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A preliminary design formula for the strength of stiffened curved panels by design of experiment method

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Abstract

In bridge construction, the use of stiffened plates for box-girder or steel beams is common day to day practice. The advantages of the stiffening from the economical and mechanical points of view are unanimously recognized. For curved steel panels, however, applications are more recent and the literature on their mechanical behaviour including the influence of stiffeners is therefore limited. Their design with actual finite element software is possible but significantly time-consuming and this reduces the number of parameters which can be investigated to optimise each panel. The present paper is thus dedicated to the development of a preliminary design formula for the determination of the ultimate strength of stiffened cylindrical steel panels. This approximate formula is developed with help of a design of experiment method which has been adapted from the current statistical knowledge. This method is first presented and its feasibility as well as its efficiency are illustrated through an

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application to the reference case of unstiffened curved panels. Then, the case of stiffened curved panels is investigated and a preliminary design formula is developed. The ease of use of this formula for preliminary design is finally illustrated in a cost optimisation problem.

Keywords: Design of computer experiments, Response surface, Cylindrical curved panels, Stiffeners, Stability, GMNIA.

1. Introduction

The interest of stiffening steel plates or panels to increase their strength under compression has been known for almost a century [1]. In the field of structural engineering, the use of such panels is a common practice, for example in bottom flanges of box-girder bridges. Recent developments of the curving process allowed for the use of curved panels in civil engineering structures where they offer attractive aesthetic and aerodynamic possibilities. The verification of these panels is yet difficult due to a lack of specifications, especially in European Standards: EN 1993-1-5 [2] gives specifications for flat or slightly curved panels with the condition $R \geq R_{lim} = b^2/t_p$ (where R is the curvature radius of the panel, b its width and t_p its thickness) and EN 1993-1-6 [3] deals only with revolution cylindrical shells. Nevertheless the curved panels in bridges have characteristics exactly between these two conditions, as illustrated in the case of the Confluences bridge in Angers, France 2011 (Fig. 1), whose radius $R = 80\text{ m}$ is much smaller than the limit of EN 1993-1-5: $R_{lim} = 1440\text{ m}$ (with $b = 4.8\text{ m}$ and $t = 16\text{ mm}$) and for which EN 1993-1-6 is not applicable neither because these curved flanges are not full revolution cylinders.

Figure 1: Stiffened curved panel of the Confluences Bridge in Angers (France, 2011)

19 From an academic point of view, the articles related to the buckling theory
 20 of curved panels are not so numerous due to the complexity of the studied
 21 problem and also due to its late application in the bridge construction. First
 22 investigations were conducted in the forties by Batdorf & Schildcrout [4] and
 23 Schildcrout & Stein [5] who showed that the stiffeners and the curvature
 24 increase the critical buckling strength. A state of the art on curved stiffened
 25 panels was then proposed by Becker [6] in 1958 in its handbook on structural
 26 stability. Based on experimental results (provided by Gall [7], Lundquist
 27 [1] and Ramberge *et al.* [8]), he confirmed that, when a stiffened flat panel
 28 is bent to a circular curve, its buckling stress is slightly increased (around
 29 6% for the tested specimen which is relatively few compared to the effect of
 30 stiffening alone or curvature alone). More recent parametric studies based
 31 on numerical examples and the finite element modelling (e.g. Cho *et al.* [9],
 32 Khedmati & Edalat [10] or Park *et al.* [11]) investigated and quantified the
 33 influence of the main parameters on the ultimate strength of curved stiffened
 34 plates. They however did not lead to a practical criterion for the evaluation
 35 of the resistance of such panels which is therefore still an open question.

36 In a former study, the authors [12] had investigated the case of unstiff-
 37 ened cylindrical curved panels under axial compression and established a set

of formulas for the evaluation of the ultimate strength (which were confirmed by [13]). These semi-analytic formulas had been fitted on a total of 524 combinations of the main parameters. Each calculus involved Geometrical and Material Non-linearity with Imperfection Analysis (GMNIA) and required between 5 and 10 minutes depending on the refinement of the mesh. Considering the fact that in the case of stiffened panels the number of parameters is considerably larger, re-employing the same methodology seemed unrealistic. It appeared hence that there is a need for a robust strategy for the choice of the set of tested models and for the measure of the approximated model accuracy. Such a strategy exists for the design of physical experiments as well as for that of computer experiments, they are known as "design of experiments methods".

In the following, the authors present thus first the characteristics of computer experiment strategies. Afterwards the feasibility and ease of use of the methodology as well as its efficiency are illustrated through an application to the reference case of unstiffened curved panels. Then, the case of stiffened curved panels is investigated and a preliminary design formula is developed. The interest of this formula for early stages of design is finally illustrated by a short example of cost optimisation.

2. Design of computer experiments

2.1. Background of the design of experiments method

Design of experiment (DOE) methods exist since the beginning of scientific experiment. The first formal theory for the design of experiments in a "modern sense" was published by Fisher [14] in the 1920s and 1930s,

62 while working on improving agricultural yield. Since the 1940s, various re-
63 searchers have promoted and developed the use of experiments strategies in
64 many other areas [15]. In the late 1970s, the theory of Taguchi [16] on qual-
65 ity improvement made the design of experiment widely used in the industrial
66 environment. In the past 20 years, advances in computational power have led
67 to the study of physical process through computer simulated experiments,
68 which tends to replace physical experiments in cases where the number of
69 variables is too large to consider performing a physical experiment or where it
70 is simply economically prohibitive to run an experiment on the scale required
71 to gather sufficient information.

