regularization. The constraint \(|c_l| = 1\) is because of \( \sum_{l=0}^{L} \sum_{m=-l}^{l} c_l c_l^T = 1 \). In this abstract, we propose Non-local NNSD (NLNNSD) which considers the non-local spatial information and Rican noise. Non-local mean has been used in image denoise [3], [4] and regularization [5]. The cost function in NLNNSD is

\[
J(c) = \frac{1}{2} \sum_{l=0}^{L} \sum_{m=-l}^{l} \left| c_l^T \mathbf{K}(\mathbf{c}) c - \mathbf{E}_l \right|^2 + \frac{1}{2} c_l^T \mathbf{L} c
\]

where \( c_l \) is the coefficient vector and diffusion signal at voxel \( x \). \( N \) is the number of voxels, \( \mathbf{NLM}(c_l) = \arg \min \sum_{x \in V} w_x d(x, c_l)^2 \) is the non-local Riemannian mean of \( c_l \) [6], \( \mathbf{NLM}(E_l) = \left[ \sum_{x \in V} p_x(E_l)^2 \right]^{-1/2} \) is the non-local mean of \( E_l \) considering Rican noise with standard deviation of \( \sigma \). \( w_x \) is the non-local weights determined by the distance of coefficient vectors, i.e.,

\[
w_x = \frac{1}{Z} \exp(-\frac{\sum_{l=0}^{L} \sum_{m=-l}^{l} G_x c_l^T Y_l^m(u) Y_l^m(u)^T c_{lm}}{2\sigma^2}), \quad \text{where } c_l \text{ and } c_{lm} \text{ are the coefficient vectors respectively in the neighborhood } N_x \text{ of } x \text{ and the neighborhood } N_y \text{ of } y, \quad G_x \text{ is the Gaussian weighting with standard deviation of } a, \quad Z_0 \text{ is the normalization factor.}
\]

To minimize Eq. (3) with the constraint \(|c_l| = 1\), we first set \( \lambda_{NL} = 0 \), and perform a Riemannian gradient descent on the sphere \(|c_l| = 1\) to minimize \( J(c) \) individually for each voxel \( x \).

\[
(J(c))^{1/2} = \mathbf{Exp}_{c_l(x)}\left(-\mathbf{dt}(\mathbf{E}_l)\right), \quad \mathbf{Exp}_c(v) = c \cos(|v|) + \frac{\mathbf{F}}{|v|} \sin(|v|)
\]

The isotropic fODF with \( c = (1, 0, 0, \ldots, 0)^T \) is chosen as the initialization. Then the non-local Riemannian mean is performed to calculate NLM\(c_l(x)\) at each voxel. Then the Riemannian gradient descent is performed again with \( \lambda_{NL} \) and the estimated non-local mean NLM\(c_l(x)\) in Eq. (3). Note that this procedure can be iteratively performed to update NLM\(c_l(x)\) and \( c_l(x) \), however in practice we found the result with just one iteration is enough.

III. ISBI HARDI Reconstruction Challenge

In the ISBI reconstruction challenge, the testing data was generated based on Numerical Fibre Generation toolbox [7]. We test the proposed NLNNSD in the data with three kind of sampling schemes: 1) single shell DTI scheme with 32 directions, \( b = 1200s/mm^2 \); 2) single shell HARDI scheme with 64 directions, \( b = 3000s/mm^2 \); 3) multiple shell DSI-like scheme with 514 directions, \( b \in (0, 4000) s/mm^2 \). For all night datasets (three schemes with three SNR 10, 20, 30), we fixed \( L = 8, \lambda_{NL} = 0, \lambda_{NL} = 1 \), and used the tensor fiber response function with FA of 0.8, mean diffusivity of 0.8. In the non-local mean of \( c_l \) and \( E_l \), we uses a 11 × 11 × 11 search window, and a 3 × 3 × 3 patch to define the weights, where the parameters \( a \) and \( h \) were tuned respectively for \( c_l(x) \) and \( E_l \) to obtain visually good results for each dataset.

References