Approximate conditions replacing thin layers

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Asymptotic delta-Parametrization of Surface-Impedance Solutions

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***Abstract*—The surface impedance methods are among the most efficient for solving time-harmonic eddy-current problems with a small penetration depth. When the solution is required for a wide range of frequencies (or material conductivities) the standard approach leads to the solution of a complex-valued problem for each frequency (or conductivity). Hereafter we introduce a close method, parametrized by the skin depth (δ), based on a formal asymptotic expansion. It provides accurate results with a reduced computational cost for a wide range of δ values.**

***Index Terms*—Surface impedance, asymptotics, parametric solutions.**

I. Surface impedances

The classical surface impedance method allows to solve approximately and quite accurately a time-harmonic eddy-current problem in a conductor (with a linear magnetic behavior), when the skin depth δ is small compared to the characteristic size D of the conducting parts of the device under study [1]. If the boundary Σ of the conductor is regular enough, one can compute the electromagnetic field in the outer domain Ω by imposing a surface impedance condition on Σ:

$$\mathbf{curl}\,\mathbf{H} = \mathbf{J}_s \text{ in } \Omega, \tag{1}$$

$$\mathbf{n} \times \mathbf{E} = Z_s \mathbf{n} \times (\mathbf{n} \times \mathbf{H}) \text{ on } \Sigma, \tag{2}$$

$$Z_s = \frac{1+\mathrm{j}}{\sigma\delta}, \tag{3}$$

with $\mathbf{H}$ the magnetic field, $\mathbf{J}_s$ the source current density, $\mathbf{E}$ the electric field, $\mathbf{n}$ the outward normal, j the imaginary unit and Z_s the so-called surface impedance that depends on the electric conductivity σ and δ. The finite element solution is straightforward, e.g. in a 2D plane case, the vector potential (A) formulation gives (A and J with only one component):

$$-\Delta A = \mu_0 J_s \text{ in } \Omega;\ A = \alpha\,\delta\,\partial_{\mathbf{n}} A \text{ on } \Sigma;\ \alpha = \frac{\mathrm{j}-1}{2}. \tag{4}$$

If the frequency (or conductivity) is modified, the solution has to be performed again.

II. Asymptotic expansion and parametrization

The solution to Problem (4) can be expanded in a formal series in power of $\alpha\delta$ as in [2]:

$$A = \sum_{i \geqslant 0} (\alpha\delta)^i A_i, \tag{5}$$

where the coefficients are real-valued solutions to elementary problems (6)–(7) independent of δ:

$$-\Delta A_0 = \mu_0 J_s \text{ in } \Omega; \quad A_0 = 0 \text{ on } \Sigma. \tag{6}$$

$$\forall i \geqslant 1,\ -\Delta A_i = 0 \quad \text{ in } \Omega; \quad A_i = \partial_{\mathbf{n}} A_{i-1} \text{ on } \Sigma. \tag{7}$$

In practice, the computation of only 2, or 3 terms (A_0, A_1, A_2) suffices to ensure high accuracy (see Section III). Furthermore, the solution (4) for any new small value δ can be simply reconstructed by combining linearly the pre-computed terms as in (5); what amounts to a considerable gain in computational time for sensitivity or parametric studies.

III. Numerical example

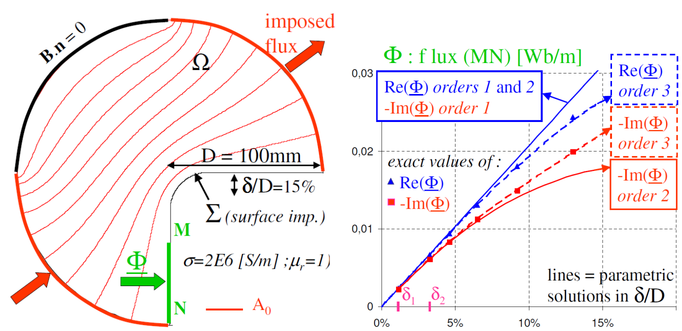

Fig. 1. Domain Ω with A_0 on the left. Flux vs. δ/D on the right.

A simple test case is represented in Fig. 1. We enforce a flux at part of the boundary Σ of a conducting angle. The first term of (5) is depicted in Fig. 1, left. The flux through segment MN, $\Phi = A(M) - A(N)$, is shown as a function of δ/D and compared to the exact solution in Fig. 1, right. The approximate flux is observed to be accurate for $\delta/D < 15\%$.

At the conference, we will detail how we compute the 3 first orders based on 2 solutions with surface impedance for 2 distinct values of δ, δ_1 and δ_2. The 3 first orders seem to provide an "accurate" behavior; see Fig. 1, right. We will also discuss error estimates, the case of a linear magnetic conductor and the 3D formulations.

References

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