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Integration of ill-known requirements in production planning

Romain Guillaume 1,2, Caroline Thierry 1, Bernard Grabot 2

1 Université de Toulouse-IRIT 5, Allées A. Machado, F-31058 Toulouse Cedex 1
Romain.Guillaume@irit.fr thierry@univ-tlse2.fr
2 Université de Toulouse, INPT, LGP-ENIT, 47 Av. d'Azerieix, BP 1629, F-65016 Tarbes Cedex
bernard@enit.fr

ABSTRACT

This paper addresses the integration of ill-know requirements (imprecision or uncertainty on quantities or due dates) in a gross requirements (expressed in terms of quantities by periods). Ill-known requirements may come from a customer of the supply chain (forecast orders) or from a make-to-order production release. The types of imperfections on these requirements are analysed and a representation model based on mathematical models of uncertainty is proposed. Moreover, a method is proposed to compute the gross requirements without loss of information. This method is illustrated on an example.

Keywords: requirement, production planning, uncertainty, imprecision, fuzzy logic, possibility theory.

1. INTRODUCTION

The manufacturing environment is made more and more complex by the network structures of nowadays supply chains, but also by the increasing uncertainty on the customers' demand. Thus, the integration of uncertainty in the planning process of the supply chain actors becomes imperative.

In present works on Supply Chains, two kinds of strategies are usually considered for managing the impact of the uncertainties in the planning process.

The first strategy focuses on the rolling horizon planning method. To cope with demand uncertainty and limit the risks of the supply chain, this commonly used technique consists in actions on the Master Plan Schedule dealing with re-planning / horizon size [1], freezing the plan [2], lot-sizing rules [3] and safety stocks [4].

The second strategy consists in integrating uncertainty in the planning process. Two main types of mathematic models have been used to represent imperfect data: stochastic model and possibility models. In the field of possibility models, a fuzzy planning can integrate uncertainties ([5], [6]), but uncertainties can also be integrated in the planning process as fuzzy constraints, to obtain a determinist plan [7].

All these studies focus on the fuzzy planning method, and not on the problematic of uncertainties caracterisation and representation to build an uncertain gross requirements.

In that context, our target is first to propose methods to characterise and represent "uncertainty" (imperfection, uncertainty and imprecision,...) on requirements. Moreover, we aim at integrating the imperfect requirements into a gross requirements (required quantities by periods) to cope with the uncertainty upstream the supply chain.

Section 2 reminds fundamental theories of uncertainty and explains our choice of models. Section 3 presents the chosen approach and section 4 proposes an illustration on an example.

2. BACKGROUND

2.1 Imperfection

Even if the word “uncertainty” is commonly used, a distinction has to be made between several concepts: uncertainty, imprecision and incompleteness. Thus, we will use the term imperfection to gather these three concepts, as in [8]:
- Uncertainty refers to the case when it is not possible to say definitely whether a particular Boolean statement is true or false.
- Imprecision denotes the impossibility to enounce knowledge in a precise and sure manner.
- Incompleteness means in most cases that an information source does not contain full information about the attributes of the real-world it describes. Some properties may be missing from the description.

Two types of mathematic model can represent imperfect data: the stochastic model and the possibility models [9]. In this paper, we consider the case when no previous observation is available to create a stochastic model. In that case, possibility models may be used for expressing information coming from expert knowledge. In that purpose, trapezoidal fuzzy sets (Figure 1) which are denoted (a, b, c, d, h) can be used without an important loss of generality.
2.2 Representation of uncertainty

For all events A from a subset S, the uncertainty is represented by two evaluation levels:
- the high level: possibility $\Pi(A)$
- the low level: certainty $N(A)$

with
$$\forall A \subseteq S, N(A) \leq \Pi(A)$$ \hspace{1cm} (1)
and
$$\forall A \subseteq S, N(A) = 1 - \Pi(A)$$ \hspace{1cm} (2)

If $A$ is certainly true $\Pi(A)=1$ and $N(A)=1$.
If $A$ is certainly false $\Pi(A)=0$ and $N(A)=0$.
If there is no knowledge available: $\Pi(A)=1$ and $N(A)=0$ (the event is fully possible but not necessary at all).

