Asymptotic delta-Parametrization of Surface-impedance Solutions

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Abstract—The surface impedance methods are among the most efficient for solving time-harmonic eddy-current problems with a small penetration depth. When the solution is required for a wide range of frequencies (or material conductivities) the standard approach leads to the solution of a complex-valued problem for each frequency (or conductivity). Hereafter we introduce a close method, parametrized by the skin depth (δ), based on a formal asymptotic expansion. It provides accurate results with a reduced computational cost for a wide range of δ values.

Index Terms—Surface impedance, asymptotics, parametric solutions.

I. SURFACE IMPEDANCES

The classical surface impedance method allows to solve approximately and quite accurately a time-harmonic eddy-current problem in a conductor (with a linear magnetic behavior), when the skin depth δ is small compared to the characteristic size D of the conducting parts of the device under study [1]. If the boundary Σ of the conductor is regular enough, one can compute the electromagnetic field in the outer domain Ω by imposing a surface impedance condition on Σ:

\[
\text{curl } H = J_s \text{ in } \Omega, \\
n \times E = Z_s n \times (n \times H) \text{ on } \Sigma, \\
Z_s = 1 + j \frac{1}{\sigma \delta},
\]

with H the magnetic field, J_s the source current density, E the electric field, n the outward normal, j the imaginary unit and Z_s the so-called surface impedance which depends on the electric conductivity σ and δ. The finite element solution is straightforward, e.g. in a 2D plane case, the vector potential (A) formulation gives (A and J with only one component):

\[
- \Delta A = \mu_0 J_s \text{ in } \Omega; \quad A = \alpha \delta \partial_n A \text{ on } \Sigma; \quad \alpha = \frac{j - 1}{2}.
\]

If the frequency (or conductivity) is modified, the solution has to be performed again.

II. ASYMPTOTIC EXPANSION AND PARAMETRIZATION

The solution to Problem (4) can be expanded in a formal series in power of αδ as in [2]:

\[
A = \sum_{i \geq 0} (\alpha \delta)^i A_i,
\]

with the coefficients are real-valued solutions to elementary problems (6)–(7) independent of δ:

\[
- \Delta A_i = \mu_0 J_s \text{ in } \Omega; \quad A_0 = 0 \text{ on } \Sigma, \quad \forall i \geq 1, - \Delta A_i = 0 \text{ in } \Omega; \quad A_i = \partial_n A_{i-1} \text{ on } \Sigma.
\]

In practice, the computation of only 2, or 3 terms (A_0, A_1, A_2) suffices to ensure high accuracy (see Section III). Furthermore, the solution (4) for any new small value δ can be simply reconstructed by combining linearly the pre-computed terms as in (5); what amounts to a considerable gain in computational time for sensitivity or parametric studies.

III. NUMERICAL EXAMPLE

A simple test case is represented in Fig. 1. We enforce a flux at part of the boundary Σ of a conducting angle. The first term of (5) is depicted in Fig. 1, left. The flux through segment MN, Φ = A(M) − A(N), is shown as a function of δ/D and compared to the exact solution in Fig. 1, right. The approximate flux is observed to be accurate for δ/D < 15%.

At the conference, we will detail how we compute the 3 first orders based on 2 solutions with surface impedance for 2 distinct values of δ, δ_1 and δ_2. The 3 first orders seem to provide an “accurate” behavior; see Fig. 1, right. We will also discuss error estimates, the case of a linear magnetic conductor and the 3D formulations.

REFERENCES