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Modeling of Rayleigh–Bénard natural convection heat transfer in nanofluids

Modélisation des transferts de chaleur en configuration de Rayleigh–Bénard dans les nanofluides

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1. Introduction

Nanofluids consist of uniformly dispersed and suspended nanometer-sized particles (10–50 nm) with very low concentration in a base fluid. A review of the state of knowledge on nanofluids was published by Wang and Mujumdar [1]. During
this last decade, many authors claim that the presence of nanoparticles in a fluid alters the flow structure and increases the natural convection heat transfer. The results presented by Khanafer et al. [2] illustrated that the suspended nanoparticles substantially increased the heat transfer rate for any given Grashof number. In addition they showed that the heat transfer rate in nanofluids increased when increasing the volume fraction of nanoparticles. Abu-Nada et al. [3] studied the effects of inclination angle on natural convection in enclosures filled with Cu-water nanofluid. They indicated that the addition of nanoparticles produced a noticeable enhancement of heat transfer and that the heat transfer rate increased with the Rayleigh number. Many experimental studies are also available in the literature. Putra et al. [4] presented an experimental investigation of the natural convection of water-based $\text{Al}_2\text{O}_3$ nanofluids inside horizontal cylinder heated from one end and cooled from the other. An apparently paradoxical behavior of heat transfer deterioration was observed in their experimental study. They reported that the presence of nanoparticles of $\text{Al}_2\text{O}_3$ in base fluid reduced the natural convective heat transfer. The degradation of natural convection heat transfer when increasing the particle concentration for a given Rayleigh number increased with the volume fraction of nanoparticles. These results are not consistent with the experimental results are presented by Putra et al. [4] and with the numerical results presented in this paper.

It appears from this literature review, that the use of nanofluid to improve natural convection heat transfer remains an open problem.

The purpose of the present study is to examine the effect of adding nanoparticles to the base fluid, near the onset of convection, on the conduction and convection heat transfer rate in a rectangular cavity heated from below (Rayleigh–Bénard configuration). In this work we show that the relations expressing the specific heat and the volumetric thermal expansion coefficients which are still used by many authors [3–8] lead to results which differ significantly from the ones obtained with thermodynamically valid expression.

2. Mathematical formulation

We consider a rectangular cavity filled with a nanofluid. The permeable horizontal walls are maintained at different and constant temperatures $T_1$ for $z = 0$ and $T_2$ for $z = H$ with $T_1 > T_2$. The vertical walls are impermeable and insulated. All the boundaries are assumed rigid. We assume that the Oberbeck-Boussinesq approximation is valid. The thermophysical properties of the nanofluid are considered constant except the density in the buoyancy term which varies linearly with the temperature: $\rho_{nf}(T) = \rho_{nf}(0)[1 - \beta_{nf}(T - T_0)]$. Here the subscript 0 denotes the reference state, the subscript $(nf)$ is relative to the nanofluids, $\beta_{nf}$ is the thermal expansion coefficient of nanofluid and $T$ is the dimensional temperature.

The governing equations for the problem can be written in dimensionless form as follows:

\[ \vec{\nabla} \cdot \vec{V} = 0 \]  

\[ \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \vec{V} = -\vec{\nabla} P + \nu_{nf} \frac{\partial^2 \vec{V}}{\partial z^2} + \nu_{nf} \nabla^2 \vec{V} \]  

\[ \frac{\partial T}{\partial t} + \vec{V} \cdot \vec{\nabla} T = \nabla^2 T \]

where $\vec{V}$ is the velocity, $P$ is the pressure and $T$ is the temperature. The dimensionless parameters governing this problem are the Prandtl number and the Rayleigh number for the nanofluid defined by:

\[ Ra_{nf} = \frac{g \beta_{nf} H^3 (T_1 - T_2)}{\alpha_{nf} \nu_{nf}} \quad \text{and} \quad Pr_{nf} = \frac{\nu_{nf} (Cp)_{nf}}{k_{nf}} \]

where $g$ is the gravitational acceleration, $\alpha_{nf}$ is the thermal diffusivity, $\nu_{nf}$ is the kinematic viscosity, $\mu_{nf}$ is the dynamic viscosity, $(Cp)_{nf}$ is the specific heat capacity and $k_{nf}$ is the thermal conductivity.

The dimensionless boundary conditions are:

\[ \vec{V} = \vec{0} \quad \text{on the boundaries} \]

\[ T = 1 \quad \text{for} \ z = 0 \ \text{and} \ T = 0 \quad \text{for} \ z = 1 \ \forall x \in [0, A] \]

\[ \frac{\partial T}{\partial x} = 0 \quad \text{for} \ x = 0, A \ \forall z \in [0, 1] \]

where $A$ is the aspect ratio of the cavity.

