Approximate conditions replacing thin layers
Laurent Krähenbühl, Patrick Dular, Victor Péron, Ronan Perrussel, Clair Poignard, Ruth V. Sabariego

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Asymptotic delta-Parametrization of Surface-Impedance Solutions

Laurent Krähenbühl\textsuperscript{1}, Patrick Dular\textsuperscript{2}, Victor Péron\textsuperscript{3} Ronan Perrussel\textsuperscript{4}, Claire Poignard\textsuperscript{5}, Ruth Sabariego\textsuperscript{6}

\textsuperscript{1}Université de Lyon – Ampère (CNRS, ECL), Écully, France
\textsuperscript{2}University of Liège, Belgium, \textsuperscript{3}Université de Pau, France, \textsuperscript{4}Université de Toulouse, France,
\textsuperscript{5}Inria Bordeaux-Sud Ouest, EPC MC2, France, \textsuperscript{6}KU Leuven, Research Institute EnergyVille, Belgium.
laurent.krahenbuhl@ec-lyon.fr

Abstract—The surface impedance methods are among the most efficient for solving time-harmonic eddy-current problems with a small penetration depth. When the solution is required for a wide range of frequencies (or material conductivities) the standard approach leads to the solution of a complex-valued problem for each frequency (or conductivity). Hereafter we introduce a close method, parametrized by the skin depth ($\delta$), based on a formal asymptotic expansion. It provides accurate results with a reduced computational cost for a wide range of $\delta$ values.

Index Terms—Surface impedance, asymptotics, parametric solutions.

I. SURFACE IMPEDANCES

The classical surface impedance method allows to solve approximately and quite accurately a time-harmonic eddy-current problem in a conductor (with a linear magnetic behavior), when the skin depth $\delta$ is small compared to the characteristic size $D$ of the conducting parts of the device under study [1]. If the boundary $\Sigma$ of the conductor is regular enough, one can compute the electromagnetic field in the outer domain $\Omega$ by imposing a surface impedance condition on $\Sigma$:

\begin{equation}
\mathbf{curl} \mathbf{H} = \mathbf{J}_s \text{ in } \Omega, \quad \mathbf{n} \times \mathbf{E} = Z_s \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \text{ on } \Sigma, \quad Z_s = \frac{1 + j}{\sigma \delta},
\end{equation}

with $\mathbf{H}$ the magnetic field, $\mathbf{J}_s$ the source current density, $\mathbf{E}$ the electric field, $\mathbf{n}$ the outward normal, $j$ the imaginary unit and $Z_s$ the so-called surface impedance which depends on the electric conductivity $\sigma$ and $\delta$. The finite element solution is straightforward, e.g. in a 2D plane case, the vector potential $(A)$ formulation gives $(A$ and $J$ with one component):

\begin{equation}
-\Delta A = \mu_0 J_s \text{ in } \Omega; \quad A = \alpha \delta \partial_n A \text{ on } \Sigma; \quad \alpha = \frac{j - 1}{2}.
\end{equation}

If the frequency (or conductivity) is modified, the solution has to be performed again.

II. ASYMPTOTIC EXPANSION AND PARAMETRIZATION

The solution to Problem (4) can be expanded in a formal series in power of $\alpha \delta$ as in [2]:

\begin{equation}
A = \sum_{i \geq 0} (\alpha \delta)^i A_i,
\end{equation}

where the coefficients are real-valued solutions to elementary problems (6)–(7) independent of $\delta$:

\begin{equation}
-\Delta A_0 = \mu_0 J_s \text{ in } \Omega; \quad A_0 = 0 \text{ on } \Sigma. \quad (6)
\end{equation}

\begin{equation}
\forall i \geq 1, -\Delta A_i = 0 \text{ in } \Omega; \quad A_i = \partial_n A_{i-1} \text{ on } \Sigma. \quad (7)
\end{equation}

In practice, the computation of only 2, or 3 terms $(A_0, A_1, A_2)$ suffices to ensure high accuracy (see Section III). Furthermore, the solution (4) for any new small value $\delta$ can be simply reconstructed by combining linearly the pre-computed terms as in (5); what amounts to a considerable gain in computational time for sensitivity or parametric studies.

III. NUMERICAL EXAMPLE

A simple test case is represented in Fig. 1. We enforce a flux at part of the boundary $\Sigma$ of a conducting angle. The first term of (5) is depicted in Fig. 1, left. The flux through segment $MN$, $\Phi = A(M) - A(N)$, is shown as a function of $\delta$/$D$ and compared to the exact solution in Fig. 1, right. The approximate flux is observed to be accurate for $\delta$/$D < 15\%$.

At the conference, we will detail how we compute the 3 first orders based on 2 solutions with surface impedance for 2 distinct values of $\delta$, $\delta_1$ and $\delta_2$. The 3 first orders seem to provide an “accurate” behavior; see Fig. 1, right. We will also discuss error estimates, the case of a linear magnetic conductor and the 3D formulations.

REFERENCES