2.3 Representation of imprecision

An imprecise information is $\nu \in A$ where $A$ is a subset of $S$ which contains more than one element. The imprecision may be expressed by a disjunction of values [9].

The imprecise information $\nu \in A$ defines a possibility distribution on $S$. $\nu \in A$ means that all values from $\nu$ out of $A$ are supposed impossible [9].
A possibility distribution $\kappa_{\nu}$ of $\nu$ is a function of $S$ in $L_{\nu}$ such as $\forall s \in S, \kappa_{\nu}(s) \in L_{\nu}$ and $\exists s, \kappa_{\nu}(s) = 1$ with $\nu$ denoting an ill-know value in $S$, $L$ scale of plausibility ($[0,1]$).

When the possibility distribution is represented with a trapezoidal fuzzy set, an imprecise information is represented by $(a,b,c,d,h)$ with $h=1$, like in Figure 1.

2.4 Representation of uncertainty and imprecision

Information can be pervaded with both uncertainty and imprecision. An uncertainty and imprecision is denoted by $\nu \in A \cup B$ where $A$ and $B$ are two disjoints subset of $S$, one of the subsets expressing the possibility that the event does not occur, the other defining the content of the event if it occurs. The possibility distribution associated to the information $\nu \in A \cup B$ is denoted $\pi(\nu)$

$$\Pi(A) = \max(\pi(\nu) | \nu \in A)$$
$$\Pi(B) = \max(\pi(\nu) | \nu \in B)$$ \hspace{1cm} (3)

where these two events are characterised by this relation:
$$\max(\Pi(A), \Pi(B)) = 1$$ \hspace{1cm} (4)

When the possibility distribution is represented with a trapezoidal fuzzy set, an uncertain and imprecise information will be represented by an union of fuzzy sets $(a,b,c,d,h)$ with $h$ the possibility of subset.

2.5 Ill-know requirements representation

The objective of this chapter is to characterise the types of imperfections on requirements.

The word “requirements” gathers here:
- forecasts from the customers,
- requirements linked to firm orders.

A requirement $\sigma$ is characterised by a required quantity $Q_{\sigma}$ and a requirement date (due date) $\tau_{\sigma}$.

The first step of the modelling phase consists in identifying the imperfections on the requirements.
Uncertainty may be present:
- on the requirement itself: a requirement which can be cancelled is uncertain;
- on the attributes of the requirement: the required quantity and requirement date can be uncertain (“I need 100 rotors blades for week 35 or for week 40”)

Imprecision is only present on the attributes of the requirement: the required quantity and requirement date can be imprecise (“I need around 100 rotor blades for week 35’’)

Uncertainties on requirements can be modelled by two values, the possibility and the certainty of presence of the requirement itself: $\Pi(\sigma)$ and $N(\sigma)$

The uncertainty on the required quantity and on the requirement date can be characterised by two values each:
- $\pi(Q_{\sigma})$ and $\Pi(Q_{\sigma})$
- $\pi(\tau_{\sigma})$ and $\Pi(\tau_{\sigma})$

The imprecision can be represented by a possibility distribution for the required quantity or/and for the requirement date:
- $\pi(Q_{\sigma})$
- $\pi(\tau_{\sigma})$
The proposed method represents the imperfection (uncertainty and imprecision) on the requirements using two fuzzy sets:

- the first fuzzy set represents the imperfection (uncertainty and imprecision) on the required quantity, so that the uncertainty of the requirement itself,
- the second fuzzy set represents the imperfection on the due date.

The considered fuzzy sets are defined as unions of trapezoidal fuzzy sets, which preserve the generality of the representation. Table 1 and 2 give the models proposed for each kind of imperfection.