72 Computer experiments differ from traditional physical experiment in their
73 deterministic character, meaning that the computer produces identical an-
74 swers for the same set of experimental parameters. The error in computer
75 experiments is no longer due to random effects which derive from the vari-
76 ability in experimental units, the order of experiments or the locations of
77 the tests. However, it was shown that in many cases, the systematic error
78 between a deterministic model and its approximation has a normal distribu-
79 tion, so that standard statistical techniques can still be applied [17]. Several
80 authors [17, 18, 19] also insisted on the fact that the selection of parameter's
81 values for computer runs is still an experimental design problem of primary
82 importance, especially considering the quantification of uncertainty of the
83 model on a statistical point of view. Indeed, as not every combination of
84 parameters can be tested, uncertainty and hazard enter the deterministic
85 process through the choice of tested combinations. The design of a computer
86 experiment is hence at the border of a physical and a statistical problem

87 which specificities are emphasized in the following section.

88 *2.2. General progress of the design of computer experiment method*

89 Schematically, a numerical model can be considered as a process: the user
90 specifies the combinations of (input) variables to the computer simulator from
91 which the responses (output) are generated. Fig. 2 illustrates this process in
92 the simple case where there are only two input values (X_1 and X_2) and one
93 response Y . Each variable can take a value from “low” to “high”. The set
94 of all domains of variation forms the “region of interest”. In correspondence
95 with each input variable (X_1^i, X_2^i), the computer program will provide one
96 result Y^i . A set of n responses will then generate by extrapolation a response
97 surface. In practice, the explicit formula for this surface is not known. The
98 aim of DOE method is to provide approximated models (response surfaces)
99 that are sufficiently accurate to replace the true response and can be used to
100 facilitate design space exploration, optimisation or reliability analyses.

101 The general steps of computer experiments are generally similar to those
102 encountered in classical experiments [20] and can be summarized as follow:

- 103 • **Step 1:** Statement of the problem.
- 104 • **Step 2:** Choice of the model for the response surface.
- 105 • **Step 3:** Selection of the input data points.
- 106 • **Step 4:** Evaluation of the approximated model.
- 107 • **Step 5:** Validation of the accuracy of the response model.
- 108 • **Step 6:** Selection of most significant terms and conclusion.

Figure 2: Principle of the computer experiment process and response surface.

109 This process is not necessarily linear and could be applied iteratively if the
 110 predicted model does not meet the desired accuracy. In the above process, the
 111 selection of the input variables (**step 3**) and the technique for approximating
 112 the response (**step 4**) are the two main issues that differ between physical
 113 and numerical experiments due to the deterministic property of computer
 114 experiments. These two issues will be developed in the following paragraphs.

115 2.3. Selecting sampling points

116 A good experimental design should minimize the number of runs needed
 117 to acquire information with a given level of accuracy. The experimental
 118 design techniques were initially developed for physical experiments. Due to
 119 the discrepancy associated with physical experimentation, classical DOEs
 120 will focus on parameter settings near the perimeter of the region of interest
 121 and take multiple data points (replicates) as shown in Fig. 3(Left). Computer
 122 experiments are determinists and are not subjected to this necessity. The
 123 objective of computer experiments is hence mainly to uniformly distribute
 124 the sampling points in the region of interest (such a design is called “space
 125 filling”) as seen in Fig. 3(Right).

126 Within the available methods of sampling [21], the following three are
 127 the most common and efficient: the Monte Carlo method (MC), the Latin
 128 Hypercube Sampling method (LHS) and the Quasi-Monte Carlo methods
 129 (QMC) which can be viewed as deterministic versions of MC methods be-
 130 cause they use deterministic points rather than random samples. Blatman
 131 *et al.* [22] showed that QMC overperforms MC and LHS, when used with
 132 polynomial response surfaces with a mean computational gain factor of 10 in
 133 order to reach a given accuracy. The QMC methods are also termed as low
 134 discrepancy procedures: sampling points are selected in such a way that the
 135 error bound is as small as possible.

Figure 3: (Left) “Classical” and (Right) “Space Filling” designs.

136 There are many ways to construct a QMC sequence but the Sobol’ se-
 137 quences are the most widely used because they are quick to construct and fast
 138 to converge [23]. They also have the advantage of preserving the uniformity
 139 of the distribution when the dimension increases: a Sobol’ sequence can be
 140 constructed from a shorter one by adding points to the shorter sequence, on
 141 the contrary to LHS, where the entire sampling process must be run again.