We assume that the nanoparticles and the base fluid are in thermal equilibrium and that the nanoparticles are uniformly dispersed within the base fluid. The thermophysical properties of the nanofluid, namely the density, specific heat capacity and volumetric thermal expansion coefficient, were calculated from nanoparticles and base fluid properties at ambient
temperature using the following formulas which are compatible with the thermodynamic laws. Introducing the particle volume fraction $\varphi$, the density of the mixture is given by the following equation:

$$\rho_{nf} = (1 - \varphi)\rho_f + \varphi\rho_s$$  \hspace{1cm} (8)

where the subscripts (s) and (f), denote respectively the nanoparticles and the base fluid. The volumetric thermal expansion coefficient $\beta_{nf}$ of a nanofluid is given by:

$$(\rho\beta)_{nf} = (1 - \varphi)(\rho\beta)_f + \varphi(\rho\beta)_s$$ \hspace{1cm} (9)

The specific heat capacity of nanofluid (Cp)$_{nf}$ is given by:

$$(\rho\text{Cp})_{nf} = (1 - \varphi)(\rho\text{Cp})_f + \varphi(\rho\text{Cp})_s$$ \hspace{1cm} (10)

To obtain $\beta_{nf}$, most of the authors previously mentioned [5,6,9] used relations similar to the relation (8) which is only valid for $\rho_{nf}$:

$$\beta_{nf} = (1 - \varphi)\beta_f + \varphi\beta_s$$ \hspace{1cm} (11)

The specific heat capacity of nanofluid (Cp)$_{nf}$ was calculated, by many authors [6,9] using the approximate relation:

$$(\text{Cp})_{nf} = (1 - \varphi)(\text{Cp})_f + \varphi(\text{Cp})_s$$ \hspace{1cm} (12)

where (Cp)$_f$ and (Cp)$_s$ are respectively the specific heat capacity of the base fluid and the nanoparticles. Polidori et al. [9] mentioned that several authors preferred to use this simpler relation to determine the specific heat and they used this relation to be consistent with them. We verified that these relations (11) and (12) did not lead to a good approximation of the nanofluid volumetric thermal expansion coefficient $\beta_{nf}$ and of the specific heat capacity (Cp)$_{nf}$ for the three varieties of nanofluids studied.

The presence of nanoparticles in a fluid increases significantly the effective thermal conductivity of the fluid and consequently enhances the conductive heat transfer characteristics. Hamilton and Crosser [10] proposed a model to calculate the effective thermal conductivity of nanofluids:

$$k_{nf} = \frac{k_s + (n - 1)k_f - (n - 1)(k_f - k_s)\varphi}{k_s + (n - 1)k_f + (k_f - k_s)\varphi}$$ \hspace{1cm} (13)

where $k_f$ and $k_s$ are respectively the thermal conductivity of the base fluid and the nanoparticles, $\varphi$ is the volume fraction of nanoparticles and $n$ is the empirical shape factor. Their experimental research showed satisfactory agreement between the theoretical predictions and the experimental results for many nanofluids in a range of volume fraction up to 30%.

The ratio of effective viscosity of nanofluids to that of their base fluid is calculated by Brinkman’s model [11] currently used in literature [2,5,6]:

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1 - \varphi)^{2.5}}$$ \hspace{1cm} (14)

### 3. Heat transfer rate

Rayleigh–Bénard (RB) convection represents a fundamental process for heat transfer in fluids. The classical RB configuration corresponds to a horizontal infinite mono-constituent fluid layer heated from below. The linear stability analysis for the case of rigid–rigid infinite horizontal boundary condition shows that the conductive solution loses its stability when the Rayleigh number $R_{nf}$ reaches the critical value: $R_{nc} = 1707.76$ associated with the critical wave number $k_c = 3.116$.

In this study, we focus only on heat transfer by natural convection in the case where the nanofluid is assimilated to mono-constituent fluid with specific thermophysical properties defined above.

To study the heat transfer by natural convection in nanofluids, we introduce the Nusselt number defined by:

$$Nu = \frac{Q_{conv}}{Q_{cond}} = \frac{h\Delta T}{(k\Delta T)/H} = \frac{hH}{k}$$ \hspace{1cm} (15)

Here $H$ is the thickness of the cell, $k$ is the thermal conductivity of the fluid and $h$ is the convection heat transfer coefficient.