### 3. GROSS REQUIREMENTS

Considering a set of imperfect requirements, we propose a three steps method to compute the gross requirements:

- design of requirements scenarios,
- computation of the different requirements scenarios,
- building of the gross requirements (required quantity per period).

**Notations:**

- $\alpha$: the index of requirement with $\alpha \in [1,0]$
- $t$: period with $t \in [1,T]$
- $c$: combination with $c \in [1,C]$
- $u_o$: periods where the requirement $o$ could exist $u_o \in T_o$
- $j_o$: fuzzy set of possible quantity of requirement $o$ with $j_o \in J_o - [1,j_o]$
- $j_o,c$: fuzzy set of possible quantity of requirement $o$ for the combination $c$.
- $c$: combinations of scenarios with $c \in [1,C]$

**General data**

$\tau_o$: possible date of requirement $o$

$Q_o$: possible required quantity of requirement $o$

$A_{i_o,j_o,c}$: fuzzy set of possible quantity of requirement $o$

$B_{i_o,t,c}$: fuzzy set of possible quantity of requirement $o$ to period $t$ when it has the possible quantity $j_o$

$\overline{B}_{i_o,t,c}$: fuzzy set of possible quantity of requirement $o$ to period $t$ when the requirement belongs to period $i$

**Decision and dependent variables**

$r_o$: possible total required quantity for period $t$

$R_t$: total required quantity for period $t$

$v_{c,o}$: possible required quantity of combination $c$ for period $t$

$V_{c,t}$: required quantity of combination $c$ for period $t$

$E_t$: set of combinations of scenarios.

$A_{c}$: set of scenario for combination $c$

$E_{o,c}$: fuzzy set of possible quantity of requirement $o$ and scenario $(i_o,v_o)$ which is on period $t$.

### 3.1 Design of requirements scenarios

Requirement scenarios are scenarios of requirement $o$ belonging to each possible period. First, we compute the possibility (equation 5) for each requirement to belong to each period:

$$\Pi(t_o \in t) = \max(\pi(t_o)(t_o \in t)) \quad (5)$$
As a second step, we select, for each requirement \( o \), the set \( T_o \) of possible periods for the requirement (equation 6) with \( u_o \in \{ U_{o \in \Pi} \} \):

\[
T_o / u_o \in T_o \text{ if } \Pi (\tau_o \in u_o) \neq 0
\]  

(6)

In the third step, we compute the requirements of the different scenarios: the possibility degree of requirement to belong to period \( t \) (\( \Pi (\tau_o \in u_o) \)) means that the requirement exists in this period with this possibility, and does not exist in the other periods of \( T_o \) with the same possibility.

Let us denote \( Q_o = \bigcup_{j_o=1}^{J_o} A_{j_o} \) with \( j_o \in [1, J_o] \) the index of the possible imprecise values of \( Q_o \) and \( A_{j_o} \) the fuzzy set which represents the imprecision on these values.

The different scenarios \( W_o \) of order \( o \) can be defined by \( W_o = (j_o, u_o) \):

\[
\forall u_o, \forall t \in T_o
\]

\[
E_{o,j_o} = B_{o,j_o} \forall u_o = t
\]

\[
F_{o,j_o} = B_{o,j_o} \forall u_o \neq t
\]

with

\[
\bar{B}_{o,j_o} = (0,0,0,0,\Pi (\tau_o \in u_o)) \\forall u_o \in T_o - \{u_o\}
\]

\[
\forall j_o \in [1, J_o], B_{o,j_o} = A_{j_o} \times (0,0,0,0,\Pi (\tau_o \in u_o))
\]

With \( U_o \) the number of period of \( T_o \), we have \( S_o = U_o \times J_o \) scenarios for each requirement \( o \).

Table 3 represents the different scenarios \( W_o \) for the requirement \( o \).

### 3.2 Computation of the total required quantity for each combination

From the table 3, we create all combinations of the requirement scenarios.