142 2.4. *Response model regression*

143 After selecting the appropriate experimental points and performing the
144 necessary computer runs, the next step is to choose an approximated model
145 and a fitting method. The approximated model must be simple and represent
146 adequately the response of the studied problem. In recent years, a lot of work
147 has been done on approximated models: polynomial response surfaces, neural
148 networks, kriging or multivariate adaptive regression splines [24]. Despite
149 the variety of approximations that is available, comparative studies of these
150 approaches are limited [25]. Depending on the complexity of the problem, one
151 of the aforementioned method might be suitable. However, the polynomial
152 response surface model is by far the simplest; it has been used efficiently in
153 a wide variety of applications and has provided good approximated solutions
154 to even very complex problems [26]. Beside, the use of polynomial response
155 surface for furthers studies such as reliability [27] and optimisation [28] is
156 relatively easy.

157 2.5. *Statistic tools for adequacy checking*

158 As mentioned in the section 2, model adequacy checking is an important
159 part of the data analysis procedure. Indeed it is necessary to ensure that
160 the fitted model provides an adequate approximation of the true system and
161 to verify that none of the model assumptions is violated. In most cases, the
162 regression model is a linear function of some unknown coefficients which are
163 identified thanks to the least square method which will be used here for its
164 simplicity and reliability.

165 3. Application to cylindrical curved panels under uniform axial 166 compression

167 In a former study, the authors [12] investigated the case of unstiffened
168 cylindrical curved panels under axial compression (see Fig. 4) and proposed
169 a set of formulas for the evaluation of the ultimate strength. These formulas
170 were established following the general European Standards procedure for all
171 kind of stability verification and will be used as a reference case to validate
172 the accuracy and relevance of the methodology proposed in previous section.
173 The strength of the panel χ was hence given as a function of the relative
174 slenderness $\bar{\lambda}$ and three parameters $\bar{\lambda}_0$, β , α_Z depending on the relative
175 curvature:

$$\chi = \frac{2\beta}{\beta + \bar{\lambda} + \sqrt{(\beta + \bar{\lambda})^2 - 4\beta(\bar{\lambda} - \alpha_Z(\bar{\lambda} - \bar{\lambda}_0))}} \quad (1)$$

176
177 These simulations, as well as those which will be conducted here, involved
178 non-linear material and second-order analyses with imperfection (GMNIA).
179 They were conducted with the software Ansys version 13 and the standard
180 quadrilateral 4-nodes element [29]. Panels were made of elasto-plastic steel
181 with linear hardening as indicated in EN 1993-1.5 C.6.c) (S355, $E = 210GPa$,
182 $\nu = 0.3$ and a slope of $E/100$). The cylindrical panels were assumed simply
183 supported on all edges and loaded by a uniform longitudinal compression
184 along the curved edges. An initial imperfection with the shape of the first
185 buckling mode and with a maximum amplitude of $1/200^{th}$ of the smallest
186 edge was also added. The study was limited to square panels, so that only
187 the thickness, the width and curvature of the panels were varied.

188 **3.1. Step 1: Statement of the problem**

189 The aim of this step is to identify in an exhaustive manner the parameters
190 of the problem and to select among them the ones which will have an influence
191 on the response and which are liable of variations in practical applications.
192 Here the quantity of interest in the panel response (the output) is the ultimate
193 strength of the panel. Basic structural engineering tells us that it is influenced
194 by the geometry of the panels (including their imperfections), their material
195 properties, their boundary conditions and the nature of the loading. All
196 these parameters could be included in the experimental program, but in this
197 first example, the objective is to validate the method and to illustrate its
198 pertinence, so that the same restrictions as in [12] will be observed:

- 199 • the imperfections are chosen following EN 1993-1-5 [2] (i.e. their shape
200 is that of the first buckling mode and their amplitude is $1/200th$ of the
201 width of the panel)
- 202 • the steel grade is S355 as generally used in modern bridges;
- 203 • the panels are simply supported on all edges;
- 204 • the longitudinal compression is uniform along the curved edges.

205 The only varying input factors are thus the dimensions of the panels:
206 their length a , width b , thickness t_p and radius of curvature R (see Fig. 4).
207 Applying the Buckingham-Vaschy's theorem, it can be demonstrated that
208 the ratio of the ultimate strength of the panel and the yield stress (σ_{ult}/f_y)
209 depends on three independent dimensionless parameters:

$$\frac{\sigma_{ult}}{f_y} = f\left(\frac{a}{b}; \frac{t_p}{b}; \frac{b}{R}\right) \quad (2)$$

Figure 4: Cylindrical curved panel under uniform axial compression (after [12]).

Now that the parameters have been defined, it is essential to define the range in which these parameters will vary. Every feasible configuration has to be included but the range of variation has to be kept as small as possible: it is directly linked with the precision of the approximated expression found at the end of the process. Here the ranges given in table 1 seem reasonable to cover most applications of such panels in bridge engineering.

Table 1: Design variables for unstiffened curved panels

Variable	Description	Variation range	Design variable
a/b	Aspect ratio	$0.6 \leq a/b \leq 1.6$	$X_1 = 2 a/b - 2.2$
t_p/b	Slenderness	$0.01 \leq t_p/b \leq 0.04$	$X_2 = 66.7 t_p/b - 1.67$
b/R	Curvature (Angle)	$0 \leq b/R \leq 1$	$X_3 = 2 b/R - 1$

As the order of magnitude of the variations of these three parameters is different, it is preferable to transform the physical parameters into centred variables X_i , ranging from -1 (low value) to 1 (high value). Their comparative influence on the response will hence be easier to catch. The three adimensional parameters X_1 , X_2 and X_3 used in the coming paragraphs are thus given in the last column of table 1.