In order to investigate the effects of volume fraction $\varphi$ on the onset of convection, the Rayleigh number for nanofluids is expressed as follows:

$$R_{nf} = \frac{(\rho\beta)_{nf} k_f (\rho\text{Cp})_{nf} \mu_f}{(\rho\beta)_f k_s (\rho\text{Cp})_f \mu_f} R_{nf}$$ \hspace{1cm} (16)

The Rayleigh number for this investigation varied from 0 to $10^4$, and the range of the volume fraction $\varphi$ varied between 0% and 8%.
Using Eq. (16) we show that $Ra_{nf} < Ra_{f}$ for all values of $\phi \in [0, 0.08]$ and for water as base fluid with three nanoparticles ($Al_2O_3$, CuO and Cu). The ratio of the nanofluid Rayleigh number to the base fluid Rayleigh number decreases with the volume fraction of nanoparticles. For a fixed value of $Ra_{f}$, the value of $Ra_{nf}$ decreases when adding nanoparticles, thus the onset of convection inside the nanofluid is delayed.

As shown in Eq. (16), in order to evaluate the nanofluid Rayleigh number function of the base fluid one, we need to understand the relations between the density, the thermal conductivity, the specific heat, the viscosity and the thermal expansion coefficient of the nanofluids and the base fluids.

For $Ra_{nf} < 1707.76$, the nanofluid assimilated to a mono-constituent fluid is quiescent and the temperature decreases linearly from $T_1$ to $T_2$.

The heat transfer rate across the fluid layer occurs by pure conduction, therefore $Nu = 1$.

Kim et al. [5] and Hwang et al. [6] showed that the heat transfer rate ratio always increases with the volume fraction of nanoparticles (Fig. 1). We also note, in this figure, that the nanofluid remains in the conductive regime while convection appears in the base fluid. The heat transfer becomes higher than one but remains lower than the ratio $k_{nf}/k_f$ of the two conductivities which is obtained both for the nanofluid and the base fluid in conductive regime. When the ratio $Q_{nf}/Q_f$ becomes higher than 1, the heat transfer rate in the nanofluid becomes higher than that in the base fluid.

Fig. 1 shows the effect of Rayleigh number on the ratio of heat transfer rate for water-based $Al_2O_3$ nanofluid for different values of the volume fraction. This variation of the ratio of heat transfer coefficients has not been observed by any author before.

Kim et al. [5] and Hwang et al. [6] showed that the heat transfer rate ratio always increases with the volume fraction without indicating the influence of the Rayleigh number.

In Fig. 1, we observe that the heat transfer can decrease or increase depending on the value of the Rayleigh number. For a water-based $Al_2O_3$ nanofluid and for $\phi = 0.08$, the ratio of heat transfer rate becomes higher than 1 when the Rayleigh number reaches values around 5000 so we obtain an enhancement of heat transfer only after this value ($Ra_f = 5000$).

So adding nanoparticles increases the heat transfer only for given values of the width cavity and the temperature difference.

### Table 1

<table>
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<th>Grid system</th>
<th>20 × 100</th>
<th>22 × 110</th>
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<td>1.5190</td>
<td>1.5188</td>
<td>1.5186</td>
<td>1.5186</td>
</tr>
</tbody>
</table>
4. Conclusion

In summary, the heat transfer by natural convection inside a horizontal cell heated from below (Rayleigh–Bénard problem) was investigated. We used three nanofluids (water-based \( \text{Al}_2\text{O}_3 \), CuO and Cu). We supposed that nanofluids are mono-constituent fluids. We used the relations giving the specific heat capacity and the thermal expansion coefficient in accordance with the laws of thermodynamics. These relations are different from the ones used by many authors. The two relations used led to significantly different results from the ones obtained by the authors aforementioned.

It is true that the addition of nanoparticles of \( \text{Al}_2\text{O}_3 \), CuO and Cu in water increases its thermal conductivity and therefore improves the conductive heat transfer in the nanofluid compared to conductive heat transfer in the base fluid. Contrary to what many authors mentioned, we showed in this work that, in the classical Rayleigh–Bénard configuration, just after the onset of convection, there is more heat transfer in the base fluid than in the nanofluid. For a fixed value of the fluid Rayleigh number, the nanofluid Rayleigh number decreases with the volume fraction of nanoparticles. Thus the nanoparticles delay the onset of convection.

Recent experimental results obtained by Donzelli et al. [13] showed the important role played by the thermodiffusion in nanofluids with suspension of highly thermophilic nanoparticles. They observed that the first bifurcation, corresponding to the onset of natural convection in Rayleigh–Bénard problem, is not a stationary fork one but oscillatory, similar to that observed in a horizontal porous layer saturated by binary solution heated from below with negative separation ratio [12,14].

So it is necessary to verify if we obtain similar experimental results in the case of commonly used nanofluids such as \( \text{Al}_2\text{O}_3 \)-water, CuO and Cu. Further theoretical and experimental research is necessary in order to be able to use nanofluids for natural convective heat transfer enhancement.

References