We have \( C = \prod_{o=1}^{O} S_o \) combinations:

Let \( E \) be the set of these \( C \) combinations \( c \). To select a restricted set of combinations, the decision maker may choose to only consider the combinations for which the possibility is higher than a given value \( \rho \). The set of combination is then reduced to \( E_\rho \) (equation 7).

\[
e_\rho = \{ c \} \in C \\text{ if } \forall u_o \in J_o, \forall u_o \in T_o
\]

(7)

Thus, only the set of required quantities which have a possibility higher than this critical value will be selected in the scenario described in table 3. Indeed, the sum of fuzzy sets has a possibility equal to the minimum possibility of the added fuzzy sets.

For a given combination of the requirement scenarios, the total required quantity combination by period \( V_{c.o} \) is the sum of the required quantities of the requirement scenarios of combination for each period (equation 8).

\[
V_{c.o} = \sum_{o=1}^{O} (E_{o,j_o})_c \\forall u_o \forall u_o = (j_o, u_o) \in A_c
\]

(8)

### 3.3 Computation of a gross requirement

A gross requirement is a set of total required quantities by period for each period of the considered horizon. The possibility of each quantity for each period is computed as the mean possibility of the possible required quantity of combination on the different requirements scenarios (equation 9).

\[
\Pi (\tau) = \sum_{c=1}^{C} \frac{\Pi (V_{c.o})}{c_p}
\]

(9)

Then, we have to normalise the result (equation 10). This consists in setting the most possible value at 1 (cf. the definition of a possibility distribution).

\[
R_i = \frac{\Pi (\tau)}{\max (\Pi (\tau))}
\]

(10)

### 4. ILLUSTRATION ON AN EXAMPLE

Each step of the method will be illustrated on an example in this section.
4.1 Data

To illustrate the method, let us consider an example composed of 8 requirements (see Table 4). From these requirements, we compute the required quantity for period P2 (from date 7 to 14) and P3 (from date 14 to 21).

Table 4. Requirements

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Requirement date</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>(100, 150, 10, 10)</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>(250, 300, 20, 20, 0.5)</td>
</tr>
<tr>
<td>( r_3 )</td>
<td>(300, 300, 15, 15, 1)</td>
</tr>
<tr>
<td>( r_4 )</td>
<td>(300, 400, 10, 10, 1)</td>
</tr>
<tr>
<td>( r_5 )</td>
<td>(300, 100, 30, 30, 1)</td>
</tr>
<tr>
<td>( r_6 )</td>
<td>(150, 150, 10, 10, 1)</td>
</tr>
<tr>
<td>( r_7 )</td>
<td>(200, 400, 0, 0, 1)</td>
</tr>
<tr>
<td>( r_8 )</td>
<td>(300, 350, 0, 20, 1)</td>
</tr>
</tbody>
</table>

4.2 Design of requirement scenarios

We can then build the different scenarios for each requirement (Table 5), resulting from the fact that requirements may be composed or different alternatives.

Table 5. Requirement scenarios

<table>
<thead>
<tr>
<th>( r )</th>
<th>( P )</th>
<th>Quantity</th>
<th>Requirement date</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 )</td>
<td>( P )</td>
<td>( (100,50) )</td>
<td>(50, 50)</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>( P_2 )</td>
<td>( (0,0,0) )</td>
<td>(0, 0, 0, 0)</td>
</tr>
<tr>
<td>( r_3 )</td>
<td>( P )</td>
<td>( (100,50) )</td>
<td>(50, 50)</td>
</tr>
<tr>
<td>( r_4 )</td>
<td>( P )</td>
<td>( (0,0,0) )</td>
<td>(0, 0, 0, 0)</td>
</tr>
<tr>
<td>( r_5 )</td>
<td>( P )</td>
<td>( (100,50) )</td>
<td>(50, 50)</td>
</tr>
<tr>
<td>( r_6 )</td>
<td>( P )</td>
<td>( (0,0,0) )</td>
<td>(0, 0, 0, 0)</td>
</tr>
<tr>
<td>( r_7 )</td>
<td>( P )</td>
<td>( (100,50) )</td>
<td>(50, 50)</td>
</tr>
<tr>
<td>( r_8 )</td>
<td>( P )</td>
<td>( (0,0,0) )</td>
<td>(0, 0, 0, 0)</td>
</tr>
</tbody>
</table>