223 **3.2. Step 2: Choice of the response surface**

224 The choice of the response surface is based on two issues: the knowledge
 225 of the physics of the problem and the desired accuracy of the approximation.
 226 Here the target response is the ultimate strength of the plate, namely the
 227 maximum load that the plate can bear when accounting for the elasto-plastic
 228 behaviour of the material. From existing standards (EC3), it is known that
 229 the strength of a flat plate is related to the slenderness by a second order
 230 polynomial which was first proposed by Winter [30]:

$$\frac{\sigma_{ult}}{f_y} = \frac{1}{\bar{\lambda}} - \frac{0.22}{\bar{\lambda}^2} \quad (3)$$

231 In (3), the slenderness $\bar{\lambda}$ is directly related to a/b and t/b (which means
 232 to X_1 and X_2) by:

$$\bar{\lambda} = \sqrt{f_y \frac{12(1-\nu^2)}{\pi^2 E}} \cdot \sqrt{\frac{1}{k_{a/b}} \cdot \frac{b}{t_p}} \quad (4)$$

233 where $k_{a/b}$ is a function of a/b

$$k_{a/b} = \begin{cases} \left(\frac{a}{b} + \frac{b}{a}\right)^2 & \text{if } \frac{a}{b} \leq 1 \\ 4 & \text{if } \frac{a}{b} \geq 1 \end{cases} \quad (5)$$

234 It can thus be concluded that a second order polynomial should provide
 235 a good approximation of the strength of a curved plate and that it will be
 236 meaningful from a physical point of view. The response surface will thus be
 237 investigated in the following form:

$$\hat{Y} \left(= \frac{\sigma_{ult}}{f_y} \right) = \beta_0 + \sum_{i=1}^3 \beta_i X_i + \sum_{j=1}^3 \sum_{i=1}^j \beta_{ij} X_i X_j \quad (6)$$

238 where $\hat{Y}$ is the approximated response, X_i are the three input variables and
 239 $\beta_{(.)}$ are the ten unknown parameters.

240 *3.3. Step 3: Selection of the input data or sampling points*

241 The selection of the input data points covers the choice of the number
242 of points and of their distribution in the investigated domain which is here
243 a hypercube in the three dimensional space. The generation of a set of
244 sampling points using a QMC method in this cube can be made easily using
245 a common statistical tool providing a function generating a Sobol' sequence
246 (Matlab here). Noting that with the Sobol' sequence, the extreme values
247 of the parameters can only be reached for an infinite number of variables,
248 some additional points located at the corners of the cube can be added to
249 the sequence to give more weight to the boundary of the domain.

250 The key issue is thus the definition of the minimal number of experiments
251 to be conducted to get a response with the desired accuracy. The number of
252 simulations n depends on the complexity of the studied phenomenon as well
253 as of the complexity of the approximated model. Yet there is not a unani-
254 mously agreed method relating the number of observations versus the number
255 of independent variables in the model. Some authors suggest $3m + 1$ points
256 [31] for a second-order polynomial approximation where $m = \frac{(p+1)(p+2)}{2}$ is the
257 number of unknown coefficients and p is the number of input variables. Fol-
258 lowing this suggestion for the present example which has 3 input parameters,
259 a second order polynomial approximation will have 10 unknown parameters
260 which could be evaluated with a good accuracy with 31 experiments. Adding
261 the corner points (in total 7 additional points as the point $(-1; -1; -1)$ is
262 by construction always part of the sequence) of the investigated domain, the
263 total number of sampling points is set to 38. An illustration of such a set is
264 shown in Fig. 5.

Figure 5: Sampling points generated by the Sobol' sequence (circles) and additional corner points (triangles).

3.4. **Step 4:** Parameters evaluation

The approximated response surface is here looked for in the form of a second-order polynomial in the three variables X_i given by Eq. 6. The unknown parameters $\beta_{(.)}$ have to be identified from the numerical experiments (here $n = 38$) which is here done by the least square method. So, from the 38 sampling points shown in Fig. 5, the ultimate strength of a curved plate can be approximated by the following expression:

$$\begin{aligned} \hat{Y} = & 0.879 + 0.002X_1 + 0.212X_2 + 0.052X_3 - 0.001X_1X_2 - 0.063X_2X_3 \\ & - 0.004X_3X_1 - 0.037X_1^2 - 0.100X_2^2 - 0.003X_3^2 \end{aligned} \quad (7)$$

3.5. **Step 5:** Evaluation of the accuracy of approximated model

To evaluate the accuracy of the approximated model, conducting an analysis of variance (ANOVA) is very useful. The coefficient of determination is first determined $R^2 = 0.977$ and then the cross-validation coefficient $Q^2 = 0.969$. The fact that R^2 is very close to 1 indicates that the regression

277 model fits well the data. The small difference between R^2 and Q^2 indicates
 278 that most observations have an influence on the regression equation and that
 279 the approximation model predicts well the observations. Moreover, the same
 280 model has been identified on a sample without additional points in the cor-
 281 ners, leading to $R^2 = 0.971$ and $Q^2 = 0.942$, the small diminution of these
 282 coefficients is a direct consequence of the diminution of the number of sam-
 283 pling points (31 instead of 38), not to the fact that the discarded points were
 284 located in the corners of the domain. It is thus concluded that, the addi-
 285 tion of corner points is not necessary to get an accurate estimation of the
 286 regression coefficients.

