# Non-Local Non-Negative Spherical Deconvolution for Single and Multiple Shell Diffusion MRI

Jian Cheng, Rachid Deriche, Tianzi Jiang, Dinggang Shen, Pew-Thian Yap

$w_y = \frac{1}{Z_y} \exp(-\frac{\sum_{j \in N_x, k \in N_y} G_a \|c^j - c^k\|^2}{2h^2})$, where $c^j$ and $c^k$ are the coefficient vectors respectively in the neighborhood $N_x$ of $x$ and the neighborhood $N_y$ of $y$, $G_a$ is the Gaussian weighting with standard deviation of $a$, and $Z_y$ is the normalization factor. $p_y$ is the non-local weight determined by the distance of $\{E_i^x\}$ with another set of $\{a, h\}$.

To minimize Eq. (3) with the constraint $\|c\| = 1$, we first set $\lambda_{NLM} = 0$, and perform a Riemannian gradient descent on the sphere $\|c\| = 1$ [6] to minimize $J(c^x)$ individually for each voxel $x$.

$$(c^x)^{(k+1)} = \text{Exp}_{(c^x)(k)}\left(-dt\frac{\nabla J(c^x)}{\|\nabla J(c^x)\|}\right), \quad \text{Exp}_c(v) = c\cos\|v\| + \frac{v}{\|v\|}\sin\|v\|$$

The isotropic fODF with $c = (1, 0, \ldots, 0)^T$ is chosen as the initialization. Then the non-local Riemannian mean is performed to calculate $\text{NLM}(c^x)$ at each voxel. Then the Riemannian gradient descent is performed again with $\lambda_{NLM}$ and the estimated non-local mean $\text{NLM}(c^x)$ in Eq. (3). Note that this procedure can be iteratively performed to update $\text{NLM}(c^x)$ and $c^x$, however in practice we found the result with just one iteration is enough.

### III. ISBI HARDI Reconstruction Challenge

In the ISBI reconstruction challenge, the testing data was generated based on Numerical Fibre Generation toolbox [7]. we test the proposed NLNNSD in the data with three kind of sampling schemes: 1) single shell DTI scheme with 32 directions, $b = 1200 s/mm^2$; 2) single shell HARDI scheme with 64 directions, $b = 3000 s/mm^2$; 3) multiple shell DSI-like scheme with 514 directions, $b \in (0, 4000] s/mm^2$. For all night datasets (three schemes with three SNR 10, 20, 30), we fixed $L = 8$, $\lambda_{NNSD} = 0$, $\lambda_{NLM} = 1$, and used the tensor fiber response function with FA of 0.8, mean diffusivity of 0.8. In the non-local mean of $c^x$ and $E_i^x$, we uses a $11 \times 11 \times 11$ search window, and a $3 \times 3 \times 3$ patch to define the weights, where the parameters $a$ and $h$ were tuned respectively for $\{c^x\}$ and $\{E_i^x\}$ to obtain visually good results for each dataset.

### References

[1] J. Tournier, F. Calamante, A. Connelly, "Robust determination of the fibre orientation distribution in diffusion MRI: non-negativity constrained super-resolved spherical deconvolution", NeuroImage 35(4) 1459–1472 (2007).

[2] J. Cheng, D. Shen, P.-T. Yap, "Non-Negative Spherical Deconvolution for Fiber Orientation Distribution Estimation", ISMRM (2013).

[3] A. Buades, B. Coll. J.M. Morel, "A non-local algorithm for image denoising" CVPR (2005).

[4] M. Descoteaux, N. Wiest-Daessle, S. Prima, C. Barillot, R. Deriche, "Impact of Rician Adapted Non-local Means Filtering on HARDI", MICCAI (2008).

[5] G. Peyré, S. Bougleux, L. Cohen, "Non-local regularization of inverse problems", ECCV, 2008.

[6] J. Cheng, A. Ghosh, T. Jiang, R. Deriche, "A Riemannian Framework for Orientation Distribution Function Computing", MICCAI (2009).

[7] T. Close, J. Tournier, F. Calamante, L. Johnston, I. Mareels, A. Connelly, "A software tool to generate simulated white matter structures for the assessment of fibre-tracking algorithms", NeuroImage (2009).