4.3 Computation of the total required quantity

From Table 5, we obtain Table 6 showing the different total required quantities for each combination of the requirement scenarios. For simplification purpose, the manager may only consider the scenarios with a possibility higher or equal than 0.6, which leads to the 16 combinations presented in Table 6.

Table 6. Combination of total required quantity

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Period 2</th>
<th>Period 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_7 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_8 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_9 )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_{10} )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_{11} )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_{12} )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
<tr>
<td>( C_{13} )</td>
<td>( B_{112} \odot B_{212} \odot B_{312} \odot B_{412} )</td>
<td>( B_{613} \odot B_{713} \odot B_{913} )</td>
</tr>
</tbody>
</table>
4.4 Computation of the gross requirements

From the previous scenarios, we may build the gross requirements, i.e., total required quantity for period 2 (Figure 2) and 3 (Figure 3).

<table>
<thead>
<tr>
<th>C12</th>
<th>B_{12} \oplus B_{13} \oplus B_{13}</th>
<th>B_{12} \oplus B_{13} \oplus B_{31}</th>
<th>B_{12} \oplus B_{13} \oplus B_{31}</th>
<th>(599, 801, 44, 44, 0, 8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C13</td>
<td>B_{12} \oplus B_{13} \oplus B_{12}</td>
<td>B_{12} \oplus B_{13} \oplus B_{13}</td>
<td>B_{12} \oplus B_{13} \oplus B_{31}</td>
<td>B_{12} \oplus B_{13} \oplus B_{31}</td>
</tr>
</tbody>
</table>

| C16 | B_{12} \oplus B_{13} \oplus B_{12} | B_{12} \oplus B_{13} \oplus B_{13} | B_{12} \oplus B_{13} \oplus B_{12} | B_{12} \oplus B_{13} \oplus B_{13} | (505, 695, 20, 20, 0, 8) |
|     | (505, 695, 20, 20, 0, 8)          | (505, 695, 20, 20, 0, 8)       | (505, 695, 20, 20, 0, 8)       | (505, 695, 20, 20, 0, 8) |

4.4 Computation of the gross requirements

From the previous scenarios, we may build the gross requirements, i.e., total required quantity for period 2 (Figure 2) and 3 (Figure 3).

Let us notice that for period 2, only one quantity has a possibility equal to 1 while for period 3, only four quantities have possibility equal to 1. It can be seen on these figures that taking into account explicitly the imprecision/uncertainty on the requirements may in some cases lead to quite rich information (showing a set of possible scenarios, and not a "picture" of one of the possibilities like in common planning). Of course, the result can be difficult to interpret if the imprecision/uncertainty still increases.

5. CONCLUSION

In this paper, uncertainty and imprecision on requirements have been analysed and a model allowing to take into account imprecision and uncertainty has been proposed. From this model, a method has been proposed for building uncertain gross requirements (i.e., quantities by period). The result stresses that the resulting uncertain total required quantity is not a simple fuzzy set (triangular or trapezoidal) but gives more information than a fuzzy number.

As a following step, two different options of planning are possible: coming back to a deterministic planning (choice the most possible quantity, stochastic optimization with fuzzy constraint) or going on with a fuzzy plan.

In this paper, the incompleteness of requirement has not been studied. As a perspective, considering this imperfection could be interesting. Indeed, in distant periods, the knowledge of the requirement may be incomplete. In that case, the evaluation of the required quantity is false. An incompleteness factor could be used to solve this problem, which can also be taken into account by the possibility framework.

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