287 *3.6. Step 6: Selection of most significant terms and conclusion.*

288 The second-order formula presented in Eq. 7 for the evaluation of the
 289 ultimate strength of curved steel panels under axial compression provides
 290 a good and best possible approximation of the real capacity of the panel.
 291 However, it is remarked that not every coefficient in Eq. 7 have the same
 292 order of magnitude. So, rather than trying to explain the model with all its
 293 terms, it can naturally be asked if some terms could be excluded from the
 294 initial model without altering significantly the accuracy of the whole model.

295 A criterion of exclusion should hence be fixed. If normality assumptions
 296 are verified (as in the present case), the t-test provides a fully reliable criterion
 297 as it relates the value of each coefficient to its estimated standard error.
 298 More simple criteria, such as arbitrary thresholds of significance are also
 299 very effective. Indeed, as the parameters all vary between -1 and 1, the
 300 contributions of the various terms can directly be analysed by comparing the
 301 coefficients which might then be neglected if their value is bellow a certain

absolute value (for example 2% of the sum of the coefficients absolute values or 5% of the maximum value of the coefficients). Fixing here this threshold to 2 % of the sum of the coefficients, the terms ($\beta_1, \beta_{12}, \beta_{13}$ and β_{33}) are found not significant (a criterion based on t-test and a 90% two sided interval would give the same results). The new model is thus given by Eq. 8; it preserves good precision with high value of $R^2 = 0.976$ and $Q^2 = 0.9674$.

$$\hat{Y} = 0.879 + 0.212X_2 + 0.052X_3 - 0.063X_2X_3 - 0.037X_1^2 - 0.100X_2^2 \quad (8)$$

Introducing the physical parameters of table 1 into Eq. 8, the ultimate strength of unstiffened cylindrical curved panels under axial compression is given by:

$$\frac{\sigma_{ult}}{f_y} = (-0.09 + 0.326(a/b) - 0.148(a/b)^2) + (40.6 + 0.314Z)(t_p/b) - (444 + 8.40Z)(t_p/b)^2 \quad (9)$$

where Z is the curvature parameter defined by $Z = b^2/Rt_p$. Eq. 9 is very similar to the classical expression of the stability problem, where the ultimate strength is represented as a polynomial function of the slenderness t/b .

Fig. 6 shows how well the expressions given by the DOE method (red squares) and by the semi-analytical method [12] (green triangles) are able to predict the numerical results (given by F.E. model). For most input values, the two models have less than 5 % of discrepancy (in absolute value) to the true numerical value. However a few observations (No. 19, No. 22 and No. 28) predicted by DOE method have higher discrepancy (from 8 % to 10 %) on the contrary to the semi-analytical method whose error remain below 5.5 %. This might be explained by the fact that, although the semi-analytical model is not richer (the calibration of $\overline{\lambda}_0, \beta, \alpha_Z$ involves only 7

parameters), its physical bases are finer which proves the crucial importance
of the choice of the parameters and response surface.

Figure 6: Comparison of FEM results with DoE and semi-analytical method [12].

These values of error should be relativised by the fact that in many complex structures such as stiffened plates, the difference between Eurocode predictions and the results of numerical simulations might reach until 20 %, sometimes in favor of safety, sometimes not [32, 33]. Moreover, as the so-called characteristic value of a member is obtained by dividing its design value (e.g. Eq. 9) by some safety factor (often taken as $\gamma_{M1} = 1.1$) the present discrepancy is indeed acceptable.

332 4. Application to stiffened curved panels under uniform axial com- 333 pression

334 The behaviour of stiffened curved panels is a more complex problem,
335 especially due to the interaction of different parameters (curvature, relative
336 rigidity of stiffeners and plate, imperfection, etc.) for which no semi-analytic
337 expression exists. So, as it has just been shown that the design of computer
338 experiment method is well adapted for studying the stability of curved plates,
339 it will be used for the development of a preliminary design formula (i.e suited
340 for hand-calculation) for the ultimate resistance of stiffened curved panels.

341 4.1. Finite element modelling

342 The stiffened panels are modelled and analysed using the commercial
343 finite element software Ansys [29]. The panels are supposed to be simply
344 supported on all edges of the panel ($u_r = 0$ in the cylinder coordinate system
345 of Fig. 7) but not on the stiffeners (unfavourable condition).

Figure 7: Boundary conditions of simply supported on all edges

346 For loading conditions, the study is here limited to a uniform compression
 347 in the longitudinal direction as it is the dominant loading in bottom flange
 348 panels. It is applied not only to the main panel, but also to the stiffeners due
 349 to their participation in the overall behaviour of the structure (Fig. 8). In
 350 fact, in a bridge, the compressive forces acting on the flange come through
 351 the diaphragms and webs that connect the upper and lower panels of the
 352 box girder. By construction, the stiffeners, in most cases, are continuous and
 353 attached by welding to diaphragms: therefore they are also subjected to the
 354 compressive load.