---

### I. Introduction

In diffusion MRI (dMRI), Spherical Deconvolution (SD) is a category of methods which estimate the fiber Orientation Distribution Function (fODF). Existing SD methods, including the widely used Constrained SD [1], normally have two common limitations: 1) the non-negativity constraint of the fODFs is not satisfied in the continuous sphere; 2) many spurious peaks are detected, especially in the regions with low anisotropy; In [2], we proposed a novel SD method, called Non-Negative SD (NNSD), to avoid these two limitations. NNSD guarantees the non-negativity constraint of fODFs in the continuous sphere $\mathbb{S}^2$, and it is robust to the false positive peaks. In this abstract, we propose Non-Local NNSD (NLNNSD) which considers non-local spatial information and Rician noise in NNSD, and apply it to the testing data in ISBI contest.

### II. Method

We represent the square root of fODF $\Phi(u)$ as a linear combination of real Spherical Harmonic (SH) basis $Y_l^m(u)$ with even order, i.e. $\Phi(u) = \left(\sum_{l=0}^L \sum_{m=-l}^l c_{lm} Y_l^m(u)\right)^2 = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \left(\sum_{l,m}^L \sum_{l',m'}^L c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta}\right) Y_\alpha^\beta(u)$, where $u \in \mathbb{S}^2$, $Q_{ll'\alpha}^{mm'\beta} = \int_{\mathbb{S}^2} Y_l^m(u) Y_{l'}^{m'}(u) Y_\alpha^\beta(u) du$ is the integral constant of three SHs which can be calculated from the Wigner 3-j symbol. Then based on the closed form of spherical convolution using SH basis, for a given axisymmetric fiber response function along $z$-axis $H(qu|(0,0,1)) = \sum_{l=0}^L h_l(q) Y_l^0(u)$, the convolved diffusion signal is

$$E(qu) = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \sum_{l,m}^L \sum_{l',m'}^L \sqrt{\frac{4\pi}{2\alpha+1}} c_{lm} c_{l'm'} Q_{ll'\alpha}^{mm'\beta} h_\alpha(q) Y_\alpha^\beta(u) = c^T K(qu) c \quad (1)$$

where for any fixed vector $q = qu$, $K(u)$ is a square matrix with the elements $K_{ll'}^{mm'}(qu) = \sum_{\alpha=0}^{2L} \sum_{\beta=-\alpha}^{\alpha} \sqrt{\frac{4\pi}{2\alpha+1}} Q_{ll'\alpha}^{mm'\beta} h_\alpha(q) Y_\alpha^\beta(u)$. Then NNSD [2] is to estimate $c$ by minimizing

$$J(c) = \frac{1}{2} \sum_{i=1}^N \left(c^T K(qu) c - E_i\right)^2 + \frac{1}{2} c^T \Lambda c, \quad \text{s.t. } \|c\| = 1 \quad (2)$$

where $\Lambda$ is a diagonal matrix with elements $\Lambda_{lm} = \lambda_{NNSD} l^2(l+1)^2$ for the Laplace-Beltrami regularization. The constraint $\|c\| = 1$ is because of $\int_{\mathbb{S}^2} \Phi(u) du = 1$. In this abstract, we propose Non-local NNSD (NLNNSD) which considers the non-local spatial information and Rician noise. Non-local mean has been used in image denoise [3], [4] and regularization [5]. The cost function in NLNNSD is

$$J(\{c^x\}) = \frac{1}{2} \sum_{x=1}^V \sum_{i=1}^N \left((c^x)^T K(qu) c^x - \text{NLM}(E_i^x)\right)^2 + \frac{1}{2} (c^x)^T \Lambda c^x + \frac{1}{2} \lambda_{NLM} \|c^x - \text{NLM}(c^x)\|^2 \quad (3)$$

where $c^x$ and $E_i^x$ are the coefficient vector and diffusion signal at voxel $x$, $V$ is the number of voxels, $\text{NLM}(c^x) = \arg\min_c \sum_{y \in V} w_y d(c, c^y)^2$ is the non-local Riemannian mean of $c^x$ [6], $\text{NLM}(E_i^x) = \sqrt{\sum_{y \in V} p_y (E_i^y)^2 - 2\sigma^2}$ is the non-local mean of $E_i^x$ considering Rician noise with standard deviation of $\sigma$. $w_y$ is the non-local weights determined by the distance of coefficient vectors, i.e.