Figure 8: Loading condition and scheme of the connection stiffener/diaphragm by welding

355 The curved panels are meshed with eight-nodes shell elements which use
 356 an advanced shell formulation that accurately incorporates initial curvature
 357 effects (this element is called SHELL-281 in [29]). They are well-suited for
 358 linear, large rotation and large strain non-linear applications and offers im-
 359 proved accuracy in curved shell structure simulations and faster convergence
 360 than plate elements as one can see in figure 9 which represents the conver-
 361 gence study from [34]. A fine mesh with more than 30 elements per panel
 362 edges is used to reduce the discretisation error.

Figure 9: Convergence study of SHELL-281 element: linear bifurcation analysis (left), non linear buckling analysis (right)

363 The panels are all made of steel which is assumed to be elasto-plastic
 364 with linear strain hardening as indicated in EN 1993-1-5 C.6 for the material
 365 non-linear second-order analyses with initial imperfections (GMNIA). The
 366 Young modulus E and Poisson's ratio are taken equal to 210 GPa and 0.3
 367 respectively. The steel grade is S355 with a yield steels equal to 355 MPa .

368 4.2. Evaluation of the ultimate strength

369 This study is limited to the case of stiffened curved panels under ax-
 370 ial compression with open section stiffeners (simple flat plates) because the
 371 curvature makes it difficult to realise a close form section of stiffener (boxed
 372 rib). Therefore, the number of input parameters is here restricted to seven as
 373 presented in table 2. The ranges of variation of these parameters are chosen
 374 in order to cover most panels used in bridge construction. As the orders of
 375 magnitude of the parameters variations are different, they are transformed
 376 the physical parameters into centred variables X_i , ranging from -1 to 1 .

377

Table 2: Design variable (dimension in meter)

Variable	Description	Variation range	Design variable
a	Length of the panel	$4 \leq a \leq 6$	$X_1 = a - 5$
b	Width of the panel	$4 \leq b \leq 6$	$X_2 = b - 5$
t_p	Thickness of panel	$0.01 \leq t_p \leq 0.02$	$X_3 = 200 \cdot t_p - 3$
$1/R$	Curvature of panel	$0 \leq 1/R \leq 0.1$	$X_4 = 20/R - 1$
d	Distance between stiffeners	$0.3 \leq d \leq 0.8$	$X_5 = 4 \cdot d - 2.2$
h_s	Height of stiffener	$0.1 \leq h_s \leq 0.2$	$X_6 = 20 \cdot h_s - 3$
t_s	Thickness of stiffener	$0.01 \leq t_s \leq 0.02$	$X_7 = 200 \cdot t_s - 3$

378 The approximated model is searched in the form of a second order poly-
 379 nomial. The total number of experiments, as suggested in section 3.3, is
 380 $n = 3 \cdot m + 1 = 109$ where $m = 36$ is the number of unknown coefficients
 381 (1 constant, 7 linear and 28 quadratic terms). Their distribution in the re-
 382 gion of interest is generated by a Sobol' sequences and the coefficients are
 383 obtained by the least square method, supposing that errors are independent
 384 and normally distributed. Like previously, the selection of significant coeffi-
 385 cients is made based on the t-test (the limit value being given for 109 tests
 386 and the bilateral 5%-95% fractile). Then all remaining coefficients (here 18
 387 coefficients) are re-evaluated using the least square method a second time.
 388 The resulting approximated model (in MN) is thus the following:

$$\begin{aligned}
 \hat{Y} = & +17.09 \\
 & -0.47X_1 + 3.58X_2 + 4.24X_3 + 7.32X_4 - 3.87X_5 + 4.83X_6 + 2.33X_7 \\
 & +1.65X_2X_4 - 1.72X_2X_5 + 1.71X_2X_6 + 0.89X_2X_7 + 1.33X_3X_4 \\
 & -0.76X_3X_5 - 1.73X_4X_5 + 0.81X_4X_6 - 1.18X_5X_6 + 0.94X_6X_7
 \end{aligned} \tag{10}$$

389 The coefficient of determination $R^2 = 0.986$ (close to 1) indicates that
 390 the value predicted by the model fits very well the data (see Fig. 10). Also,
 391 the small difference between the values of $Q^2 = 0.978$ and R^2 shows that
 392 there are few undue influence on the regression equation. The accuracy of
 393 the response is also checked by the relative mean absolute error (RMAE):
 394 4.2 % (which indicated that a mean error of 4 % is expected), the relative
 395 minimum and maximum error: -14.0 % and $+13.5$ % respectively. The
 396 above equation is thus fairly acceptable in bridge constructions and residual
 397 errors can be easily covered by using a safety factor.

Figure 10: Comparison of the ultimate strength of the FEM and DOE

398 Considering now every coefficients independently, it is found that the 5 %
 399 two-sided confidence interval of the regression coefficients is ± 0.20 *MN* for
 400 the constant term, ± 0.36 *MN* for the linear terms and ± 0.62 *MN* for the
 401 quadratic terms. As generally observed, the uncertainty on linear terms is
 402 almost twice smaller than that on quadratic terms: the direct influences of
 403 the parameters are better known than those of their interactions.

404 These statistical remarks being made, it is remarkable that all parameters
 405 are found significant in Eq. 10. In decreasing order, the most significant
 406 parameters are the curvature (X_4), the height of the stiffeners (X_6), the
 407 thickness of the panel (X_3), the distance between stiffeners (X_5), the width
 408 of the panel (X_2), the thickness of the stiffeners (X_7) and finally the length
 409 of the panel (X_1) whose influence is very limited (no more than ± 3 % of the
 410 total strength). Quite obviously, increasing the curvature, the thickness of
 411 the plate or the height and thickness of the stiffeners increases the strength
 412 of the panel, while increasing the length of the panel or the distance between
 413 stiffeners decreases it. Then the fact that the strength grows with the width
 414 is not so immediate but can be easily understood considering that when the
 415 width of the panel increases, the distance between the centre of gravity of
 416 the panel and the curved plate increases due to curvature and by there the
 417 global inertia of the curved panel increases.

418 There are then multiple interactions which combine effects are more dif-
 419 ficult to analyse. Indeed increasing the curvature, the height and thickness
 420 of the stiffeners or the thickness of the panel has always a positive effect
 421 on the strength because, for these parameters, the linear term dominates
 422 clearly the quadratic terms. Then concerning the distance between the stiff-
 423 eners, in most cases diminishing it leads to an increase of the strength but
 424 not mandatory as for slender panels with small curvature and small stiff-
 425 eners it might lead to a smaller strength (indeed the coefficient of X_5 is
 426 $-3.87 - 1.72X_2 - 0.76X_3 - 1.73X_4 - 1.18X_6$ and varies between $-9.26 MN$
 427 and $1.52 MN$). In the same way, in most cases increasing the width of the
 428 panel leads to an increased strength but not for slender panels when the spac-

ing between stiffeners is too large (the coefficient of X_2 being $3.58 + 1.65X_4 - 1.72X_5 + 1.71X_6 + 0.89X_7$, it varies between $-2.39 MN$ and $9.55 MN$). Very likely in the last two cases, these changes of the coefficient sign correspond to a change in the buckling mode from column to plate or from global to local.

Anyway, it must be recalled that even in these extreme cases, the error in the prediction of the strength is not larger than in other cases (cf. Fig. 10) and that according to all statistical criteria mentioned above, the approximated model given by expression (10)) is able to predict correctly the ultimate strength of stiffened curved panels in the interested domain. It can thus be easily inserted in an optimisation pattern as illustrated in the coming section.

4.3. Cost optimisation of curved stiffened panels

The cost optimisation scheme proposed here is based on a cost objective function similar to the ones used by [35]. It assumes that the manufacturing cost of a stiffened curved panel K defined by the parameters X_i is the sum of the material costs K_m (the steel cost) and of the fabrication costs K_f which can be defined as follow:

$$K(X_i) = K_m + K_f = k_m \rho V + k_f \sum T_i \quad (11)$$

where ρ is the steel density, V is the total volume of the curved panel, k_m and k_f are characteristic coefficients of material and fabrication costs. T_i denotes manufacturing times:

- T_1 : time for preparing, cutting and assembling the pieces:

$$T_1 = \Theta_d \sqrt{\kappa \rho V} \quad (12)$$

with Θ_d a factor characterising the impediment for welding and κ the number of elementary pieces to be welded;

451 • T_2 : time for welding and T_3 : additional time for maintenance of the
 452 machine which might be considered as $0.3T_2$, so that:

$$T_2 + T_3 = 1.3 \sum C_i a_{wi}^2 L_{wi} \quad (13)$$

453 where L_{wi} is the length of the i^{th} weld, $a_{wi} = \max(0.4t_s, 4mm)$ its width
 454 and C_i a coefficient depending on the welding technique which is here
 455 taken equal to 0.2349 for Shielded Metal Arc Welding.

456 The constraint equation is then given by the stability requirement of the
 457 panel:

$$g(X_i) = \frac{N_{app}}{N_{ult}/\gamma_{M1}} - 1 \leq 0 \quad (14)$$

458 where N_{app} is the applied load, N_{ult} the capacity of the panel estimated by
 459 Eq. (10) and γ_{M1} is a safety factor.

460 The panel which is proposed here for optimisation has fixed overall di-
 461 mensions: its length a is 6 m , its width b is 4 m and its curvature radius
 462 R is 20 m). It is subjected to a uniform axial compression $N_{app} = 12 \text{ MN}$.
 463 The objective is thus to determine the parameters (thickness of the panel t_p ,
 464 thickness t_s and height h_s of the stiffeners and distance between stiffeners d)
 465 which will minimize the cost of the panel (11) and verify the constraint equa-
 466 tion (14). To make the problem more realistic, it is also considered that the
 467 variables are not continuous but discrete (which poses no problem to Matlab
 468 optimisation algorithm), so that the solution is looked for in the following
 469 domain:

- 470 • $t_p \in [0.01; 0.02]$ by steps of 1 mm ;
- 471 • $d \in [0.3; 0.8]$ by steps of 5 cm ;

472 • $h_s \in [0.1; 0.2]$ by steps of 1 *cm*;

473 • $t_s \in [0.01; 0.02]$ by steps of 1 *mm*;

474 Concerning then the definition of cost coefficients, as no precise data were
 475 available for k_f and k_m , it was decided to present the results in an adimen-
 476 sional form considering different values of the ratio k_f/k_m . For $k_f/k_m = 0$,
 477 only material cost is taken into account, while for large values of k_f/k_m , man-
 478 ufacturing cost prevail (reasonable values in northern countries lie between
 479 1 and 2). The results of the optimisation procedure are shown in table 4.3
 480 (where n is the total number of stiffeners).

Table 3: Results of the optimisation procedure for $N_{app} = 12$ *MN*

k_f/k_m	t_p	d	n	h_s	t_s	K
0.0	0.014	0.55	7	0.16	0.015	3450
0.5	0.015	0.75	5	0.18	0.016	4250
1.0	0.016	0.90	4	0.19	0.017	5000
2.0	0.018	0.90	4	0.17	0.015	6200

481 About the method first, it must be noticed here that the set of optimised
 482 parameters corresponding to each ratio k_f/k_m was obtained almost imme-
 483 diately thanks to the preliminary design formula developed in section 4.2
 484 whereas it would have taken hours or even days using directly finite element
 485 simulations. Concerning the results then, following remarks can be drawn:

486 • The number of stiffeners is higher when only material costs are con-
 487 sidered. It is however not maximal ($n_{max} = 12$) which shows that

488 increasing reasonably the thickness of the plate is very efficient from a
489 weight point of view.

- 490 • For higher values of k_f/k_m , stiffeners becomes logically stiffer to reduce
491 their number and the number of welds.
- 492 • It is often more economical to increase the panel thickness than to
493 increase the number of stiffeners which confirms the conclusion of [33].

494 5. Conclusion

495 Stiffened curved panels in civil engineering structures have high sensitivity
496 to instability phenomenon. Analytical or semi-analytical studies are often
497 not feasible as the problem depends on many parameters such as the panel's
498 curvature or the panel configuration with its stiffener and semi-rigid supports.
499 There is hence a need for a robust strategy when attempting to develop
500 approximated models for such problems. The proposition of such a strategy
501 was the aim of the first part of the present paper and this, through a turnkey
502 methodology based on the theory of the design of experiment method. The
503 efficiency of the method was first reviewed. Some particular points which
504 differentiate the ordinary physical experiments from computer experiments
505 were discussed. Afterwards this methodology was applied to the case of
506 unstiffened curved panels for which solutions were already available in order
507 to evaluate the accuracy of the method and its relevance. A huge gain of time
508 was noticed when using the DOE method: only 38 simulations were needed
509 in the first application against 524 observations in [12] for determining the
510 capacity of a curved panel. Also the general accuracy of the model in the

511 form of a second-order polynomial was comparable to that obtained with
512 more standard heuristic methods. Moreover, as the experiment designer had
513 existing knowledge of the problem, the input values were adequately chosen
514 and the physical interpretation of the results was easy and satisfying, despite
515 the simplicity of the model. The strategy proposed here provides thus a
516 reliable alternative method for the prediction of the ultimate strength of
517 curved panels.

518 Confident in the methodology, the authors then developed a fully reliable
519 formula for preliminary design of stiffened curved panels. The accuracy of
520 the formula was demonstrated and the influence of various design parameters
521 was discussed. A simple cost optimisation problem was finally presented to
522 illustrate the potential of the formula.

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527 **Nomenclature**

528	α_Z	Parameter characterising the imperfections sensitivity
529	β	Parameter characterising the asymptotic behaviour of the panel
530	β_0	Constant term and average value of the approximated response
531	β_i	Coefficient characterising the effect of the variable X_i
532	β_{ij}	Coefficient characterising the interaction of the variables X_i and X_j
533	χ	Reduction factor for the panel buckling according to EC3
534	$\bar{\lambda}$	Relative slenderness of the panel according to EC3
535	$\bar{\lambda}_0$	Slenderness separating plastic buckling from elasto-plastic buckling
536	ρ	Steel density
537	σ_{ult}	Ultimate strength of the panel
538	a	Length of the panel
539	b	Width of the panel
540	d	Distance between stiffeners
541	f_y	Yield stress of the panel
542	h_s	Height of stiffeners
543	K_f	Fabrication costs
544	k_f	Fabrication cost per volume unit

545	K_m	Material costs (steel cost)
546	k_m	Material cost per volume unit
547	$k_{a/b}$	Parameter characterising the influence of the aspect ratio
548	m	Number of unknown coefficients in the approximated model
549	n	Number of simulations or numerical experiments
550	N_{app}	Normal force applied to the panel
551	N_{ult}	Capacity of the panel
552	p	Number of input variables
553	R	Curvature radius of the panel
554	T_i	Manufacturing time of the i^{th} operation
555	t_p	Thickness of the panel
556	t_s	thickness of stiffeners
557	V	Total volume of the curved panel
558	X_i	Generic name of the i^{th} input variable
559	X_i^j	j^{th} value of the i^{th} input variable
560	$Y, \hat{Y}$	Response and approximated response
561	Y^j	j^{th} value of the response
562	Z	Curvature parameter defined by $Z = b^2/Rt_p$

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565 sive strength of flat and slightly curved sheet and stiffener combinations.
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